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Classifying Medical Projection Techniques based on Parameterization Attribute Preservation

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1. Introduction

In many areas of medicine, visualization researchers can help by contributing to to task simplification, abstraction or complexity reduction. As these approaches, can allow a better workflow in medical environments by exploiting easier communication through visualization, it is important to question their reliability and their reproducibility. Therefore, within this short paper, we investigate how projections used in medical visualization, can be classified with respect to the handled data and the underlying tasks. Many of these techniques are inspired by mesh parameterization, which allows for reducing a surface from $\mathbb{R}^3$ to $\mathbb{R}^2$. This makes complex structures often easier to understand by humans and machines. In the following section, we will classify different algorithms in this area (see Table 1) and discuss how these mappings benefit medical visualization.

2. Mesh Parameterization

Finding a bijective mapping between two triangular mesh surfaces with the same topology is called mesh parameterization. Often used for texture mapping in computer graphics and visualization, several other applications which exploit this projection exist. Exploiting various properties of such a parameterization to solve different problems are discussed in the following sections, categorized after their preservation characteristics. By considering medical tasks, we examine two selected characteristics, a conformal one which preserves angles and an equiareal one preserving area while mapping from the objects surface to its corresponding parameter domain. Combining these two characteristics creates an isometric parameterization without distortion, preserving lengths while mapping from one space to another. As this can be important, when performing quantification within a medical diagnosis step, we will take this characteristic as well into account. Thus, grouping mapping algorithms in the three categories *conformal*, *equiareal* and *(quasi-)isometric* rather than *projection* and *reformation* allows an inspection in which fields such parameterizations simplify problems.

Since isometric parameterization is only possible on zero Gaussian curvature surfaces, either angles or the area gets distorted through mapping. It depends on the final application which properties should be preserved to produce a satisfactory result. Two classic examples of relevant projections would be the *stereographic* (conformal) and the *Lambert* (equiareal) one which unwrap the earth onto a flat surface.

2.1. Conformal

A conformal mapping between two sets of meshes preserves angles locally around a point, but not necessarily the curvature nor size of the considered segment. Applications using this kind of parameterization usually aim at preserving curvature as well as angles to capture the bending of the underlying geometry. Minimizing distortion as well as resolving occlusions is aimed for, to maintain the general shape of present structures and features. Methods such as brain cortex and colon flattening make use of this property to detect structures which could be abnormal or allow a surface based registration between data sets as well as statistical comparison. The tubular structures of the colon or vessel trees can be represented on a flat map to be used as a navigation and localization support for surgeons, especially when structures are complex.

Colon unfolding. Unfolding techniques are used for virtual colonoscopy, whereby the curvature along the surface gives information about possible polyps which build up at the inner wall of the colon and can turn into cancer. The preservation of structure is important to identify a polyp manually or with automated detection algorithms. Haker et al. [HATK00] calculate the conformal map using a finite element approach (FEM) and emphasize high-curvature areas except Haustral folds with colorization. High genus surfaces can be handled by Hong et al. [HGQ*06] also using FEM. The work of Zeng et al. [ZMKG11] describes a mapping to a cuboid instead of a plane by analyzing the volumetric colon wall.

Brain surface flattening. Brain visualization is another field where conformal mapping finds application. Measuring asymmetry, deformation or neural activity of the highly folded three-dimensional structures in the human brain are important fields in medical visualization. Being angle-preserving allows a surface based registration through features to compare cortical regions. However, significantly different maps generated between varying approaches can make a comparison rather difficult as shown by Balasubramanian et al. [BPS10].

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Method (Name)	Reference(s)	Preservation Type
Colon Unfolding	[HATK00], [HGQ*06], [ZMKG11]	conformal
Brain Flattening	[AHTK99], [GWC*04], [WSY*12]	conformal
Flattening Treelike Structures	[MK16]	quasi-conformal, shape-preserving
Colon Unfolding	[BWKG01]	equiareal
Brain Flattening	[ZHGH11], [SZS*13]	equiareal
Tumor Maps	[RWS*10]	equiareal
Brain Flattening	[BPS10], [KSZ14]	quasi-isometric
Vessel Flattening	[ZHT05]	quasi-isometric (two-stage process)

Table 1: *Discussed techniques categorized after their mesh parameterization preservation characteristics*

The algorithm of Angenent et al. [AHTK99] produces a flat map using FEM to solve a second order elliptic partial differential equation to find the mapping function since the brains topology resembles a sphere. Wang et al. [WSY*12] use Ricci flow to generate a singularity free mapping. Anatomical landmark curves are used to match surfaces for asymmetry studies to allow the detection of differences in cortical morphometry.

