



EUROGRAPHICS2017

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EUROPEAN ASSOCIATION FOR COMPUTER GRAPHICS

Topology Optimization for Computational Fabrication

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Topology Optimization for Computational Fabrication

Part 3: Controllable Topology Optimization – Geometric Features

Dr. Jun Wu

TU Delft

Complexity is free



TU Delft & MX3D, 2015



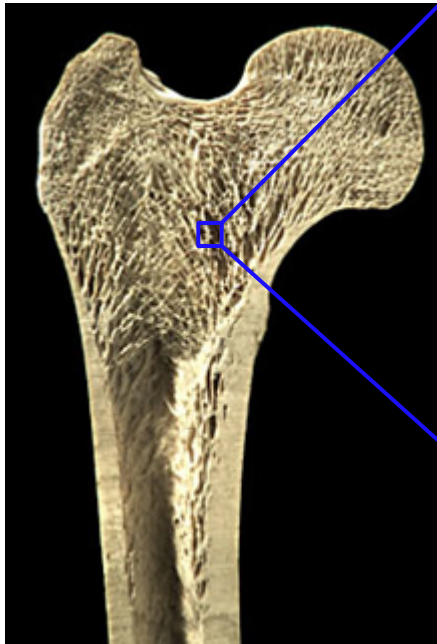
Joshua Harker



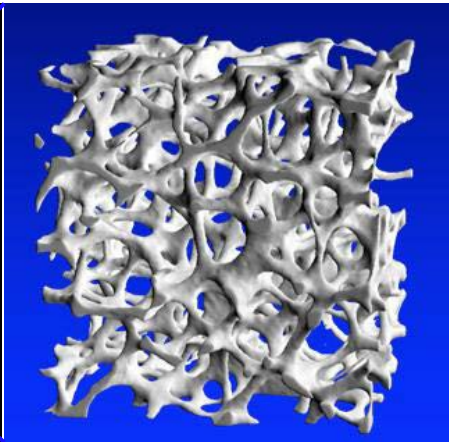
Scott Summit

Complexity is free? ... Not really!

Tiny details



Paul Crompton



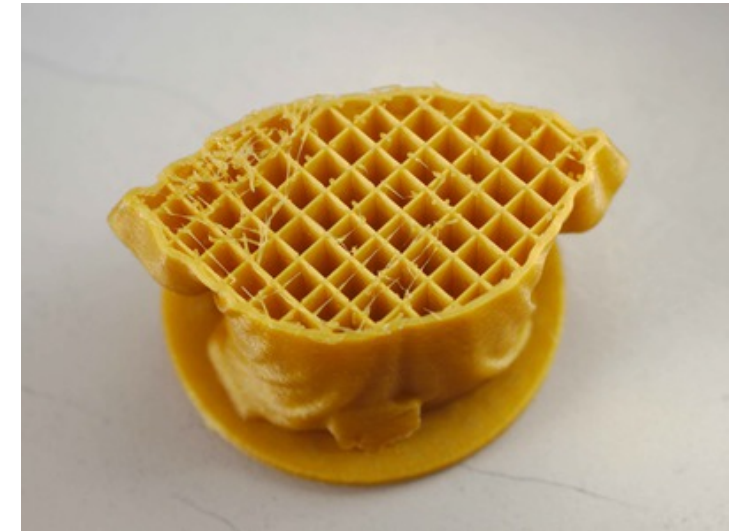
Ralph Müller

Supports



Concept Laser GmH

Infill



mpi.fs.tum.de

Outline

- Geometric feature control by **density filters**
- Geometric feature control by **alternative parameterizations**

Geometric feature control by density filters (An incomplete list)

