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Short Paper

StretchDenoise: Parametric Curve Reconstruction with Guarantees by Separating Connectivity from Residual Uncertainty of Samples

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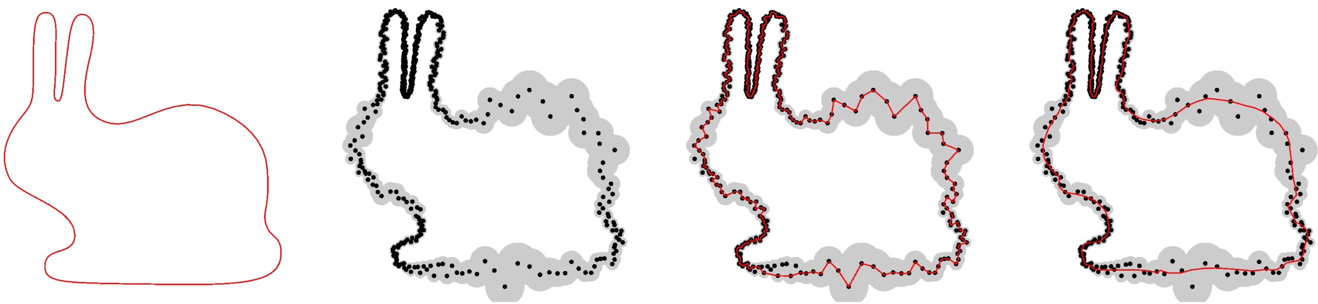

(a) Original smooth curve (b) Samples with noise extent (c) Pass #1: Connected manifold (d) Pass #2: Denoised curve

Figure 1: *Our parameter-free method reconstructs features while effectively removing noise by a two-pass approach.*

Abstract

We reconstruct a closed denoised curve from an unstructured and highly noisy 2D point cloud. Our proposed method uses a two-pass approach: Previously recovered manifold connectivity is used for ordering noisy samples along this manifold and express these as residuals in order to enable parametric denoising. This separates recovering low-frequency features from denoising high frequencies, which avoids over-smoothing. The noise probability density functions (PDFs) at samples are either taken from sensor noise models or from estimates of the connectivity recovered in the first pass. The output curve balances the signed distances (inside/outside) to the samples. Additionally, the angles between edges of the polygon representing the connectivity become minimized in the least-square sense. The movement of the polygon's vertices is restricted to their noise extent, i.e., a cut-off distance corresponding to a maximum variance of the PDFs. We approximate the resulting optimization model, which consists of higher-order functions, by a linear model with good correspondence. Our algorithm is parameter-free and operates fast on the local neighborhoods determined by the connectivity. This enables us to guarantee stochastic error bounds for sampled curves corrupted by noise, e.g., silhouettes from sensed data, and we improve on the reconstruction error from ground truth. Source code is available online. An extended version is available at: https://arxiv.org/abs/1808.07778

CCS Concepts

•***Computing methodologies** → **Shape modeling; Point-based models;***

1. Introduction

Reconstructing closed curves from noisy samples is considered an important problem in computational geometry by itself. Furthermore it has applications in image analysis, computer vision and reverse engineering. An example use case is the extraction of silhouettes from sensed depth images [BBP16], which consist of noisy points, to segment the color data once reconstruction and denoising have generated clear contours. Existing curve reconstruction and denoising methods [Lee00, GG07, MTSM10, DGCSAD11, KH13] often rely on Gaussian smoothing, which creates nice visual output but may oversmooth features. Also the actual noise extent is not considered, even if sensor device properties are known [Köp17], in order to (stochastically) guarantee the error of acquisition. However, recovering the connectivity requires knowing the extent of noise, and the high frequencies of the signal, the noise, can in turn only be estimated well if the baseline of the signal, the connectiv-

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ity, is known. This mutual dependency is why such algorithms often output curves which are not manifold, or over-smooth features. We therefore propose a two-pass approach:

First, to break up the mutual dependence of connectivity and noise, we apply FITCONNECT [OW18], an algorithm which manages to reconstruct the connectivity by testing for consistent manifold fittings of circular arcs as curve segments on increasing scales. For a closed curve, it outputs a polygon with samples as vertices that are sparsely chosen in proportion of the size of noise clusters and therefore recover features. These vertices are augmented with normals, and the neighborhood of samples contributing to its local curve fit. This allows us to order and associate the noisy samples along the reconstructed connectivity, in a single-parametric space, with their Hausdorff distances as residuals separated from the underlying low-frequency manifold connectivity.