Vessel tree flattening. The flattening of treelike structures is investigated in the work of Marino et al. [MK16]. A viewpoint dependent map is computed which is only quasi-conformal but shape-preserving, thus allows an easier recognition of three-dimensional structures in the two-dimensional map. This is furthermore aided by an occlusion free arrangement of the different branches. Virtual bronchoscopy is a good example where two-dimensional maps have benefits in terms of navigation and localization.

As an alternative to a conformal mapping, the area between the objects surface and the parameter domain could also be preserved. The next section gives an overview of fields of applications which use area-preserving mapping.

2.2. Equiareal

When it comes to problems where it is crucial for surgeons to precisely judge the size of an area in order to apply the correct treatment for the patient, area preserving mappings should be preferred. Such fields can be tumor treatment or estimating the ratio between healthy and diseased tissue.

For polyp detection during virtual colonoscopy it is important that all polyps are still present in the generated map. Stretched areas can destroy the quality of features or the typical appearance of abnormal deformations and therefor lead to false or missed detections during classification tasks.

Bartroli et al. [BWKG01] present an area-preserving mesh parameterization which achieves its results through an iterative approach to obtain an equiareal mapping using nonlinear 2D scaling and sampling.

Brain flattening. Brain flattening methods use equiareal mappings to perform a classification of gray and white matter when for instance studying the Alzheimer's Disease. Zou et al. [ZHGH11] developed a surface flattening algorithm by first using an approach by Gu et al. [GWC*04] to generate an initial conformal mapping. With the help of Lie advection, applied in a second step, they then achieve area-preservation. Su et al. [SZS*13] generate their 2D map based on Monge-Brenier theory [Bre91].

Radiofrequency tumor ablation techniques destroy tumor cells using heat applied to the unwanted tissue. A crucial parameter is the duration of the treatment to be sure that no malignant cells survive the procedure. Rieder et al. [RWS*10] present a visualization scheme to create tumor maps which display the distribution of healthy and dead tissue of a tumors surface at a specified safety margin around its center. To achieve area-preserving properties a Mollweide projection is used.

Combining angle- and area-preserving parameterization leads to an isometric transformation which is a equidistant mapping and preserves lengths.

2.3. Isometry

Perfect isometric mapping which preserves angles, areas, and the geodesic distance on the surface edges is usually not possible with the data dealt with in medical visualization since *Gauss's "Theorema Egregium"* states that only zero Gaussian curvature K surfaces can be mapped isometrically to the parameter domain [DCDC76]. With

$$K = \kappa_1 \cdot \kappa_2$$

and κ_1 and κ_2 being the principal curvatures of a surface, at least one of both needs to be zero for the surface to be isometric. This is mostly possible for developable surfaces like planes or cylinders, otherwise distortion in at least on of the intrinsic parameters is present in the mapping. Thus, the methods presented here are quasi-isometric and try to eliminate the distortion of the parameterization as much as possible while reducing complexity of the data.

Equidistant mapping in brain flattening techniques allows for distance and area measurements across the surface in the parameter domain. This could also be done in 3D space. However, Euclidean calculations and complex analysis are better understood in planar environments [BPS10]. Balasubramanian et al. [BPS10] present a set of criteria which should be fulfilled to achieve quasi-isometry. Additional to that they propose an algorithm which satisfies these requirements. Khosravi et al. [KSZ14] consider not only one extracted surface of the cortex but multiple layers and track features changing between these. The distortion of the distances between corresponding points in neighboring iso-surfaces together with the points on the 2D map is minimized to achieve quasi-isometry.

Beside a conformal mapping approach for vessel flattening, an additional two-stage approach was developed by Zhu at el. [ZHT05]. Regarding quasi-isometry, building up on a first mesh parameterization step has the drawback that it weakens the first executed mapping by distorting it with the following step to a quasi-isometric solution by applying both parameterization steps after another. It is also debatable if such an approach can be called quasi-isometric.

3. Discussion and Future Work

Existing visualization techniques like for example the Volumetric Bulls Eye Plot [TBB*07] or Aneurism Maps [NGB*09] could be made conformal or equiareal in order allow surgeons and medical staff judging the patients condition in a more reproducible manner instead of relying on their experience when distortions can alter the visualization in an unwanted way.

Another interesting thought could be the finding of a generalized data-aware approach, which optimizes the characteristics with respect to data and task. In the best case a quasi-isometric solution to join conformal and area-preserving attributes exists which could close the gap when otherwise multiple algorithms are needed.

We see this short paper as a first step into the direction of investigation preservation in the context of medical projections. Shape [MK16] or context [MC10] as well as mass [ZYHT07] preserving techniques also exist, and could also be investigated with respect to which problems in medical applications suits which mesh parameterization best.

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