Reference



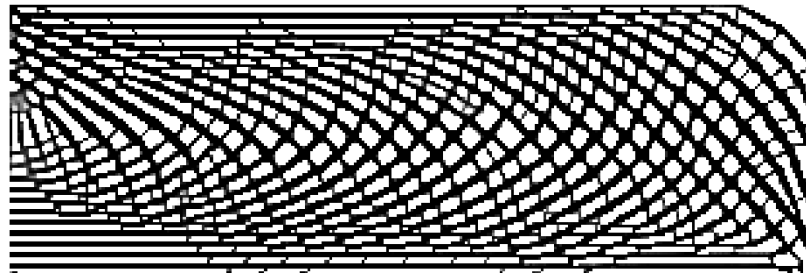
Minimum feature size, Guest'04



Coating structure, Clausen'15

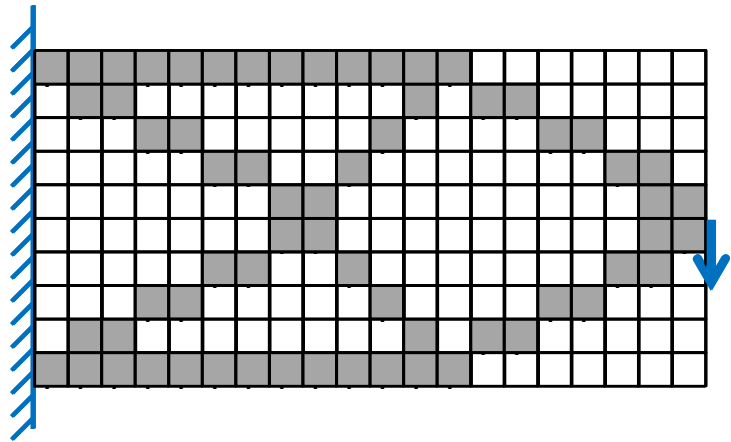


Self-supporting design, Langelaar'16



Porous infill, Wu'16

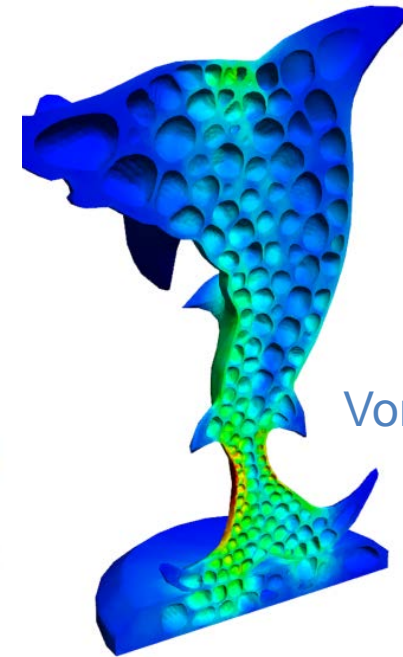
Geometric feature control by alternative parameterizations (An incomplete list)