Secondly, we move the vertices of the reconstructed polygon to find the *most probable curve* fitting the noisy samples. We maximally straighten the curve while keeping it within the error bounds, specified based on sensor noise models, for example. If a cut-off PDF is used, a probability of being within the ground truth can be guaranteed. At the same time we keep the samples' Hausdorff distances balanced between the in- and outside of the curve to avoid area shrinking.

Our contributions are:

- A two-pass reconstruction approach that uses prior connectivity to *enable a simpler and more efficient denoising model* while *conserving features emerging over the noise extent* (Figure 1).
- A *parameter-free* denoising method with *stochastic guarantees*.

2. Problem Definition

As input we take a set of noisy points S sampling a closed smooth curve C. We obtain the connectivity by running the algorithm FITCONNECT [OW18], which fits a linear piece-wise curve to the samples, i.e., a polygon P with vertices $V \subseteq S$. To do so, FITCONNECT iteratively fits increasing k-neighborhoods of noisy samples with circular arcs until adjacent fits become mutually consistent. In that process it eliminates samples in noisy clusters which are redundant w.r.t. connectivity. For the remaining points it blends the arcs along their determined normals as a simple post-processing step to approximate the original curve. In this paper, we omit this step in order to apply our own denoising method, which assumes the following input: Each vertex $v_i \in V$ has a neighborhood N_i, which is a list of samples in S ordered by their projection onto its fit, as well as a normal n_i and a maximum noise extent r_i detected by FITCONNECT (r_i is zero if the sample can be interpolated without requiring fitting to local noise). In case a noise cut-off radius r_i is available from another source, e.g., if a sensor noise model is known, we will take these values as input instead. With $d(x,P)$ being the Hausdorff distance between a point x and polygon P, we define its signed variant as:

$$\hat{d}(x) = \begin{cases} d(x,P), & \text{if } x \text{ on or outside } P. \\ -d(x,P), & \text{if x inside P.} \end{cases} \quad (1)$$

Noise from sensed data is often modeled as a Gaussian probability distribution function (PDF). In our use case – silhouettes extracted from sensed data and projected onto the view plane as point sets – we only consider lateral noise and define a simplified isotropic radial PDF, since this corresponds closely to the x- and y-axis distribution of sensed data [Köp17]:

$$f_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left\{-\frac{(x-\mu)^2}{2\sigma^2}\right\}, \sigma > 0 \quad (2)$$

This guarantees the sample to lie within a cut-off radius r with probability Π, which depends on a user-defined maximum allowed σ.

To achieve a curve that optimally both denoises and fits the noisy samples, we pursue the following three goals:

1. **Eliminate high frequencies (noise)** by regularizing the curve in the sense of straightening it where no features protrude over the noise extent. We achieve this maximal denoising of the curve by minimizing the angles of the polygon in the least-squares sense:

$$\arg\min_V \sum \|\alpha_i\|_2^2, \alpha_i = \angle\overrightarrow{v_{i-1}, v_i}, \overrightarrow{v_i, v_{i+1}} \quad (3)$$

2. **Balancing the curve** with respect to the number of samples that lie inside and outside. This is achieved by setting the desired mean signed distance to P to zero:

$$\sum_i^{|S|} \hat{d}(s_i) = 0. \quad (4)$$

Using the signed distance prohibits area shrinking.

