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A Simple Diffuse Fluorescent BBRRDF Model

Alisa Jung[1] Johannes Hanika[1] Steve Marschner[2] Carsten Dachsbacher[1]

[1] Karlsruhe Institute of Technology, Germany
[2] Cornell University

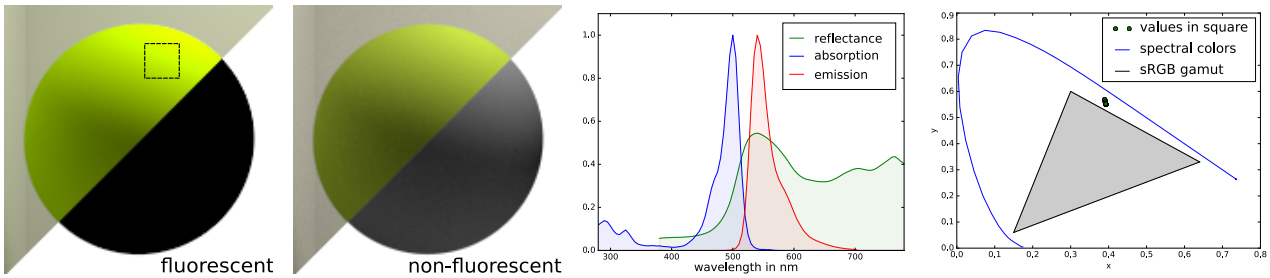

Figure 1: *A fluorescent and non-fluorescent sphere with the same base reflectance, and their RGB-space blue channels (lower triangles) with increased exposure. The fluorescent sphere's blue channel is in fact negative. Fluorescence can produce saturations that cannot be captured in RGB, like the values of the dotted square (right plot). Even a spectral renderer would only be able to produce such saturations at the cost of a lower brightness. The absorption and emission spectra (fluorescein) of the left sphere and the non-fluorescent reflectance spectrum (color checker green) of both spheres are shown second to right.*

Abstract

Fluorescence — the effect of a photon being absorbed at one wavelength and re-emitted at another — is present in many common materials such as clothes and paper. Yet there has been little research in rendering or modeling fluorescent surfaces.
We discuss the design decisions leading to a simple model for a diffuse fluorescent BBRRDF (bispectral bidirectional reflection and reradiation distribution function). In contrast to reradiation matrix based models our model is continuous in wavelength space. It can be parameterized by artificially designed spectra as well as by many publicly available physical measurements. It combines fluorescence and non-fluorescent reflectance, as most real-world materials are not purely fluorescent but also reflect some light without changing its wavelength.
With its simple parameterization the BBRRDF is intended as a starting point for any physically based spectral rendering system aiming to simulate fluorescence. To that end we show how to continuously sample both incident and exitant wavelengths from our BBRRDF which makes it suitable for bidirectional transport, and we discuss energy and photon conservation in the context of fluorescence.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Three-Dimensional Graphics and Realism—Color, shading, shadowing, and texture

1. Introduction

Fluorescence is an important aspect of many real-world materials. Being able to render subtle effects, e.g. increasing the apparent brightness of paper or white textiles, as well as more noticable effects, such as in safety vests or fluorescent paint, can help us achieve a more realistic appearance.
However, rendering fluorescence poses many challenges. Since fluorescence modifies the wavelengths of photons we need a spectral renderer, which is more expensive than traditional RGB rendering. In addition we need realistic models for fluorescent materials.
First steps have been made in analytically modeling simple fluorescent surfaces with a reradiation matrix representing the energy shifts between discrete incident and exitant wavelength pairs [Gla94] [WWLP06], and in acquiring fully measured BBRRDFs [HHA*10]. However, measuring such BBRRDFs is especially complex, since it requires measuring reflectance for multiple incident-excitant wavelength pairs in addition to multiple angle

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combinations. On the other hand, modifying individual absorption and emission properties of a reradiation matrix is hard.
Our goal is to derive a simple analytical BBRRDF based on the physical processes of fluorescence, such that its parameters can be described by publicly available measurements and are easily modifyable. We will also analyse photon and energy conservation of fluorescence and our BBRRDF.

2. Related Work

One of the first works on fluorescence in the context of rendering comes from Glassner [Gla94] who derived the *full radiance equation*, an extended version of the rendering equation including fluorescence. His fluorescent surfaces are parameterized by empirically modeled reradiation matrices and only pass energy from shorter to longer wavelengths.
Wilkie et al. [WTP01] show how to support fluorescence and polarization at the same time. They use a reradiation matrix with entries coming from measured data. In a later work, Wilkie et al. [WWLP06] investigate the qualitative behaviour of fluorescent and specular reflectance. They show how to include fluorescence to analytical BRDF models that can be split into a diffuse and specular component, and also base their fluorescent component on the reradiation matrix.

