Procedural Modeling of Suspension Bridges

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Figure 1: *Recreation of a Brooklyn Bridge-like street view, from an artist-provided city and a procedurally generated bridge.*

Abstract

In this paper we introduce a method for designing a class of engineering structures, namely suspension bridges. These bridges are ubiquitous in the industrialized countries, often appearing in known city landscapes, yet they are complex enough that hand-based modeling is tedious and time consuming. We present a method that finds the right proportions for such a structure through an optimization method that tries to distribute the tower positions while maintaining cable width to be a finite number. By simultaneously optimizing the span and sag of the cables of a bridge, we optimize the geometry and soundness of the structure. We present the details of our technique together with examples illustrating its use, including comparisons with real structures.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling— I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—

1. Introduction

One of the main challenges in computer graphics is the creation of realistic models of human-made structures. At the same time, there is an increasing need to develop interactive, user-friendly editing tools allowing a broader range of public to generate new content.

The current approach to 3D modeling is to manually create 3D geometry using tools like Autodesk Maya or 3ds Max. This process is time consuming, tedious and repetitive, but gives to the artist full control of the final 3D model. However, sometimes there are situations where reference images are not available, so this approach can be difficult to reconcile with a demand for visual realism.

In the last decade, procedural modeling has emerged as a powerful technique for generating architectural geometry [WWSR03] [MWH*06]. Later, Lipp et al. [LWW08] introduced a visual editing paradigm with direct, fine-grained local control of all aspects of the grammar for individual buildings, but the underlying paradigm did not change: the user still is expected to generate rules that are applied to sets of shapes, resulting in new product shapes. This again, requires the user to know the intrinsic parameters of the structure to create, or infer them in a trial and error process. We present a method to automatically compute feasible dimensions for a suspension bridge, while leaving control in the designerÕs hands for deciding the most relevant parameters of the model. This method, based on practices in the field of structural engineering, is intended to quickly create novel and physically realistic suspension bridge structures, using simple optimization techniques and a minimum of user effort. It is important to clarify that in this work we are not going to deal with aerodynamic stability or responses of earthquakes, which is left as future work.

Resulting from the application of our proposal, we can enumerate our main contributions:

- an automatic mechanism for the computation of the constructive bridge parameters given very simple and intuitive inputs.
- an automatic way of generating structurally feasible procedural bridges.
- we present a measure that determines how close a model is to being structurally feasible. It is enabled by a formulation that agrees closely with engineering constructive procedures, and it matches with available data.

2. Previous work

Very little has been published in the graphics literature on the problem of the automatic generation of man-made structures beyond buildings and houses.

The current trend in procedural building modelling is to use grammar-based procedural techniques that have shown promising results, as shown by Wonka et al. [WWSR03] and later improved by Müller et al. [MWH*06]. Later, Lipp et al. [LWW08] introduced a real-time interactive visual editing paradigm for shape grammars, allowing the creation of rulebases from scratch without text file editing.

Müller et al. [MZWVG07] and Koutsourakis et al. [KST*09] present methods to automatically recover shape grammars from real-building photographs by combining the grammars with image-based analysis. Aliaga et al. [ARB07] presented Style Grammars for quick visualization of buildings and structures. In that work, they proposed an automatic grammar derivation system from existing buildings.

In the context of plant modeling, static analysis has been used to balance the weight of branches for creating realistic tree structures [HBM03]. The problem of creating truss structures (a common and complex category of buildings) in Computer Graphics was first presented by Smith et al. [SHOW02]. In that work, they also used an optimization procedure to simultaneously find the location of the joints and the strengths of individual beams in a truss structure. However, their technique can only be applied to sets of rigid bars, which precludes the inclusion of funicular structures as presented here.

Whiting et al. [WOD09] studied the problem of procedural modeling structurally-sound masonry buildings. Their method automatically tunes a set of user-chosen degrees of freedom to obtain buildings that are structurally sound. We also aim at building structurally sound suspension bridges, but we use engineering standard procedures to obtain feasible structures in a much shorter period of time.

2.1. Representing Bridge Structures

In the following explanations, we refer to Figure 2 for the definitions. Let K be the length of the main span of the bridge (the distance between the two towers), f be the sag (the vertical distance between the anchor point of the main cable at the towers and the lowest point of the cable).

