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Finite Time Steady Vector Field Topology

A. Friederici[1], T. Günther[2], C. Rössl[1] and H. Theisel[1]

[1]University of Magdeburg [2]ETH Zurich

Abstract

Vector Field Topology describes the asymptotic behavior of a flow in a vector field, i.e., the behavior for an integration time converging towards infinity. For some applications, a segmentation of the flow into areas of similar behavior for a finite integration time is desired. We introduce an approach for a finite-time segmentation of a steady vector field and equip the separatrices with additional information on how the separation evolves at each point with ongoing integration time. We analyze this behavior and its distribution along a separatrix, and provide a visual encoding for the 2D and 3D case. The result is an augmented topological skeleton. We demonstrate the approach on several artificial and simulated vector fields.

1. Introduction

Vector Field Topology has been established a standard approache to visualizing steady vector fields, see e.g. [**?**, LHZP07, PPF*11]. Its main idea is to separate the field into regions of similar asymptotic flow behavior. This way, even complex flow structures can be represented by a low number of graphical primitives. In addition to this separation, Vector Field Topology has an attractive property in terms of computation: The segmentation as a whole is determined by only few points (critical points, boundary switch points) and only few lines or surfaces called *separatrices* have to be computed.

This work is an extension of [FRT15]. We present a method for quantifying the separation on separatrices based on finite-time information. We then set them in relation to their behavior when integrating towards infinity and provide visual encodings both for the 2D and 3D case.

2. Computation

In a steady field $\mathbf{v}(\mathbf{x})$, we integrate the Vector Field Topology using the flow field $\phi(\mathbf{x},\tau)$. To compute a separation value in each point along a stream line $\phi(\mathbf{x}_1,\tau)$, one possibility is to integrate a secondary stream line in the close neighborhood and measure the distance after a set integration time. This however requires the integration of several lines and depends on the initial offset. For separatrices as a special case of stream lines, it can be shown that the computation simplifies to

$$s(\mathbf{x},\tau) = \int_0^\tau \mathbf{w}(\phi)^T \mathbf{J}(\phi)\,\mathbf{w}(\phi)\,dr$$

with $\mathbf{w}(\mathbf{x})$ being the normalized orthogonal field of $\mathbf{v}(\mathbf{x})$. This allows for a single integration only. For more details, we refer to [FRT15]. While s will not neccessarily capture the maximal distortion of the field, the value will be close to maximal and thus give a good approximation.

In 3D fields, all computations stay basically the same. Instead of lines, separatrices are now surfaces originating in the saddle planes. In terms of computation, we need to integrate and triangulate these surfaces. The perpendicular field $\mathbf{w}(\mathbf{x})$ is defined locally as the normal of the separatrix plane.

3. Visualization

In the linear neighborhood of a saddle, $s(\mathbf{x})$ converges to a linear function $a\tau$. Instead of a direct visual mapping, we chose to display the difference function $b(\mathbf{x})$ to this final linear behavior as $\lim_{\tau\to\infty} s(\mathbf{x},\tau) = a\,\tau + b(\mathbf{x})$ which can be evaluated iteratively as $b(\mathbf{y}) = b(\mathbf{x}) + s(\mathbf{x},\tau_\mathbf{y}) - a\,\tau_\mathbf{y}$.

For a more intuitive visualization and to set the different values of a into relation, we further map $b(\mathbf{x})$ to the visualized function

$$h(\mathbf{x}) = a\,e^{k\,b(\mathbf{x})}$$

where $k > 0$ is a free parameter steering the visual prominence of deviations from linear behavior. As $b(\mathbf{x}) = 0$ in the linear neighborhood of a saddle, $h(\mathbf{x}) = a$ in the same region. Figure 1 shows a simple dataset and the functions s, b and h computed on it.

2D Visualization

For steady 2D vector fields, we map $h(\mathbf{x})$ to transparent walls standing on the planar separatrices. The field is displayed by a LIC texture. The height of the wall represents the function values of $h(\mathbf{x})$, while the color represents positive (red) or negative (blue) separation. Critical points are raised to cylinders and colored based on their kind (sink, source or saddle). Figure 2 shows 4 example datasets.