Hullin et al. [HHA*10] formalize the BBRRDF as

$$f(\lambda_i,\omega_i,\omega_o,\lambda_o) = \frac{\mathrm{d}^2 L_o(\omega_o,\lambda_o)}{L_i(\omega_i,\lambda_i)\cos(\theta_i)\,\mathrm{d}\omega_i\,\mathrm{d}\lambda_i}, \qquad (1)$$

which also is the base of our model. They measured full BBRRDFs for several materials, including angular dependencies, and use reradiation matrices to store spectral information. In contrast, our BBRRDF is analytical and purely diffuse without angular dependencies, and builds on measured spectra of (fluid) fluorescent samples.

Aside from projects in the context of rendering, fluorescence is extensively researched in natural science [Lak06]. The processes causing fluorescence are well understood, and the spectral measurements of hundreds of fluorescent samples are available online (e.g. [tug]).

3. Physical Background

Fluorescence revolves around the energy states of atoms and molecules. For molecules, each of those energy states consists of many closely spaced vibrational and rotational energy levels. Usually molecules are in some vibrational level at the ground singlet state S_0. When a photon hits a fluorescent molecule, it can be absorbed (10^{-15} seconds), thereby raising one of its electrons to a higher energy level in an excited state S_1 (or higher). Next the electron quickly (10^{-12}s) relaxes to the lowest vibrational level of S_1 without emitting radiation. Now several things can happen to that electron (for a detailed explanation see [JM66]). One possibility is fluorescence, which is when the electron (after about 10^{-8}s) relaxes to one of the energy levels in S_0 while emitting a photon corresponding to the energy difference between the new energy level and the base of S_1. The ratio of emitted to absorbed photons is called the *quantum yield* and is generally wavelength independent. The overall process is depicted in a Jablonski diagram in figure 2.

Due to the loss of energy when relaxing to the base of S_1, the emitted photon usually has lower energy (i.e. a longer wavelength) than the absorbed photon.

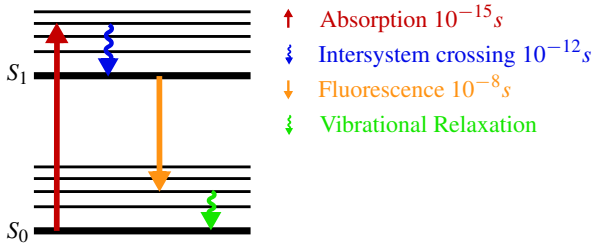

Figure 2: *Jablonski diagram for fluorescence. Straight (wavy) lines represent radiative (nonradiative) transitions.*

The probability of a photon being absorbed as well as the distribution of emitted wavelengths depends on the spacing of the energy levels. The *absorption spectrum* describes the fraction of energy that is absorbed for each incident wavelength of a light beam. The quantum yield tells us how many absorbed photons get re-emitted, and the *emission spectrum* tells us how the emitted energy is distributed over wavelengths.

Note that since emission always starts from the lowest level of S_1, the emission spectrum is independent of the absorbed wavelength. This is called *Kasha's rule* and holds for most substances. This means that we do not need to store a full reradiation matrix to describe fluorescence, but only the spectra. It also means that if the absorption and emission spectrum overlap, it is possible, though unlikely, for photons to be emitted at their original or even shorter wavelengths. This does not violate conservation of energy, since it also modifies the energy content of the scene.
The direction of the emitted photon is independent of its incident direction, making fluorescence a diffuse process. It is important to note that most fluorescent real-world materials are also able to reflect light with its original wavelength.

4. Our BBRRDF

4.1. Assumptions

Our BBRRDF model is based on physically measurable properties of fluorescent surfaces or fluid samples. However, in order to create a general model we ignore effects that only occur for certain solvents, concentrations of the fluorescent substance, temperatures, clumped molecules or other factors. We further ignore phosphorescence, photobleaching, or substances whose quantum yield depends on the absorbed wavelength.

4.2. The BBRRDF Model

Our BBRRDF model relies on 5 parameters: the fluorescent absorption and emission spectra $a(\lambda_i)$ and $e(\lambda_o)$, the non-fluorescent reflectance spectrum $r(\lambda)$, a parameter Q for the fraction of emitted energy to absorbed energy (similar to the quantum yield), and a concentration parameter $c \in [0,1]$.
This last parameter determines the fraction of fluorescence in the material. It does not correspond to a physical property, but can be

interpreted as a surface's relative amount of fluorescent molecules. We introduce c to scale down the absorption spectrum, since most available fluorescence spectra online are scaled to one, which would mean absorbing every photon with the absorption spectrum's peak wavelength.
We assume that fluorescent and non-fluorescent molecules are distributed homogeneously in the surface, and that a photon either interacts with a fluorescent or non-fluorescent molecule. In case the photon hits but is not absorbed by a fluorescent molecule it passes through it and interacts with a non-fluorescent molecule instead. We chose this model since fluorescent molecules are (near) transparent to photons with wavelengths where their absorption spectrum is low or 0.
Our BBRRDF is diffuse, as is fluorescence itself. We assume that all photons are emitted in the upper hemisphere; emitting in all directions and discarding photons emitted into the material could be realized e.g. by adjusting Q to be $Q/2$.