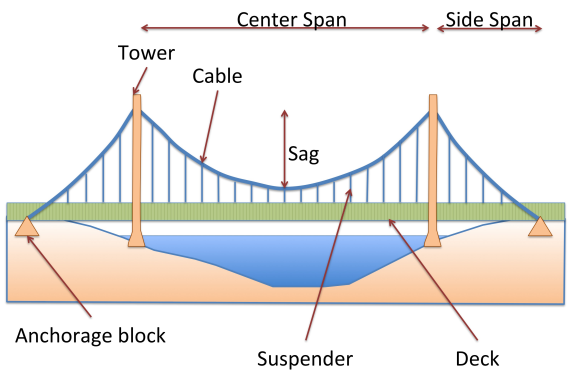

Figure 2: *General structure of a suspension bridge.*

If concentrated vertical loads are applied on a cord, fastened at its ends and considered weightless, it will assume a definite polygonal form dependent upon the relations between the loads. This polygon receives the name of *funicular polygon* (from Latin, *funiculus*, "of or like a cord or thread"). If the loads are continuously distributed, as when hanging free under its own weight, the funicular polygon becomes a continuous curve.

Let ω be the load per horizontal linear unit at any point having abscissa x. In general, we can distinguish between the dead load ω_D (given by the bridge itself) and the live load ω_L, but here we will use the total load computed as $\omega = \omega_D + \omega_L$. Then, the following differential equation of the equilibrium curve can be obtained [Mel13]

$$H\frac{d^2y}{dx^2} = -\omega \quad (1)$$

where H is the horizontal component of the cable tension.

- For a *uniformly* distributed load, i.e. for a constant load ω, if we take the origin of coordinates at the lowest cable point, the integration of the previous equation will give

$$y = \frac{\omega x^2}{2H} \quad (2)$$

Hence, in this case the equilibrium curve is a *parabola*. In this case H is

$$H = \frac{\omega K^2}{8f}$$

- If the load is not constant per horizontal unit, but per unit length of the cord, then the equilibrium curve takes the form of a common catenary:

$$y = \frac{1}{2c}(e^{cx} + e^{-cx} - 2)$$

with $c = g/H$, and g the gravity.

In the following, we will assume a constant load per horizontal length unit. If needed, form Equation 2 we can find the total length of the cable as

$$L = K\left(1 + \frac{8}{3}n^2\right)$$

Quite often bridge designers refer to the sag/span ratio $n = f/K$ to describe a suspension bridge. In general, this ratio is approximately about $1/16$ [Che98].

At any point along the cable, we can compute the tangent of the angle ϕ the cable makes with the horizontal axis as

$$tg(\phi) = \frac{8fx}{K^2}$$

and then the vertical component of the cable tension can be computed from $V = H \cdot tg(\phi)$ and the cable tension as $T = H \cdot sec(\phi)$. Thus, the largest stress in the cord (at the anchor points) can be calculated as

$$T_{max} = \frac{\omega K^2}{8f}\sqrt{1+\left(\frac{4f}{K}\right)^2} \qquad (3)$$

2.2. Optimizing Bridge Structures

In theory, suspension bridges can be infinitely long if an infinitely strong cable can be provided. However, the real limit of span attainable with a suspension bridge is determined by the condition that the cable shall have a finite cross section A. In fact, one can directly compute the maximum span practicable for suspension bridges [RBBB94], but this would give us an unrealistic result which can be too large for our purposes. Thus, the standard bridge designing procedure [OSH99] is to find H from Equation 1, and then find T_{max} from Equation 3. Using a safety factor (usually 3), the required ultimate strength of the cable is computed and, from a set of tabulated values, one, or a group of, galvanized bridge ropes is selected that will meet the required strength. Once the cross section A is computed, it must be verified that it is an acceptable value, and if not, the computations must be restarted with a new set of K and f values [CL05]. While optimizing a suspension bridge, it must be noted that not only the central span must be considered, but also the side spans. So, for the bridge to cover a given total length l, it must be computed the cable cross section A both for the central and for each side span, keeping the maximum value. Thus, if $A(*Span)$ is the cross section for either the central or any of the side-spans, our basic function is

$$max(A(CentralSpan), A(SideSpan))$$

This function can be considered as an energy function, which can be computed for a generic bridge as shown in Figure 3. As expected, for a value equal to half the length of the bridge it takes minimum values with C^0 continuity. This is not a problem, as we do not want the *optimum* values, but a set of values that would allow the construction of a feasible bridge. Feasible bridges are those that are *lower* in the landscape than the brown line shown in the figure. In that figure we also show a possible path for an optimization run, from the initial value to the final one that satisfies the imposed constraints.

