International Conference on Artificial Reality and Telexistence
Eurographics Symposium on Virtual Environments (2023)
J.-M. Normand, M. Sugimoto, and V. Sundstedt (Editors)

4D Exploring System for Intuitive Understanding of 4D Space by Extending Familiar 3D Interfaces

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Abstract

With the advancement of VR technology and the increasing demand for high-dimensional data, a variety of intuitive visualization and interaction methods for high-dimensional data have been proposed. In this paper, we propose a new 4D space interaction system aimed at not only being more intuitive but also making it easier for non-experts to understand 4D space. The proposed system functions as a familiar system for exploring 3D space, and when exploring the 4D space, a user is able to directly leverage the operations used for navigating the 3D space. This feature is achieved through a combination of displaying cross-sections of 4D space by slicing through 3D screens and intuitive operation using motion controllers. By moving back and forth between exploring the 4D space and the 3D cross-sections, a user can observe and experience the relationships, aiding in the understanding of 4D space. From the maze exploration experiments, not only were promising results obtained, but interesting insights were also garnered regarding the field of high-dimensional space perception, an area with many unresolved aspects.

CCS Concepts

• ***Human-centered computing*** → *Scientific visualization; Virtual reality;*

1. Introduction

At the end of the 19th century, Victor Schlegel initially attempted to visualize 4D objects using Schlegel diagrams [Sch83]. Shortly thereafter, in the early 20th century, Albert Einstein introduced the general theory of relativity [Ein22], establishing the concept of higher dimensions as indispensable in the field of science [PW87]. In the field of abstract mathematics, there are also many scientific subjects, such as complex functions and 3-dimensional manifolds, that are suitable for embedding into 4D space. However, the development of visualization techniques for 4D figures remains limited. Although the depiction of higher-dimensional figures in lower dimensions through the methods such as projection and slicing influenced artists of the time [Hen13], these visualizations held little scientifically meaningful information for most non-experts. Some researchers even doubted that humans, living in a 3D space, could comprehend 4D space [AWC*09, Wan14a, Fra14, Wan14b]. Nevertheless, considering that manipulating and observing 3D solids can enhance human understanding of geometry [CMS13], it is expected that superior visualizations and interaction systems for four dimensions could promote comprehension of this higher-dimensional space.

In recent years, advancements in computer graphics and virtual reality technologies have not only enabled the visualization of various 4D figures but also led to the proposal of numerous new interaction methods [Agu06, ZH07, CFHH09, MSH18, MOI*19, phi20, Bos20]. The growing demand for higher-dimensional data due to the progress in information science has also increased the importance of understanding higher-dimensional concepts. Therefore, based on the idea that intuitive comprehension of 4D space requires appropriately connecting understanding of 3D space with 4D concepts, we propose a novel 4D interaction system.

The system provides a novel interface for freely exploring 4D space, analogous to exploring 3D space in a first-person perspective video game. The output visuals are rendered on a 3D screen within the VR space, with translational movement and rotation executed through input from motion controllers. These output and input can be limited to lower dimensions by displaying a 2D cross-section of the output visuals on the 3D screen and by ignoring input in the fourth dimension. As a result, users gain access to a user-friendly system for exploring 3D space, which is extended to 4D in an extremely natural manner. The dimensionality of the input and output can easily be switched, and the 3D screen and its cross-section can be displayed simultaneously, enabling users to seamlessly learn and explore 4D space as an extension of 3D space. Connecting 3D and 4D in such a manner represents an unprecedented approach.

In this paper, we propose a new system that naturally integrates 3D interactive space with 4D interactive space together with the underlying theory and verify the system operation through a maze exploration experiment.

DOI: 10.2312/egve.20231319

2. Related Work

The idea of extrapolating to higher-dimensional objects based on lower-dimensional ones has long been emphasized. In "Flatland," [Abb84] inhabitants of a 2D world encounter residents of a 3D world and are depicted creating a world of 4 dimensions or higher.

The interaction techniques using VR technology are diverse. Miwa and colleagues [MSH13, MSH18] proposed a system that displays a vanishing point on the 3D projection of a 4D object, and rotates the projection by grabbing and moving the vanishing point. They used this system to test recognition capabilities of 4D space. This system provides intuitive 4D rotation in any direction by matching users' hand movements with changes in the projection. Matsumoto and others [MOI*19] proposed a system that realizes 4D rotation by arranging four 3D projections corresponding to the four axes of 4D space, and rotating each projection in 3D. This system makes operation easier to understand by making a single operation identical to that of a 3D solid, although the direction of rotation is limited. Ten Bosch [Bos20] proposed a system involving physical computations of 4D space using a 3D cross-section of 4D space as the main drawing. Although the approach using a cross-section involves limitations in the information and interaction methods that can be presented, it excels in its ability to focus on 3D drawings and interactions.

