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Appendix to Visibility-Driven Processing of Streaming Volume Data: Proof of correctness of the PVV.

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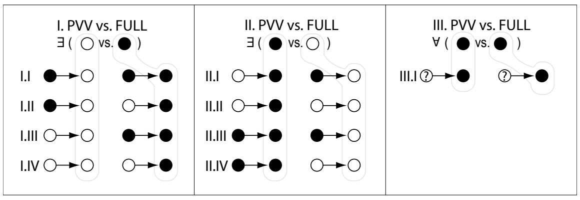

Figure 1: *This table indicates possible transitions of a voxel caused by filtering and compares them for a PVV-filtered and fully-filtered dataset. Empty circle means transparent and black non-transparent, question mark means that the transparency state is irrelevant. Arrow means the transition during filtering. The table has three columns, each of which focuses on existence of a particular pair of voxels.*

Our goal is that the visualization that is generated more efficiently based on visibility-driven filtering (PVV-visualization) is identical to the visualization of the entirely filtered dataset. In the paper, we presented a solution to the correct PVV that takes into account reduced occlusion and increased visibility of voxels caused by a filtering operation. Now we want to prove that if we solve these two problems, i.e., reduced occlusion and increased opacity, the visualizations will be identical. We start with an obviously true statement: if all voxels in both datasets map to the same color/opacity, and if the same rendering method is used, the visualizations will be identical. These are then our general assumptions which must hold:

- PVV-filtering and full filtering are performed on the same dataset
- the same filtering operation is applied on both datasets
- the opacity transfer function is same for the generation of the PVV and for both visualization

We infer the proof from the implication $P \Rightarrow Q$ where statement P is "Two visualizations are not identical" and statement Q as $I \bigvee II \bigvee II$ with:

1. less non-transparent voxels in the PVV-visualization
2. more non-transparent voxels in the PVV-visualization
3. same amount of non-transparent voxels in both visualizations, but some voxels in the PVV-visualization that are visible contain unfiltered values

Our goal is to show that Q is always false with our technique and with our general assumptions. Then this implication is only true when P is also false, which means, the visualizations are identical. In Figure 1, we list all possible relevant transitions of a transparency state of a voxel which are related to statements I, II and III. We will discuss each case individually and show that this case either never occurs or we have handled it.

1. Less non-transparent voxels in PVV-visualization.

Let us assume that the PVV visualization has less transparent voxels than the original visualization. This could have happened because of one of the following reasons:

§ **1.1** At least one voxel in PVV-filtered dataset became transparent while its corresponding voxel in the fully-filtered dataset stayed non-transparent. In the fully filtered dataset, all voxels are filtered per definition. The second part of the statement implies that the effect of the filtering operation on this voxel is that it stays non-transparent. However, the first part of the statement says that this voxel in the PVV-filtered dataset changed its value after filtration to non-transparent. That implies that it was transparent before filtration and that it was included in the PVV. This contradicts the general assumption that the input datasets are identical.

§ **1.2** At least one voxel in PVV-filtered dataset became transparent while its corresponding voxel in the fully-filtered dataset became non-transparent. This means that voxels both in the PVV-filtered and in the fully-filtered dataset changed their value. This further implies that both voxels were filtered. This is in contradiction with the general assumption: if the input data, the transfer function and the filtering method are the same.

§ **1.3** At least one voxel in PVV-filtered dataset stayed transparent while its corresponding voxel in the fully-filtered dataset stayed non-transparent. This indicates that voxels both in the PVV-filtered and in the fully-filtered dataset kept their value which maps to different visibility. This again violates the general assumption that the input data and the transfer function are the same.

§ **1.4** At least one voxel in PVV-filtered dataset stayed transparent while its corresponding voxel in the fully-filtered dataset became non-transparent. Second part of the statement implies that the filtering operation changes the voxel value so that it is mapped to be non-transparent. If in the PVV-filtered dataset the corresponding voxel stays transparent after the filtering operation, it implies that it was not included in the PVSV and it was not filtered. This is the case which we call "increased opacity" and our method addresses it.

2. More non-transparent voxels in PVV-visualization.

Another reason for two visualizations are different, might be that one visualization contains more non-transparent voxels. In the same fashion as we investigated the case of less non-transparent voxels, we will investigate this case. More non-transparent voxels in the PVV dataset can occur for the following reasons:

§ **2.1** At least one voxel in PVV-filtered dataset became non-transparent while its corresponding voxel in the fully-filtered dataset became transparent. This statement implies that the datasets are not identical, since in one case, a transparent voxel is filtered and in other case a non-transparent voxel is filtered. The general assumption is violated.

§ **2.2** At least one voxel in PVV-filtered dataset became non-transparent while its corresponding voxel in the fully-filtered dataset stayed transparent. This statement implies that the filtering operation does not change the transparency state since the voxel in the fully-filtered remains transparent. However, there is a change of the transparency status of the PVV-filtered voxel. In this case, the filtration changed the transparency status from transparent to non-transparent. This implies that either the original voxel value was not the same or the filtering operation was not the same. Both implications violate the general assumption.

§ **2.3** At least one voxel in PVV-filtered dataset stayed non-transparent while its corresponding voxel in the fully-filtered dataset became transparent. This statement implies that the filtration operation changes a voxel from non-transparent to transparent. In the PVV-dataset, the voxel was not filtered by implication of the statement and the general assumption. If the voxel was not filtered, then it was not in the PVV. The reason why a non-transparent voxel is not in the PVV is that it is occluded. This voxel will become visible only if all occluders become transparent. Our method addresses this case.

§ **2.4** At least one voxel in PVV-filtered dataset stayed non-transparent while its corresponding voxel in the fully-filtered dataset stayed transparent. This case cannot occur since it is violating the general assumption about the identical input datasets.

3. Same number non-transparent voxels in the PVV-visualization, but there is at least one pair of non-transparent voxels with different values.

If there is a pair of transparent voxels with different values, it is of no importance. These do not matter, because they do not contribute to the visualization. To continue with the proof, we must refute that "at least one voxel that is visible in the final visualization was not in the PVV." This problem boils down to the reduced occlusion problem which we have also addressed.

Our theoretical verification shows that the results should be identical. In addition to this analysis, we also conducted a bank of experiments for each potential error case to exclude to the possibility of numerical inaccuracies affecting the outcome. In all our tests, there were no numerical differences between the images generated from the fully filtered volumes and those obtained with our visibility-driven approach.