In general, values of cable cross section area vs. ultimate strength are tabulated (e.g. [Lex11]), but we performed a simple linear regression for standard galvanized steel bridge strands, obtaining an excellent linear regression of the form $T_{break} = mA$ with A the area of the cable cross section (T_{break} in units of $2000lb$, and A in square inches, but conversion is simple as $1lb = 2.2046kg$ and $1m = 0.0254in$) and m was found to be 76.669 with a correlation coefficient of 99.989%.

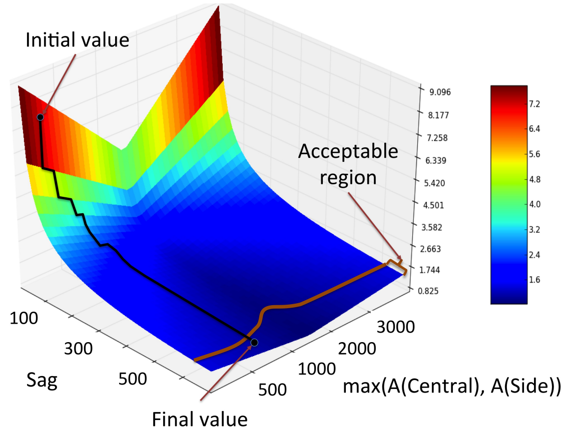

Figure 3: *Energy landscape for our two-parameter structure: span and sag. The brown line marks the feasible region: values below represent all feasible bridges, although we are only interested in the first one that satisfies this condition.*

Finding actual information of the load, both dead and live, of real bridges is feasible only for a few examples, like the Golden Gate bridge in San Francisco, USA. Thus, here we propose that the total load of a bridge will be linearly related with the traffic it will hold. In particular, we propose that the load per unit length ω is a factor times the number of effective lanes in the bridge. We compute the effective lanes by considering the total number of lanes among all decks, plus the train railways as one lane each. We estimated this factor from the date of the Golden Gate bridge data, so we decided to call it *GoldenFactor* and in our case it has a value of $5300kg/lane$. This value gives a perfect matching for the Golden Gate bridge, but it turns out to be also a good approximation for the other bridges in Table 1, as we will see in Section 2.3

We performed tests with a few numerical optimization methods, like the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method [NW07], or the L-BFGS-B algorithm, which is a constrained variant of the previous one [BLNZ95]. We obtained the best results with a simple modification of Powell's method [PVTF02] to find the minimum of our objective function, which consists in adding an early termination criteria as soon as a function is evaluated below a user-provided threshold. In our case, this threshold was set to the maximum feasible cable diameter, which resulted in very good results for existing bridges, as can be seen in Section 2.3. We set the bounds for the optimizations with the following criteria:

- A span cannot be larger than the real bridge length.
- A span must be larger than a minimum distance that, in our case, was selected to be the distance between the river shores at a given depth.
- A sag must be smaller than half the current span.

There is no need to add a lower bound for the sag as the function rises rapidly to infinite as the sag goes to zero, as shown in Figure 3. In the case of the BFGS and Powell's method, we added the constraints in the form of penalizing terms to the target function. Initialization was set to the real bridge length minus a few meters for the span, and to a few meters high for the sag.

	Country	Main Span	Side Span	Length	REAL length	Height above road	Cables	Lanes	Trains	cable diameter
Golden Gate Bridge	USA	1280	343	2737	1966	152	2	6	0	0.92
Akashi Kaikyō Bridge	Japan	1991	960	3911	3911	217.08	2	6	0	1.12
Humber Bridge	UK	1410	280(N)/530(S)	2220	2220	125.5	2	4	0	0.68
Manhattan Bridge	USA	448	221	2089	890	61.3	4	7	4	0.54
Brooklyn Bridge	USA	487	283.464	1833.68	1053.9	43.12	4	6	0	0.38

Table 1: *Figures of some of the most famous bridges: the Akashi Kaikyō Bridge is the longest bridge currently built, while Humber Bridge was the longest from 1981 until 1998 (now it is the 5th). The "Length" column indicates the "official" length of the bridge, while the column "REAL length" shows the length of the suspended part of the bridge, including the central span and both side spans. All length measurements are in meters.*