In contrast to these systems that either opted for a 3D-centric intuitive interface or a 4D-centric free interface, our proposed system provides an interface that combines both advantages.

3. Overview

A VR system is implemented using Unity 2021.3.11f1 and XR Interaction Toolkit 2.2.0, and operates with compatible headsets [Pic22]. Performance verification and experiments were primarily conducted by running a standalone application built with PICO Integration 2.1.3 on a PICO 4 headset. Figure 1 depicts the PICO 4 controllers and the buttons utilized in the system.

Figure 2 illustrates the overview of the system. The system simulates the 4D space, including a 4D hypercamera. The hypercamera captures a section of the 4D space and converts it into a 3D image. Subsequently, the system projects the 3D image onto a 3D screen within the VR space, allowing the user to observe the 3D image from any desired direction through a head-mounted display. Moreover, the user can control the position and orientation of the hypercamera within the 4D space via input from a motion controller. Consequently, the user can explore the 4D space through the hypercamera.

Upon user input, the system slices the 4D space to present a cross-sectional 3D space. In this case, instead of converting the 4D space into a 3D image, the hypercamera converts the cross-sectional 3D space into a 2D image, and the system projects the 2D image onto a 2D screen within the VR space. The movement of the hypercamera is restricted within the cross-sectional 3D space, and the user explores the 3D space through the hypercamera while observing the 2D screen within the VR space. While this interface for exploring the 3D space can be seen as inefficient, it closely corresponds with the interface for exploring the 4D space as explained later, allowing the user to leverage their experience exploring the 3D space directly in the exploration of the 4D space.

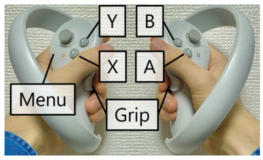

Figure 1: *PICO 4 controllers and its buttons.*

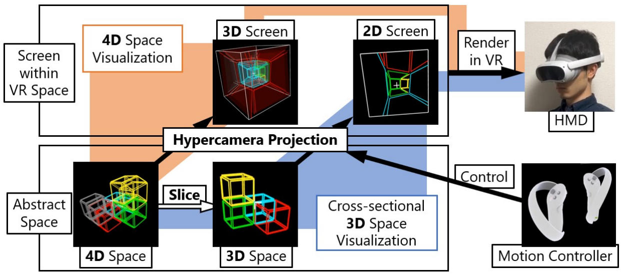

Figure 2: *Overview of the system.*

4. Interaction System

In this section, we delineate the 4D exploration system that facilitates the projection of 4D entities into a 3D virtual environment.

4.1. Projection

The methodology for converting 4D spatial coordinates to 3D coordinates bears a striking resemblance to the approach employed for transforming 3D spatial coordinates into 2D coordinates. To allow not just the objective observation of 4D figures but also the exploration within a 4D environment, the system is compatible with perspective projection. Just as the position and orientation within a 3D space are represented by 4D homogeneous coordinates, the position and orientation within a 4D space are denoted by 5D homogeneous coordinates. Analogous to a camera that projects a 3D space onto a 2D screen, a hypercamera, responsible for projecting a 4D space onto a 3D screen, possesses parameters that determine the field of view and clipping plane in addition to the position and orientation. Consequently, the matrix $\boldsymbol{T}$ to transform 5D homogeneous coordinates to 4D homogeneous screen coordinates is expressed as follows;

$$\boldsymbol{T} = \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ \hline & & \boldsymbol{p} & & 1 \end{array}\right) \left(\begin{array}{cccc|c} & & \boldsymbol{x}_p & & 0 \\ & & \boldsymbol{y}_p & & 0 \\ & & \boldsymbol{z}_p & & 0 \\ & & \boldsymbol{w}_p & & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \end{array}\right)^{-1} \left(\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \frac{k}{f-h} & -\frac{k}{h} \\ \hline 0 & 0 & 0 & \frac{fk}{f-h} & 0 \end{array}\right) \tag{1}$$

where $\boldsymbol{p}$ represents the position coordinates of the hypercamera, $(\boldsymbol{x}_p, \boldsymbol{y}_p, \boldsymbol{z}_p, \boldsymbol{w}_p)$ denote the coordinate axes describing the hyper-

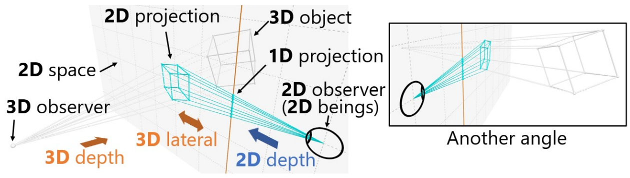

Figure 3: *Analogy: Observation of the 2D projection of a 3D object by a 2D being.*

camera's orientation, and k, h, f correspond to the screen size, front clipping hyperplane, and back clipping hyperplane, respectively.

