Course notes

Inverse Computational Spectral Geometry

EUROGRAPHICS 2021 Tutorial

This tutorial will be built around top-tier publications that constitute the solid theoretical and applicative foundations on which we will organize the course, and that cover all the existing analysis and results on inverse computational spectral geometry. For each part of the course (outlined in the following list), we provide all the important references.

1) **Introduction:** Motivation and historical overview.
    a) M. Kac, “Can one hear the shape of a drum?” *The American Mathematical Monthly*, 73(4), 1966.
    b) M. H. Protter, “Can one hear the shape of a drum? Revisited” *SIAM Review,* 29(2), 1987.
    c) B. Osgood, R. Phillips, P. Sarnak, “Compact isospectral sets of surfaces”, *Journal of Functional Analysis*, 80(1), 1988.
    d) C. Gordon, D. Webb, S. Wolpert, “Isospectral plane domains and surfaces via Riemannian orbifolds”, *Inventiones mathematicae*, 110, 1992.
    e) C. Gordon, D. Webb, “Isospectral convex domains in euclidean space”, *Mathematical Research Letters*, 1(5), 1994.
    f) P. Buser, “Inverse Spectral Geometry on Riemann Surfaces”, *Birkhäuser Trends in Mathematics, 1997.*
    g) T. Driscoll, “Eigenmodes of isospectral drums”, *SIAM Review*, 39(1), 1997.

2) **Problem foundations:** Shape-from-spectrum as a classical problem in mathematical physics, inverse eigenvalue problems in matrix calculus, shape-from-metric and shape-from-intrinsic operators.
    a) M. Chu, G. Golub, “Inverse Eigenvalue Problems: Theory, Algorithms, and Applications”, *Oxford University Press*, 2005
    b) D. Boscaini, D. Eynard, D. Kourounis, M.M. Bronstein, “Shape-from-Operator: Recovering Shapes from Intrinsic Operators”, *Computer Graphics Forum*, 34(2), 2015.
    c) A. Chern, F. Knoeppel, U. Pinkall, P. Schroeder, “Shape from metric”, *TOG 63*, 2018.

3) **Background:** The forward direction of classical spectral geometry processing.
    a) I. Chavel, “Eigenvalues in Riemannian geometry”, *London: Academic Press*, 1984.
    b) P. Buser, “Geometry and spectra of compact Riemann surfaces”, *Birkhäuser Basel*, 1992.
    c) M. Reuter, F.-E. Wolter, N. Peinecke, “Laplace-spectra as fingerprints for shape matching”, *Proc. of the ACM symposium on solid and physical modeling*, 2005.
    d) M. Reuter, F.-E. Wolter, N. Peinecke, “Laplace–Beltrami spectra as ‘Shape-DNA’ of surfaces and solids”, *Computer-Aided Design*, 38, 2006.

4) **Inverse spectral geometry in CG:** Motivations, applications, and examples in graphics.
    a) L. Halbeisen, N. Hungerbuhler, “Reconstruction of weighted graphs by their spectrum”, *European Journal of Combinatorics*, 21(5), 2000.
    b) Z. Zhang, Q. Li, Z. Huang, J. Wu, J. Tenenbaum, W. Freeman, “Shape and material from sound”, *NIPS*, 2017.
    c) D. Aasen, T. Bhamre, A. Kempf, “Shape from Sound: Toward New Tools for Quantum Gravity”, *Physical Review Letters*, 110, 2012.

5) **Computational techniques:** Existing approaches based on formulating an optimization problem, numerical methods and machine learning-based techniques.
    a) L. Cosmo, M. Panine, A. Rampini, M. Ovsjanikov, M.M. Bronstein, E. Rodolà, “Isospectralization, or how to hear shape, style, and correspondence”, *Proc. CVPR* 2019.
    b) Huang, R., Rakotosaona, M.J., Achlioptas, P., Guibas, L.J. and Ovsjanikov, M., 2019. OperatorNet: Recovering 3d shapes from difference operators. *Proc. CVPR* 2019.
    c) R. Marin, A. Rampini, U. Castellani, E. Rodolà, M. Ovsjanikov, S. Melzi, “Instant recovery of shape from spectrum via latent space connections”, *Proc. 3DV* 2020.

6) **Applications:** Inverse spectral geometric pipelines addressing practical problems in graphics.
    a) A. Rampini, I. Tallini, M. Ovsjanikov, A. M. Bronstein, E Rodolà, “Correspondence-Free Region Localization for Partial Shape Similarity via Hamiltonian Spectrum Alignment”, *Proc. 3DV* 2019.
    b) A. Rampini, F. Pestarini, L. Cosmo, S. Melzi, E Rodolà, “Universal Spectral Adversarial Attacks for Deformable Shapes” *Proc.* CVPR 2021

7) **Open problems and future directions:** Main limitations of current approaches, next steps and open problems.

8) **Conclusions and Q&A.**