We derive our BBRRDF in two steps, starting with fluorescent events. Given an incident beam of light with wavelength λ_i, a fraction of $c \cdot a(\lambda_i)$ of the beam is absorbed by fluorescent molecules, and a fraction Q of this absorbed energy is re-emitted. The emitted energy is distributed over wavelengths λ_o according to $e(\lambda_o)$, which must be a normalized distribution (i.e. $\int_\Lambda e(\lambda)\,\mathrm{d}\lambda = 1$). The resulting fluroescent BBRRDF component is

$$f_{\text{fluor}}(\lambda_i, \omega_i, \omega_o, \lambda_o) = \frac{ca(\lambda_i)Qe(\lambda_o)}{\pi}. \tag{2}$$

The remaining energy gets reflected according to r:

$$f_{\text{nonfluor}}(\lambda, \omega_i, \omega_o) = \frac{(1 - ca(\lambda))r(\lambda)}{\pi}. \tag{3}$$

We sum those two cases up to get our full BBRRDF:

$$f_r(\lambda_i, \omega_i, \omega_o, \lambda_o) = \frac{\delta_{\lambda_i,\lambda_o}(1 - ca(\lambda_i))r(\lambda_i) + ca(\lambda_i)Qe(\lambda_o)}{\pi}. \tag{4}$$

Note that this BBRRDF is reciprocal in terms of ω_i, ω_o, but fluorescent BBRRDFs are generally not reciprocal in terms of wavelengths. When rendering fluorescence it is therefore important to pay attention to which wavelength is incident (arriving light) and exitant (light leaving the surface).

4.3. Physically Reasonable Parameters

For physically correct behaviour c, $Q, a(\lambda)$, and $r(\lambda)$ should be in $[0, 1]$, although practically they are usually lower than 1 except for the peak of a. Note that $c = 0$ results in a non-fluorescent BRDF with the provided reflectance spectrum.
Important qualitative properties are that in many substances the emission spectrum is the mirror image of the last peak of the absorption spectrum if plotted over wave*number*, and the emission spectrum's peak is always at a longer wavelength.

4.4. Construction of a reradiation matrix

Previous research on rendering fluorescence uses a reradiation matrix to store the spectral component of a BBRRDF. If our spectra are evaluated for discrete wavelengths $\lambda_0, \ldots, \lambda_n$ in discrete vectors $\mathbf{a} = (a(\lambda_0), \ldots, a(\lambda_n))^T$, $\mathbf{e}, \mathbf{r}$, using the diagonal matrix $R = (r_{i,j})$ with $r_{i,i} = (1 - c\mathbf{a}_i)\mathbf{r}_i$, our model can be converted to a reradiation matrix M as follows:

$$M = R + c \cdot \mathbf{a} \cdot Q \cdot \mathbf{e}^T. \tag{5}$$

5. Wavelength sampling

If we want to render fluorescence in a spectral Monte Carlo renderer, we need to be able to sample new wavelengths from our BBRRDF. For example, in a spectral path tracer or photon mapper fluorescence can be simulated by starting a path with some random wavelength and optionally changing its wavelength at each fluorescent surface encountered. One aspect of sampling from our BBRRDF is the δ component representing non-fluorescent interactions. We therefore separate sampling into 2 steps:
First, we sample whether there is a fluorescent interaction. We do that by setting the probability of a fluorescent event on a light path (P_l) or camera path (P_c) to be the fraction of fluorescently "reflected" energy to the overall reflected energy for the given incident (light paths) or exitant (camera paths) wavelength respectively. This relative reflected energy is the integral of the fluorescent portion f_{fluor} of as well as the whole BBRRDF f_r over all exitant (incident) wavelengths. Noting that $\int_\Lambda e(\lambda)\,\mathrm{d}\lambda = 1$ this results in the probabilities

$$P_l = \frac{ca(\lambda_i)Q}{ca(\lambda_i)Q + (1 - ca(\lambda_i))r(\lambda_i)} \tag{6}$$

for a fluorescent event on a light path, given an incident wavelength λ_i, and

$$P_c = \frac{c\int_\Lambda a(\lambda)\,\mathrm{d}\lambda Qe(\lambda_o)}{c\int_\Lambda a(\lambda)\,\mathrm{d}\lambda Qe(\lambda_o) + (1 - ca(\lambda_o))r(\lambda_o)} \tag{7}$$

for a fluorescent event on a camera path, given an exitant wavelength λ_o.