The objects to be rendered are drawn on a 3D screen within the VR space according to the transformed 3D coordinates and are subsequently projected onto a 2D screen by a camera associated with the headset. To elucidate this scenario, Figure 3 presents a lower-dimensional analogy. Consider a 2D individual (an inhabitant of a hypothetical 2D plane) residing on the xy plane within a 3D space described by the xyz coordinate system. This 2D individual can only observe objects within the plane through a 1D retina. However, analogous to humans residing in a 3D space who can comprehend the structure of 3D objects through 2D retinal images, the 2D individual can grasp the structure of 2D objects via 1D retinal images. Now, consider a 3D object external to the plane, and project it onto the plane with a point on the opposite side serving as the focal point. In this case, the 2D individual can perceive a 2D image similar to that observed by a human viewing a 3D object from the focal point. Analogously, in the present system, a 3D projection of a 4D space is observed by projecting it from a lateral direction pertaining to the 4D individual onto a 2D VR screen.

4.2. Operation

In our system, a user explores the 4D space by moving and rotating the hypercamera using conventional motion controllers. A rigid body situated in 4D space possesses four translational degrees of freedom and six rotational degrees of freedom in 4D space. To control the total of ten degrees of freedom, the present system utilizes the information on the coordinates and orientations of the motion controllers held in both hands, each with six degrees of freedom.

Figure 4 illustrates an example of the system's operation. In the images excluding the ones labeled "Default" in the top left, each illustrates the inputs and their results stemming from the "Default" state. The images annotated with "3D" are generated through the subsequently described slice representation. By pressing the A or X button, and moving the controller, users can initiate movement or rotation in the direction of motion. The magnitude and direction of inputs are indicated by white arrows or rings over the controller. Following the conventional input format for gamepads, the left-hand input corresponds to movement, while the right-hand input pertains to rotation. Advancement and retreat are coordinated with the twisting motion of the left hand. Additionally, the user can grasp the screen and move it to a desired position and direction by pressing the grip buttons.

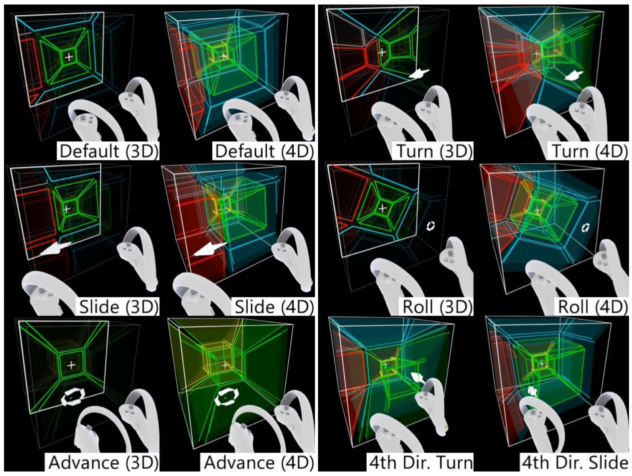

Figure 4: *Operation system.*

4.3. Slice and Restriction to 3D

The user can toggle the on and off states of slicing and operation restriction by pressing the B or Y button. The position of the slice is fixed at a location that passes through the center of the screen, facing directly towards the user in the initial state. While the slice is being displayed, the original 3D projection is also shown in a semi-transparent manner.

Input by the controller is ignored only in directions orthogonal to the slice. The direction that can be controlled with this operation will be referred to as the "fourth direction" hereafter in this paper. In the bottom right of Figure 4, examples of operations in the fourth direction is illustrated.

5. Evaluation

We conducted a preliminary experiment of maze exploration to evaluate the validity of the system by ten subjects who had no prior knowledge of 4D space.

5.1. Participants

The participants were university students in the sciences aged from 18 to 22 with no specialized knowledge of 4D space. To verify the effects of slicing and operation restrictions, subjects were randomly divided into two groups, one that could use these features (Group A) and one that could not (Group B).