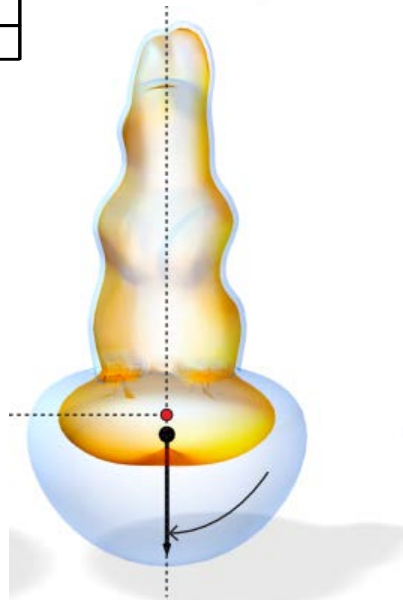
Reference: Voxel discretization



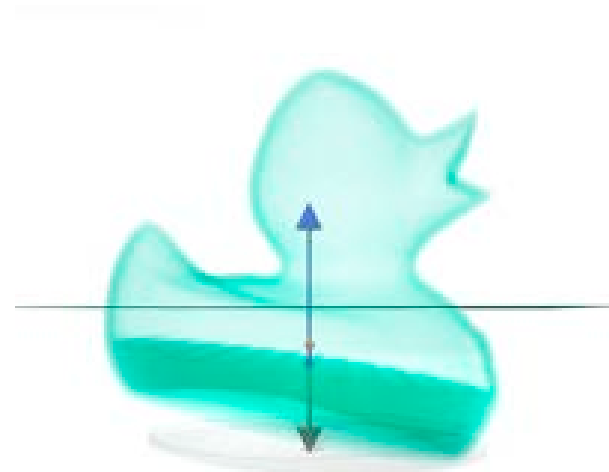
Skin-frame, Wang'13



Voronoi cells, Lu'14



Offset surfaces, Musialski'15



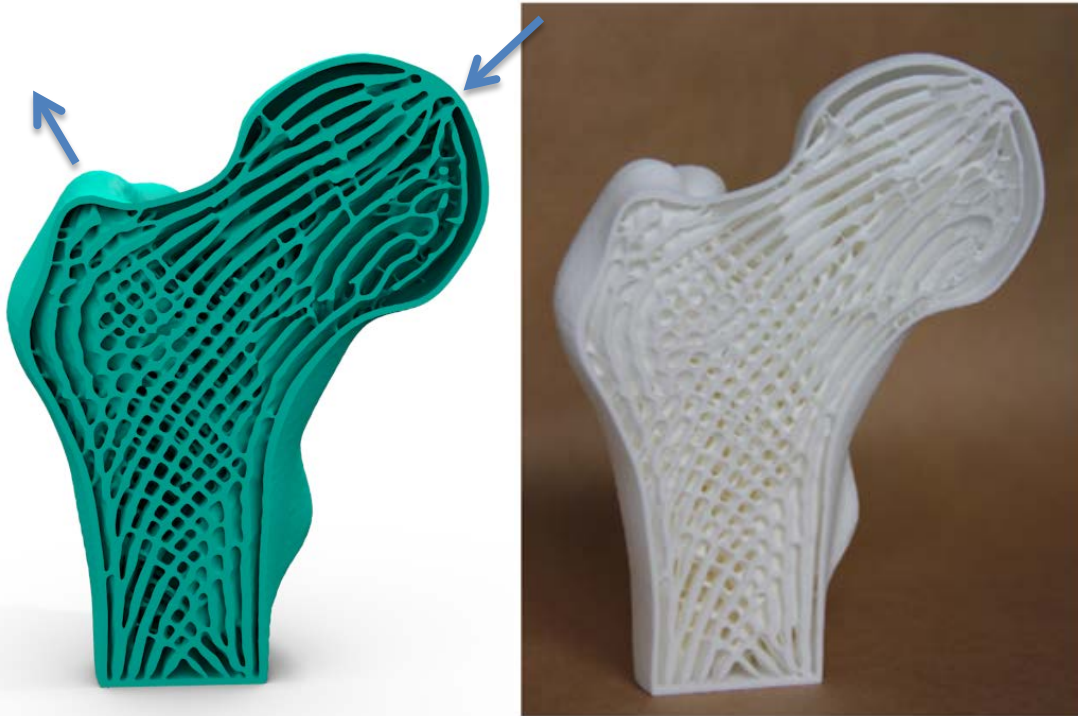