3. **Bounding the curve** within the discs $D_i(v_i, r_i)$ of the maximum permitted distance from samples, in order to preserve the features recovered by FITCONNECT:

$$\{\forall s_i \in S : d(s_i, P) \leq r_i\} \quad (5)$$

This results in the stochastic guarantee of the samples having been produced by the curve with probability Π.

Note that we do not consider outlier points, for example introduced by sensing errors. Those are not connected to P by FITCONNECT since they lie too far from the curve to be mutually consistent with inlier points. Thus, we assume V to be free of outliers.

3. Denoising Algorithm

The above-mentioned constrained optimization model poses some challenges: It allows too much freedom, and is formulated globally, both of which make it difficult to solve it effectively and in reasonable run time. Moving the polygon vertices V freely in $\mathbb{R}^2$ would result in higher-order functions in the minimization problem as well as in the constraints and bounds, making it slow to solve and becoming trapped inside local minima. Since the curve polygon is locally mostly tangential to the normals anyway, free movement is too lenient and we restrict the problem by allowing vertices v_i to move only along their normals n_i. This allows us to model all functions as linear ones, enabling fast solving for the minimum, and we do not expect a significant deviation from the minimum of the exact model specified above.

3.1. Adapted Model

We adapt and detail the above-mentioned model in the following ways to obtain linear functions:

Let $\mathbf{v}'_i = \mathbf{v}_i + x_i\mathbf{n}_i$, with $x_i \in \mathbf{x}$ as a vector of displacement scalar values and $\mathbf{n}$ as the normalized normals at $\mathbf{v}$.

1. **Angles:** We approximate the non-linear computation of an angle between incident edges of a vertex $\mathbf{v}'_i$ by its linear distance to the baseline $\mathbf{b}$ of its neighbor vertices, weighted by its reciprocal length to get relative values proportional to angles:

$$y(i) = \frac{d(\mathbf{v}_i, \mathbf{b})}{||\mathbf{b}||}, \mathbf{b} = (\mathbf{v}_{i-1}, \mathbf{v}_{i+1}), \approx \alpha_i = \angle(\mathbf{v}_{i-1}, \mathbf{v}_i), (\mathbf{v}_i, \mathbf{v}_{i+1}) \tag{6}$$

Both angle and the weighted distance correspond at their zero values. Since these values are summed up as squares before minimizing, we expect the non-linear mapping to have little impact. When we move a $\mathbf{v}_i$ to $\mathbf{v}'_i = \mathbf{v}_i + x_i\mathbf{n}_i$, this affects not only α_i but also adjacent α_{i-1} and α_{i+1}, multiplied by the dot product of their normals $\mathbf{n}_{i-1}, \mathbf{n}_{i+1}$ with $\mathbf{n}_i$,and therefore:

$$H(i-1, i) = \mathbf{n}_{i-1}^T \mathbf{n}_i \frac{d(\mathbf{v}'_i, (\mathbf{v}_{i-2}, \mathbf{v}_{i-1}))}{||\mathbf{b}||} \tag{7}$$

$$H(i, i) = \mathbf{n}_{i-1}^T \mathbf{n}_i \frac{d(\mathbf{v}'_i, \mathbf{b})}{||\mathbf{b}||} \tag{8}$$

$$H(i+1, i) = \mathbf{n}_i^T \mathbf{n}_{i+1} \frac{d(\mathbf{v}'_i, (\mathbf{v}_{i+1}, \mathbf{v}_{i+2}))}{||\mathbf{b}||} \tag{9}$$

We can then substitute into Equation 3 to approximately express the linear squares minimization of angles in terms of $\mathbf{x}$:

$$\arg\min_{\mathbf{x}} ||\mathbf{Hx} - \mathbf{y}||_2^2 \tag{10}$$

as a sparse diagonal matrix with 3 non-zero colums per row.