Second, in case of a fluorescent event we sample a new wavelength from the emission spectrum for light paths, or from the absorption spectrum for camera paths. Otherwise the path is continued with the same wavelength. Note that splitting up sampling in two steps results in the same probabilities and probability density functions (pdf) as directly constructing the proportional pdf from our BBRRDF.

6. Energy and Photon Conservation

At this point it is important to keep in mind that the absorption and emission spectra are acquired by measuring the absorbed and emitted energy respectively, and not by counting emitted photons. This is relevant because a photon with, say, wavelength 700*nm* has half the energy of a photon with 350*nm*. In the context of rendering, we usually require a BRDF to be energy conserving in order to be physically valid. The process of fluorescence creates no additional photons, but it is not always energy conserving in the typical sense – on rare occasions a photon may be emitted at a slightly lower wavelength than was absorbed. Note that the overall system energy stays the same, since the energy of the fluorescent molecule decreases accordingly.

Our BBRRDF is energy conserving, i.e. it fulfills

$$\forall \lambda_i \in \Lambda, \omega_i \in \Omega : \int_\Lambda \int_\Omega f_r(\lambda_i, \omega_i, \omega_o, \lambda_o)\, \mathrm{d}\omega_o\, \mathrm{d}\lambda_o \leq 1 \qquad (8)$$

if $c, Q, a(\lambda), r(\lambda) \in [0,1] \forall \lambda$, and $\int_\Lambda e(\lambda)\, \mathrm{d}\lambda = 1$, keeping in mind that the BBRRDF was defined using units of Watts and not photon counts. Equivalently, in an energy conserving reradiation matrix the sum of each row must be ≤ 1.
However, using those constraints our BBRRDF is not photon conserving. For example, for $c = 1, Q = 1$, and the spectra in figure 1, all incident energy at the absorption spectrum's peak of 500*nm* is absorbed and then emitted at almost exclusively longer wavelengths, requiring more photons to be emitted than were absorbed. Interestingly, using an emission spectrum at shorter wavelengths in our model, though unrealistic, would still result in an energy conserving BBRRDF.

7. Results

Figure 3 shows that fluorescent surfaces can convert light from the ultraviolet range to visible wavelengths, and how Q and c influence the appearance of our BBRRDF given the spectra from figure 1.
Figure 4 shows several achievable appearances using real-world spectra and constant UV and visible lighting.

8. Limitations and Future Work

Our BBRRDF models a purely diffuse surface. Possible extensions include combining diffuse and specular behaviour, or even including fluorescence to microfacet or multilayer models. Using our BBRRDF as the diffuse component in such models should be straightforward. We would also like to measure actual fluorescent surfaces and volumetric samples to confirm the intuitions of our approach and apply it to fluorescent volumes, as well as to further investigate the qualitative appearance of such surfaces which in reality are hardly purely diffuse. Another challenge lies in finding further parameter restrictions, maybe including an energy based equivalent of the quantum yield, that assure photon conservation in addition to energy conservation, both for our BBRRDF and reradiation matrices.

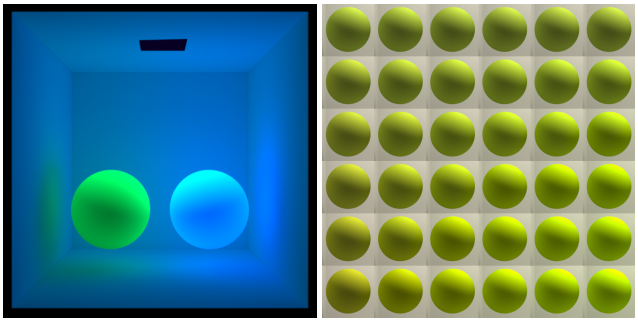

Figure 3: *Left: Two fluorescent spheres illuminated by a UV light. Right: $Q, c \in [0, 0.2, 0.4, 0.6, 0.8, 1.0]$, Q increasing from left to right, c increasing from top to bottom.*

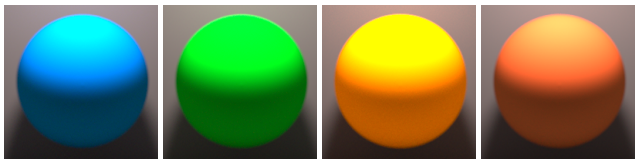

Figure 4: *Our BBRRDF parameterized with real-world spectra such that respective emission and reflectance spectra have similar colors.*

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