Ray representation, Wu'16



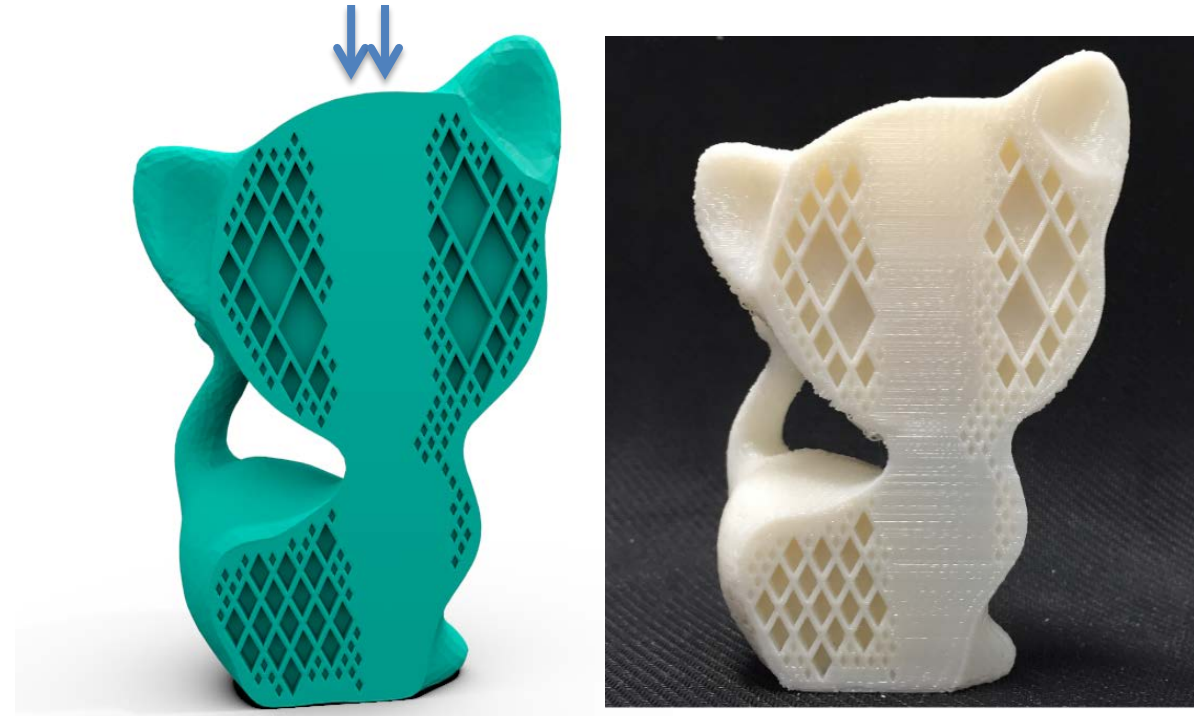
Adaptive rhombic, Wu'16

Outline

- Geometric feature control by **density filters**
- Geometric feature control by **alternative parameterizations**