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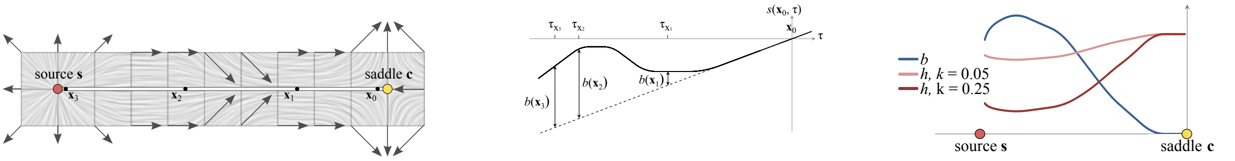
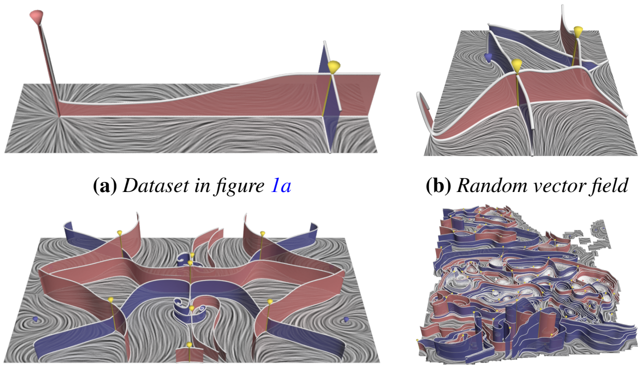


(a) *Simple* 12×3 *field, bilinear interpolation* **(b)** *Graph of* $s(\mathbf{x}, \tau)$ *on field in (a)* **(c)** *Graphs of b and h for varying k*

Figure 1: *s, b and h in a simple field. Spatial distance in (a) and integration time τ are not proportional, as can be seen at points* x_1, x_2 *and* x_3.



(a) *Dataset in figure 1a* **(b)** *Random vector field*

(c) *Rayleigh-Bénard convection cell dataset* **(d)** *South Pacific Ocean*

Figure 2: *2D visualization on datasets of increasing complexity*

3D Visualization

In 3D fields, the mapping to wall heights becomes infeasible. Instead, we map $h(\mathbf{x})$ using color and opacity: Areas of high relative separation are displayed light and opaque while areas of low h fade out. Saddles are displayed as octahedral with eigenvector signs mapped to red and blue color. Figure 3 shows an example. Additionally, the analysis of the surface can be enhanced by several methods. Figure 4 shows a comparison of other techniques for surface visualization. In the left image, angle-based transparency [HGH*10], layer adaptivity [CFM*13] and silhouettes were computed. The center image maps importance to opacity and silhouettes were added. Finally, we applied opacity optimization [GSE*14] to emphasize the important structures (right). Shadows are realized via Fourier Opacity Mapping [JB10] in all cases.

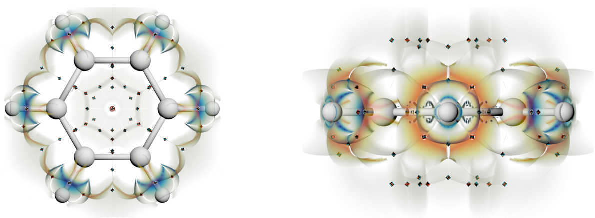


Figure 3: *Magnetic field around a Benzene molecule*

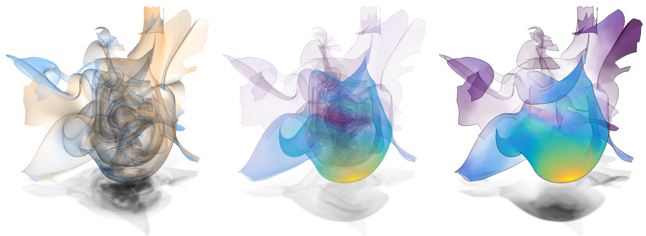


Figure 4: *Alternative visualizations of 3D separatrices*

4. Conclusions

The proposed approach shares the benefits of steady vector field topology, which is stable and well-defined. It also shares the disadvantages of general topological methods: When the topological skeleton has too many primitives, the visualization becomes to complex to be feasible, see for example figure 2d. On a small enough skeleton however the separation functions add new information on the behavior and relevance of separating structures.

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