2. **Balance:** When we move a vertex v_i, this displaces its two adjacent edges $e_{i,prev}(v_{i-1}, v_i)$ and $e_{i,next}(v_i, v_{i+1})$. In turn, this affects the Hausdorff distance of the samples S_e closest to an edge e. We consider the initial distance of samples as orthogonal to the edge:

$$b_i(\mathbf{e}) = \sum_{s_j}^{S_e} (\mathbf{s}_j - \mathbf{v}_i)^T \mathbf{n}_\mathbf{e}, \mathbf{n}_\mathbf{e} = \perp\mathbf{e} \tag{11}$$

and clamped unit values of samples' positions along the edge since they will move more in terms of x_i the closer they are to v_i, with a factor of $[0,1]$:

$$c_i(\mathbf{e}) = \sum_{s_j}^{S_e} \frac{(\mathbf{s}_j - \mathbf{v}_i)^T \mathbf{e}}{||\mathbf{e}||^2} |_{[0,1]} \tag{12}$$

so that we can express the displacement of samples in terms of x_i along n_i approximatively by substituting Equations 11 and 12 into Equation 4. This computes the distances of the samples $x_i c_i$ from the moving edge minus their initial displacement b_i:

$$\sum_i^{|S(v_i)|} x_i[c_i(\mathbf{e}(s_i))] - b_i(\mathbf{e}(s_i)) = 0 \tag{13}$$

Note that while our initial (constant) displacement corresponds to the Hausdorff distance as being orthogonal to the edge, we use distance along the vertex normal to approximate this quadratic term by a linear one. Since the linear term (non-orthogonal distance of point to line) is an upper bound of the quadratic term (Hausdorff distance), x_i values will not diverge.

3. **Bounds:** We set lower and upper bounds:

$$\{\forall i \in |S| : -r_i \leq x_i \leq r_i\} \tag{14}$$

Note that this would also permit using anisotropic PDFs.

Our adapted model now contains:

- A least-squares minimization (Equation 10)
- A linear system (Equation 13) with a single row and
- Lower+upper bounds (Equation 14).

Concisely we formulate this as:

$$\begin{aligned} &\text{minimize } \mathbf{Hx} - \mathbf{y} \\ &\text{subject to } \mathbf{Cx} - \mathbf{b} = \mathbf{0} \\ &\text{and } -\mathbf{r} \leq \mathbf{x} \leq \mathbf{r} \end{aligned} \tag{15}$$

and we solve this as a *constrained least squares* problem, using Lagrangian multipliers [Sel13].

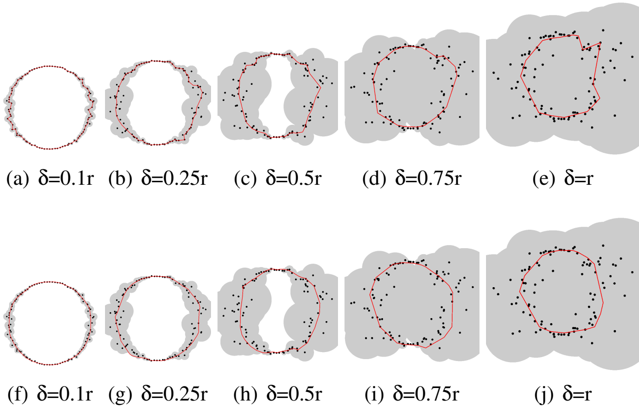


Figure 2: *Top: Blending of fitted circular arcs as in* FITCONNECT *post-processing. Bottom: Our denoising based on* FITCONNECT *connectivity, but considering individual noise extents per sample.*

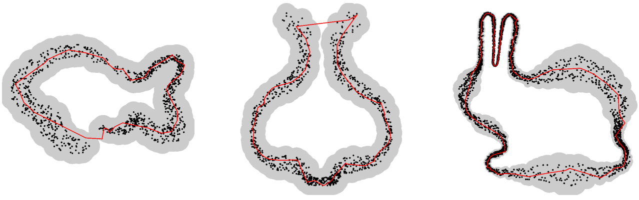
4. Results

We have analyzed a large number and wide variety of point sets with our method. This includes (1) data sets from related work in order to compare and show our improvements, (2) synthetic data sets to measure the reconstruction error with respect to ground truth in order to demonstrate the guarantees, and (3) real data, i.e., segmented silhouettes from noisy sensed data. Open source code that replicates all result figures and tables of this paper is available online at `https://github.com/stefango74/stretchdenoise`