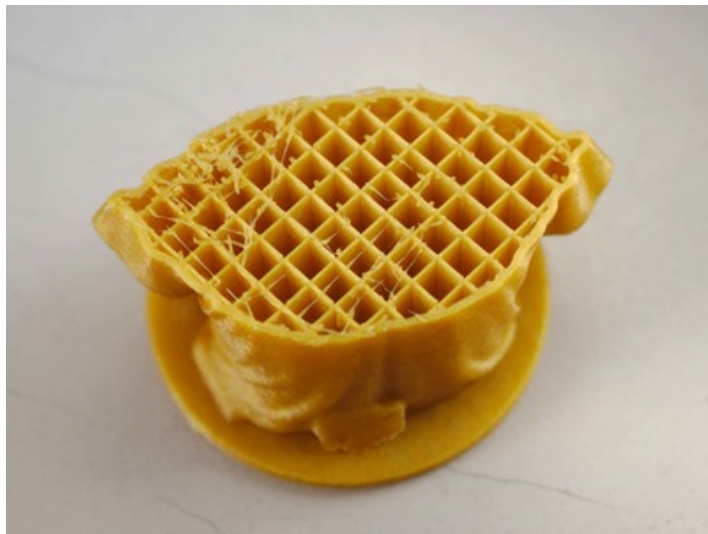
Bone-inspired infill



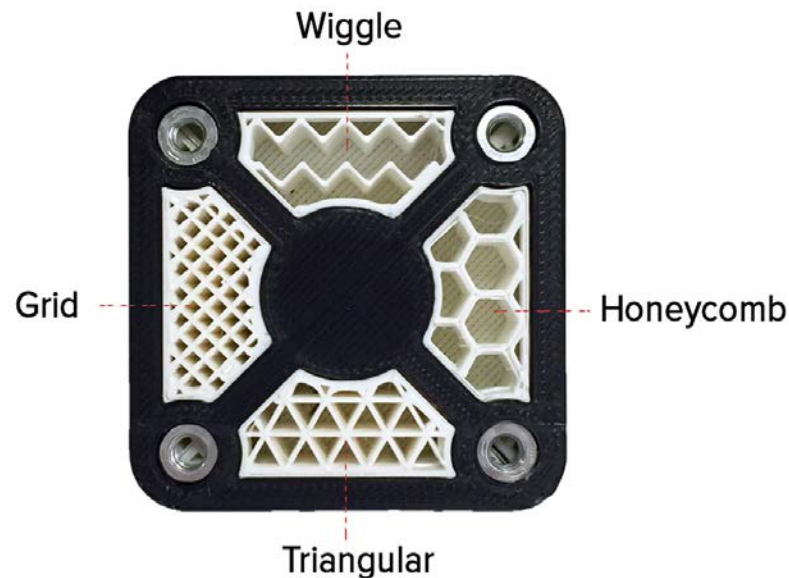
Self-supporting infill

Infill in 3D Printing

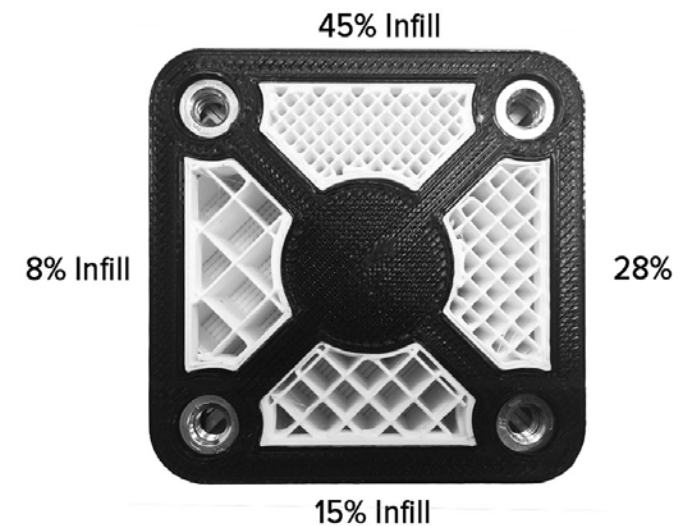
- A user-selected **regular** pattern, with a volume percentage
- A rough balance between
 - Physical properties (mass, strength), and
 - Cost (material usage, print time)



Infill



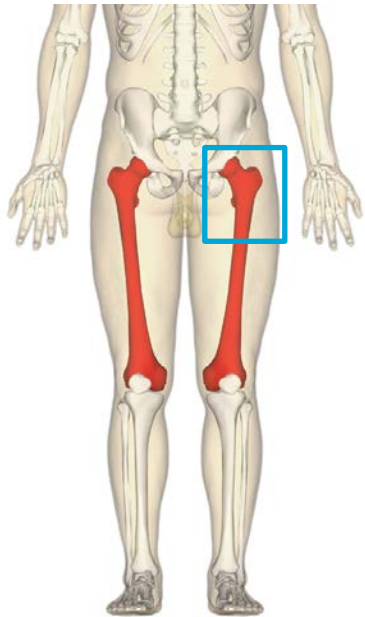
Different infill patterns



Different infill percentages

Infill in Nature

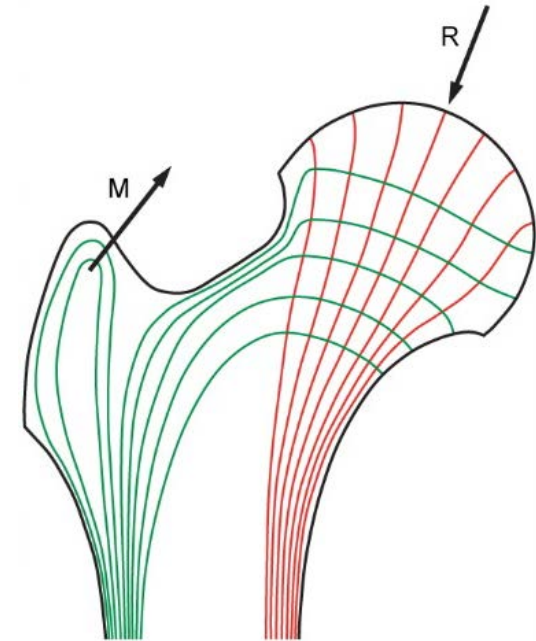
- Trabecular bone
 - Porous structures, oriented with the principle stress direction
 - Resulted from a natural optimization process
 - Light-weight-high-resistant