The procedure to generate the actual bridge from these parameters is more or less straightforward. In particular, we implemented a method that takes a (linear) street segment as input in OSM format [Ope09], and creates from it the decks and, if needed, the supporting structures for multiple decks (e.g. for the Manhattan Bridge). Then, the street segment is sampled along its length to locate the anchorage blocks and the towers, which are positioned taking into account the span length obtained from the already described procedure. If multiple towers are allowed, the suspended length of the bridge is divided in the side spans, and as many central spans as needed given the computed span length. Finally, the cables are created following Equation 2 with the obtained sag, span and cross section as control parameters. The suspender cables are added at regularly spaced distances from the anchors and towers, from the decks up to the main cables. It is important to note that our procedural implementation does not require the construction of the whole bridge from scratch every time a parameter changes during optimization, as only the affected parts need to be re-computed: the cables and the position of the towers, but the towers themselves or the decks do not need to be rebuilt. Of course, any other procedural mechanism would work, like the one described by Benes et al. [BSMM11], which could be used as well.

2.3. Results and Discussion

The procedure presented in this paper has been implemented as a module of the **skyline**Engine system [RP10], which works on top of SideFX's Houdini3D modeler [Sid10]. The first thing to verify is the concordance of our computations with actual bridge measurements. In Table 2, in the last column, we can find the results of evaluating the functions already described with the actual span and sag for their respective number of lanes and cables. We can see that concordance is high in spite of the crude approximation represented by the GoldenFactor, showing a concordance between 0% for the Golden Gate bridge (which is not surprising, as the GoldenFactor was computed for this particular bridge) to about to 24% for the Humber bridge. This is probably because this bridge is asymmetrical, its south side span being almost twice the northern side span. The other bridges show even smaller errors, so we can say that this estimation is effective enough in the context of Computer Graphics.

The other columns in Table 2 show the result of using this cable function in an optimizing procedure. Here, we feed the optimizer with an initial span and sag values, as described in Section 2.2. We let the algorithm optimize these values until a value smaller than a prescribed maximum cable area was found. As described, we added an early termination criteria, to stop as soon as an evaluation satisfied this criterion, but

	Span	Sag	D	iterations	D_{direct}
Golden Gate Bridge	1010.90	81.40	0.97	1 (early)	0.92
Akashi Kaikyō Bridge	1953.72	234.63	1.13	1 (early)	1.19
Humber Bridge	1110.00	138.74	0.69	2 (full)	0.89
Manhattan Bridge	474.35	100.0	0.44	2 (early)	0.49
Brooklyn Bridge	474.35	100.0	0.38	2 (early)	0.45

Table 2: *Results from the optimization for our set of known bridges: Final span, sag, cable diameter and number of iterations. Early/full in the last column refers to an early quit because a feasible bridge was achieved, or the final result of the converged optimization. Column D_{direct} refers to the value obtained for the diameter when the functions are evaluated with the exact span and sag for the respective bridges. Lengths measured in meters.*

we did not had to make use of this criterion except for the Humber bridge, which has a special asymmetrical structure, as already mentioned. Observe the similarity of the results for the Manhattan and Brooklyn bridges, which is to be expected as these bridges share a very similar structure, differing only in the number of lanes each carries, which is the reason for their different cable diameters.

Figure 4: *Manhattan and Brooklyn bridges on a New York map, obtained with the described procedure.*

In Figure 4 we can see a part of New York with Manhattan and Brooklyn bridges on it, with their values obtained with the presented method. The values used are in Table 2. In Figure 5 we can see three steps in the optimization of Akashi Kaikyō Bridge, from the initial values (span = 1500, sag = 10, diameter = 6.42), at the end of the first iteration (span = 1953.72, sag = 166.37, diameter = 1.31), and the final optimized value (span = 1955.49, sag = 244.42, diameter = 1.11). Both images are simple screen captures of our procedural modeling framework. As can be seen from these results, we have presented a method that can produce structurally feasible suspension bridges from a few very intuitive parameters: the number of lanes the bridge will hold, its total length, the number of cables it will have and a maximum acceptable cable cross section area. All the other constructive