Figure 2 shows how our method is able to recover the circle curve from very large extents of noise (up to its entire radius) and denoise

	Input			Blend			Ours		
δ	max	mean	RMS	max	mean	RMS	max	mean	RMS
0.1	0.076	0.016	0.023	0.073	0.013	0.020	**0.023**	**0.006**	**0.008**
0.25	0.183	0.039	0.059	0.109	0.024	0.034	**0.069**	**0.020**	**0.027**
0.5	0.367	0.079	0.117	**0.126**	**0.041**	**0.053**	0.140	0.042	0.055
0.75	0.553	0.118	0.175	0.188	**0.053**	**0.069**	**0.162**	0.056	0.073
1	0.741	0.155	0.230	0.233	0.079	0.098	**0.145**	**0.054**	**0.065**

Table 1: *Comparison of the error of the noisy input samples versus* FITCONNECT *blending and our denoising method, as Hausdorff distances from the original circle. The noise δ varies as shown in Figure 2 and all values are in terms of the circle radius.*



(a) (b) (c)

Figure 3: *Reconstruction of highly noisy point sets. Left and center: from a noisy curve construction algorithm (point sets courtesy of Lee [Lee00]), with assumed uniform noise extent. Right:* BUNNY *with approximate noise extent of* $\delta = \frac{1}{3} lfs$.

it effectively, compared to simple blending of the fitted circular arcs that FITCONNECT performs as post-processing. Table 1 shows how well both approaches reduce the input noise, and that our method mostly denoises much better, reducing the input noise (mean or RMSE) typically by a factor of 2-3.

Figure 3 shows the results of comparing our denoising method on point sets with uniform very high noise. Note that the compared algorithm [Lee00] only works on open curves whereas FITCONNECT reconstruction closes the curve (see Fig. 13+14 in [Lee00]). Further, it is iterative as opposed to ours, requires parameter tuning, and while its regression analysis will produce a nice-looking smooth curve, it is likely to over-smooth fine features.

Runtime for our unoptimized denoising algorithm part varies between 0.003 and 0.2 milliseconds for the shown point sets.

(a) THING (b) THING (c) THING

Figure 4: *Segmented silhouette of sensed 3D object denoised with the individual noise extents per sample detected by* FITCONNECT *but a minimum noise extent of derived from range image properties of the samples. Left: Sensed RGBD point cloud. Center:* FITCONNECT *connectivity overlaid on RGB image. Right: Our denoised reconstruction overlaid on RGB image. Note that some deviations are due to imprecise silhouette extraction not being part of our method.*

In Figure 4, we show segmented silhouettes of sensed 3D objects where the noise extent is computed from the range image properties of the samples' (x,y,z) position. Note that the extracted silhouettes show some deviations to the objects' real boundaries in the images, due to the used silhouette extraction algorithm.

5. Conclusion

We have shown that our two-pass method successfully enables reconstructing a curve from arbitrarily noisy points within a stochastically guaranteed distance to the original curve while at the same time retaining the features emerging over the local noise extent. The error between the reconstructed and original curve is guaranteed in terms of the input noise, which can be provided either by sensor-specific properties, or estimates from FITCONNECT. Our method is parameter-free since we model the requirements of a most probable curve as minimization, equality and bounds respectively. We successfully apply a technique that we developed ourselves to solve this constrained optimization problem effectively and efficiently. One sample application is determining silhouettes of objects in sensed data, however the underlying assumptions extend directly into 3D where reconstruction is a much more interesting and challenging problem. Our non-optimized denoising algorithm runs fast enough for practical use, it can be verified using the open source available online. Further extensions aside from reconstruction of surfaces for 3D objects include a sharp corner detector to optimize in-between segments locally, e.g., straight lines of man-made objects, as well as handling open curves.

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