wikipedia.org



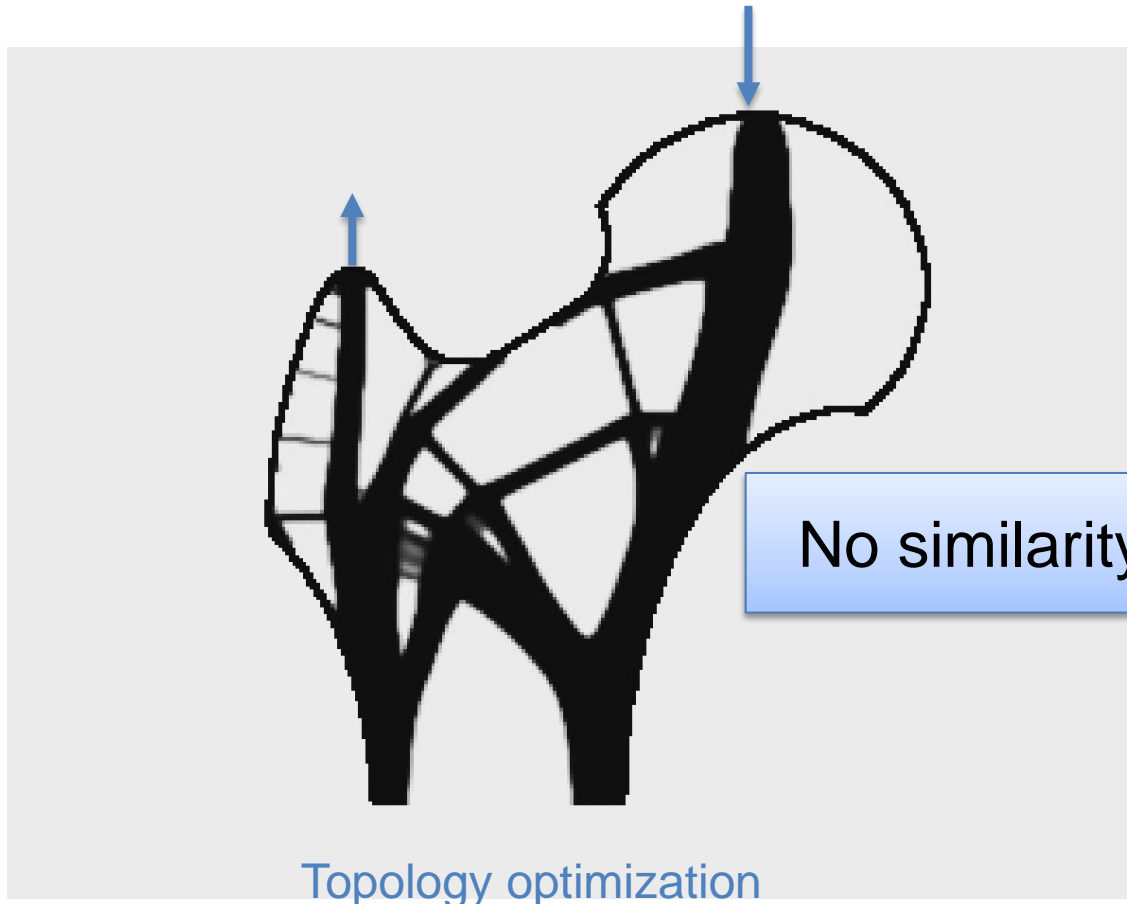
Cross-section of a human femur



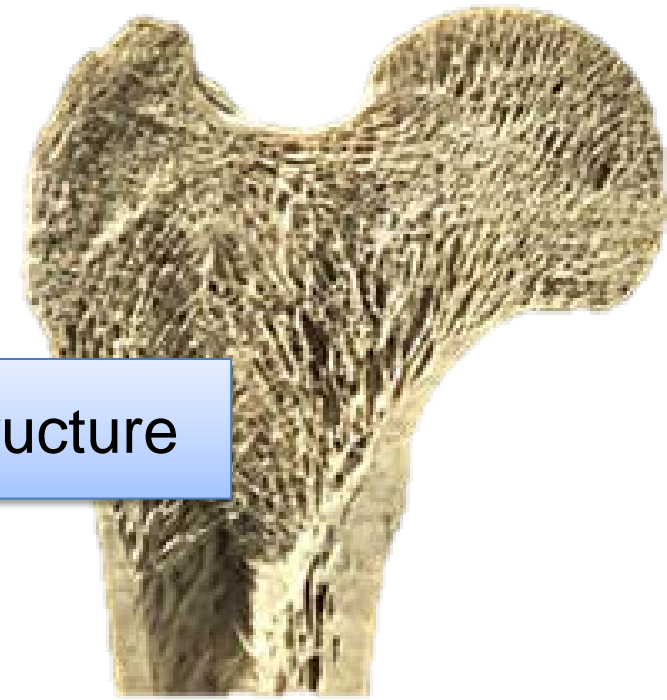
Principle stress directions

Optimize bone-like structures as infill for AM?

Topology Optimization Applied to Design Infill



No similarity in structure



Infill in the bone

Topology Optimization Applied to Design Infill

- Materials accumulate to “important” regions
- The **total** volume $\sum_i \rho_i v_i \leq V_0$ does not restrict local material distribution



Infill by standard
topology optimization

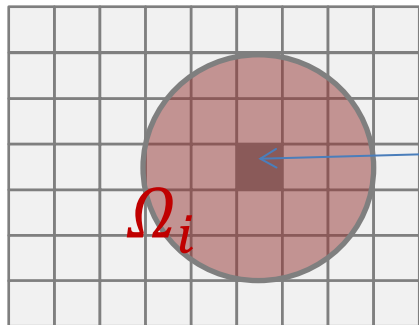


Infill in the bone

Approaching Bone-like Structures: The Idea

- Impose **local constraints** to avoid fully solid regions

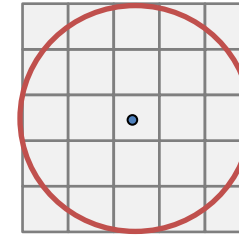