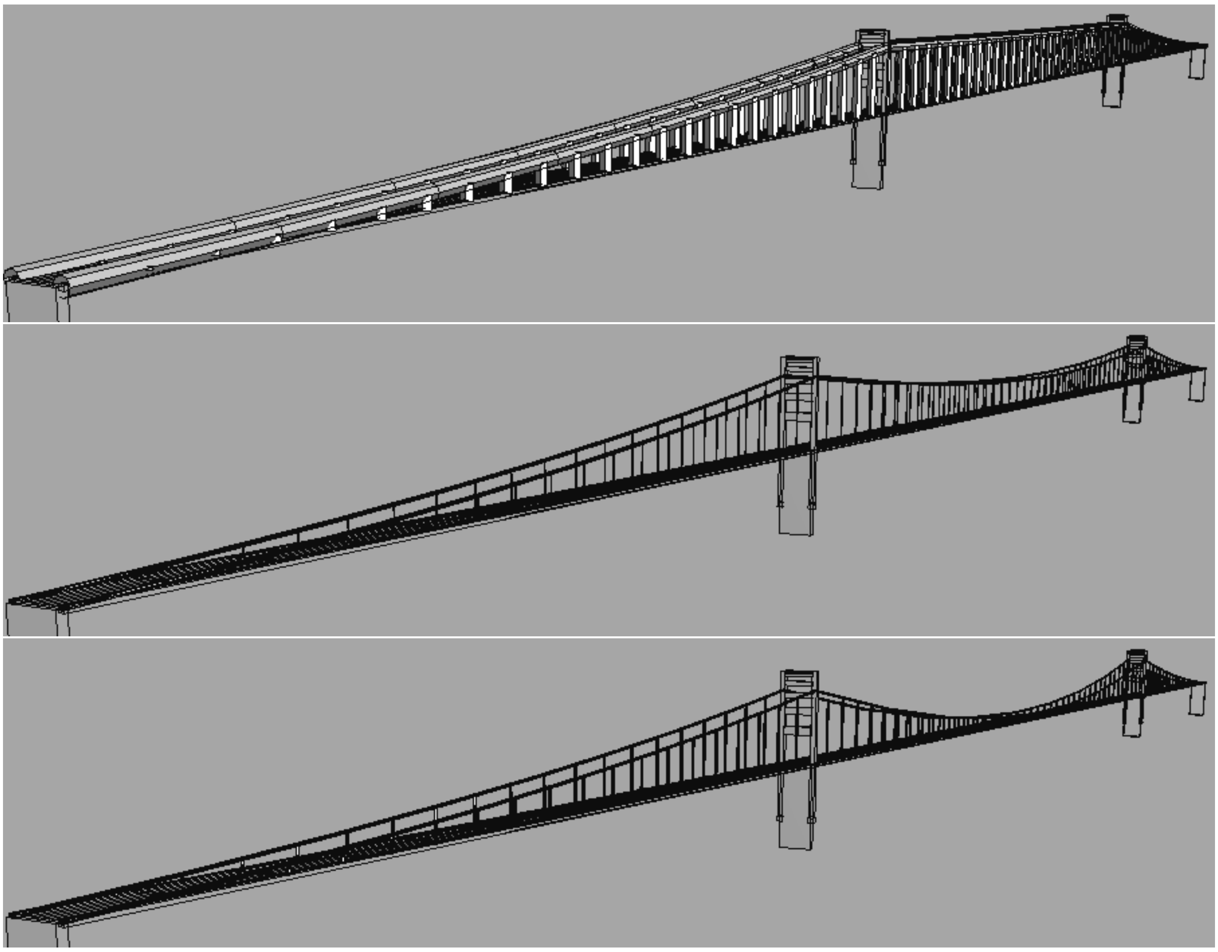

Figure 5: *Three steps in the optimization for Akashi Kaikyō Bridge: initial values, at the end of the 1st iteration, and final result.*

parameters are automatically obtained with an optimization procedure.

Although our algorithm is based on standard engineering procedures, one limitation it presents is the need to have more or less accurate information regarding the bridge length, the number of lanes it will have, the number of cables it will use, and the maximum feasible cable cross section area. In our experience these last parameters are easy to fix after a couple of trials, and, as the optimization step is almost immediate (less than a second), the whole process is quite fast.

2.4. Future Work

As we mentioned in the introduction, throughout this work we have not dealt with aerodynamic stability or with dynamic response analysis, needed to compute the responses of earthquakes. This is a complex topic which involves carefully tuned simulations that are beyond the scope of this paper and are left for future work.

Also, from the approach proposed in this paper, several new lines for future research are possible. First of all, although the work by Smith et al. [SHOW02] covers a wide range of truss structures, funicular structures like the ones studied in this paper have not been studied enough. There are several other classes of bridge structures based on cables, like cable-stayed bridges, which need special considerations. Also, cables are used in other architectonic structures, like sport stadiums, monuments or special buildings like the Milwaukee Art Museum (MAM) by the architect Santiago Calatrava, which is located on Lake Michigan in Milwaukee, Wisconsin. It is precisely those structures that are the most easily recognizable by the final user, so their recreation becomes vital when reconstructing a real urban landscape.