5.2. 4D Maze

The generation and rendering algorithms for the 4D maze are based on McIntosh's 4D Maze [McI14]. A 4D maze consists of hypercubes, each connected cell to cell. A hypercube is an analog of a cube in 4D space, consisting of eight cube-shaped cells. The hypercamera can traverse the interior of the hypercube and can travel between adjacent hypercubes connected by cells. A 2x2 space of hypercubes is not created.

Proceedings published by Eurographics - The European Association for Computer Graphics.

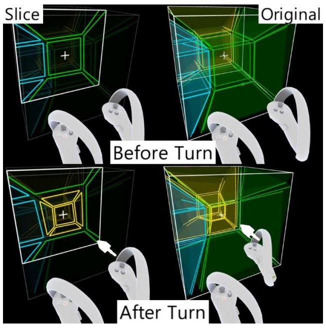

Figure 5: *Turning towards the fourth-dimensional direction*

The maze consists of five hypercubes twisted in four dimensions and aligned in a single line, colored sequentially from start to finish in gray, red, blue, green, and yellow. Due to its four-dimensional structure, it is impossible to reach the goal with operation restrictions activated. In the situation shown in Figure 5, the path to the yellow goal room extends in the fourth direction, and the yellow room does not intersect with the slice. To reach the goal, it is necessary to deactivate the operation restrictions and reorient in the fourth direction by making a hand-thrusting motion. Figure 4 also depicts the interior of the maze.

After navigating through the maze, a new maze is randomly generated by pressing the menu button.

5.3. Experiment

Both groups aimed to explore the maze and solve it within one minute in three consecutive attempts without using slicing and operation restrictions.

Participants were verbally briefed in advance on the objectives and the method of operation, and were shown a demonstration of solving a maze. The configuration of the maze in the demo differs from the actual maze, requiring the participants to comprehend the strategy for finding the route and moving in the appropriate directions based on their own experiences. Participants in Group A are permitted to use slicing and are informed about how to activate it; thus, it is expected that they would more readily understand the method of operation and the characteristics of the four-dimensional space through the use of slicing. Following the briefing, the time taken from the commencement of operations to the achievement of the goal is measured. Additionally, after the completion of the measurements, interviews were conducted regarding thoughts and impressions during the experiment.

5.4. Results and Discussions

Table 1 shows the time each participant took to solve the maze. The results showed significant individual differences, and no significant difference was observed between the two groups. To clearly verify the difference, not only a sufficient number of subjects but also a more complex or multistage task setting and analysis of the correlation with various individual abilities may be required. The subjects in Group A first activated the slice to adopt restricted operations.

Table 1: *Results*

Group	A					B				
Subjects	1	2	3	4	5	6	7	8	9	10
Score (sec)	182	201	1922	292	320	879	258	457	3524	1632

Interestingly, the subjects in Group B, despite the absence of slices, initially began performing restricted operations too. This seems to be because they tried operations similar to those in a 3D maze due to the clear concept of maze exploration, the similarity of the front view, and the intuitive operation. However, the time they took to adapt to such operations seemed to be shorter for Group A.

In the end, the ability to master the operation in the fourth direction determined the time for both groups. In situations where operation in the fourth direction was necessary, there were two barriers: understanding that the desired room is in the fourth direction and actually executing the operation. Subjects who could not perform the latter and tried 3D operations for the shift in the fourth direction were confused by the unexpected results. Moreover, some participants were only able to perceive the screen in two dimensions and, even after receiving additional explanations after the experiment, were unable to master the operation in the fourth dimension for some time. Some subjects in Group A commented that slicing and operation restrictions helped them coordinate and understand the fourth direction.

Finally, while everyone in Group A responded that the slice function was helpful in understanding the operation, all subjects responded negatively to the question of whether they felt they understood four-dimensional space.

6. Conclusions

In this study, we constructed an intuitive 4D operation system by extending a general 3D operation system. Furthermore, by presenting cross-sectional views of the 3D projections of 4D space, we enabled various applications, such as pre-training using the 3D operation system and comparing 4D space projections. Although the experiment did not provide significant evidence that the functions of this system contribute to high-dimensional tasks, many interesting observations on the experience of high-dimensional space were obtained.

In the future, we will carefully consider the task settings, increase the sample size, and objectively investigate the effects of slicing and operation restriction functions. Also, by comparing with different tasks and other systems, we will examine in detail the mechanisms related to the performance of this system and the understanding of four-dimensional space.

Acknowledgements

This work was partly supported by JSPS KAKENHI, Grant Number 22K19817, and Tateisi Science and Technology Foundation, Grant Number 2237001.

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