Min: $c = \frac{1}{2} U^T K U$
 s.t. : $KU = F$
 $\rho_i \in [0,1], \forall i$
 ~~$\sum_i \rho_i \leq V_0$~~
 $\hat{\rho}_i \leq \alpha, \forall i$



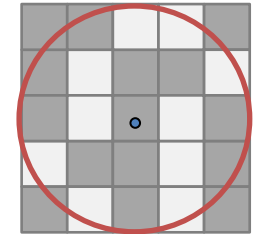
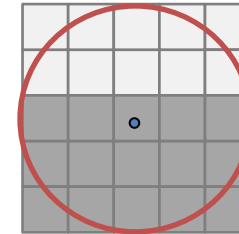
$$\hat{\rho}_i = \frac{\sum_{j \in \Omega_i} \rho_j}{\sum_{j \in \Omega_i} 1}$$

Local-volume measure

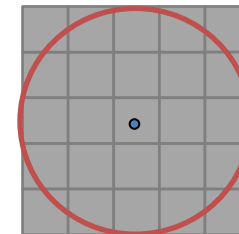
$$\hat{\rho}_i = 0.0$$



$$\hat{\rho}_i = 0.6$$



$$\hat{\rho}_i = 1.0$$



Constraints Aggregation (Reduce the Number of Constraints)

$$\hat{\rho}_i \leq \alpha, \forall i$$



$$\max_{i=1, \dots, n} |\hat{\rho}_i| \leq \alpha$$



$$\lim_{p \rightarrow \infty} \|\rho\|_p = (\sum_i (\hat{\rho}_i)^p)^{\frac{1}{p}} \leq \alpha$$

Too many constraints!