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References

[ARB07] ALIAGA D. G., ROSEN P. A., BEKINS D. R.: Style grammars for interactive visualization of architecture. *IEEE Transactions on Visualization and Computer Graphics 13*, 4 (2007), 786–797. 2

[BLNZ95] BYRD R. H., LU P., NOCEDAL J., ZHU C.: A limited memory algorithm for bound constrained optimization. *SIAM J. Sci. Comput. 16* (September 1995), 1190–1208. 3

[BSMM11] BENES B., STAVA O., MECH R., MILLER G.:

Guided procedural modeling. In *Computer Graphics Forum (Eurographics 2011), to appear* (2011). 4

[Che98] CHEN W. F.: *Structural Engineering Handbook*. 1998. 3

[CL05] CHEN W. F., LIU E. M.: *Handbook of Structural Engineering*, second edition ed. 2005. 3

[HBM03] HART J. C., BAKER B., MICHAELRAJ J.: Structural simulation of tree growth and response. *The Visual Computer 19*, 2-3 (2003), 151–163. 2

[KST*09] KOUTSOURAKIS P., SIMON L., TEBOUL O., TZIRITAS G., PARAGIOS N.: Single view reconstruction using shape grammars for urban environments. In *ICCV* (2009). 2

[Lex11] LEXCO C. M.: Strand, structural strand & bridge rope - astm a586 & a603, 2011. http://www.lexcocable.com/. 3

[LWW08] LIPP M., WONKA P., WIMMER M.: Interactive visual editing of grammars for procedural architecture. *ACM Transactions on Graphics 27*, 3 (Aug. 2008), 102:1–10. 1, 2

[Mel13] MELAN J.: *Theory of arches and suspension bridges*. Myron C. Clark, Chicago, Ill, 1913. D. B. Steinman, translator. 2

[MWH*06] MÜLLER P., WONKA P., HAEGLER S., ULMER A., VAN GOOL L.: Procedural modeling of buildings. *ACM Trans. Graph. 25*, 3 (2006), 614–623. 1, 2

[MZWVG07] MÜLLER P., ZENG G., WONKA P., VAN GOOL L.: Image-based procedural modeling of facades. *ACM Trans. Graph. 26* (July 2007). 2

[NW07] NOCEDAL J., WRIGHT S. J.: *Numerical Optimization*, second ed. Springer, New York, 2007. 3

[Ope09] OPENSTREETMAP: OpenStreetMap, 2009. http://www.osm.org. 4

[OSH99] OKUKAWA A., SUZUKI S., HARAZAKI I.: Suspension bridges. In *Bridge Engineering Handbook*, W.F. Chen Lian Duan W.-F. C., (Ed.). 1999. 3

[PVTF02] PRESS W. H., VETTERLING W. T., TEUKOLSKY S. A., FLANNERY B. P.: *Numerical Recipes in C++: the art of scientific computing*, 2nd ed. Cambridge University Press, New York, NY, USA, 2002. 3

[RBBB94] RAYMOND M. C. W., BIXBY W. H., BURR E., BIXBY C. W. H.: *Report Of Board Of Engineer Officers As To Maximum Span Practicable For Suspension Bridges*. 1894. 3

[RP10] RIDORSA R., PATOW G.: The skylineengine system. In *XX CONGRESO ESPAĎOL DE INFORMçTICA GRçFICA, CEIG2010* (2010), pp. 207–216. 4

[SHOW02] SMITH J., HODGINS J., OPPENHEIM I., WITKIN A.: Creating models of truss structures with optimization. In *Proceedings of the 29th annual conference on Computer graphics and interactive techniques* (New York, NY, USA, 2002), SIGGRAPH '02, ACM, pp. 295–301. 2, 5

[Sid10] SIDEFX: Houdini 11, 2010. http://www.sidefx.com. 4

[WOD09] WHITING E., OCHSENDORF J., DURAND F.: Procedural modeling of structurally-sound masonry buildings. In *ACM SIGGRAPH Asia 2009 papers* (New York, NY, USA, 2009), SIGGRAPH Asia '09, ACM, pp. 112:1–112:9. 2

[WWSR03] WONKA P., WIMMER M., SILLION F., RIBARSKY W.: Instant architecture. *ACM Transaction on Graphics 22*, 3 (July 2003), 669–677. Proceedings ACM SIGGRAPH 2003. 1, 2