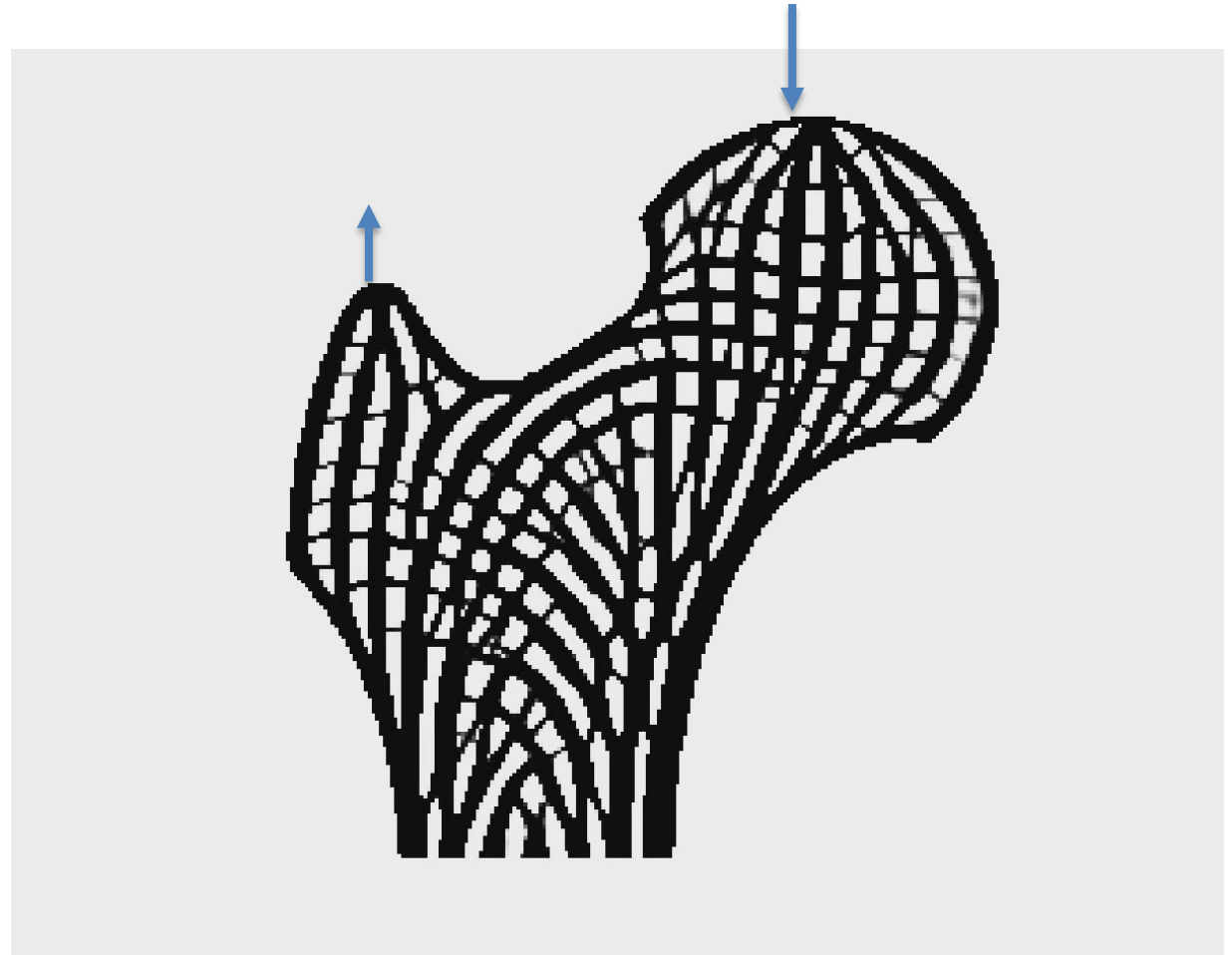
A single constraint
But non-differentiable

A single constraint
and differentiable
Approximated with $p = 16$

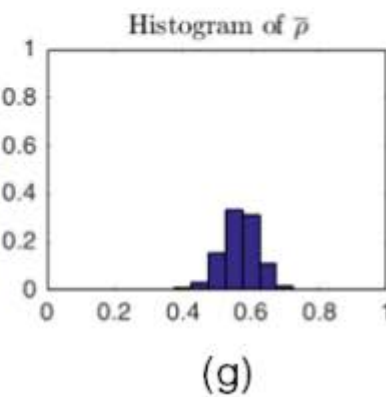
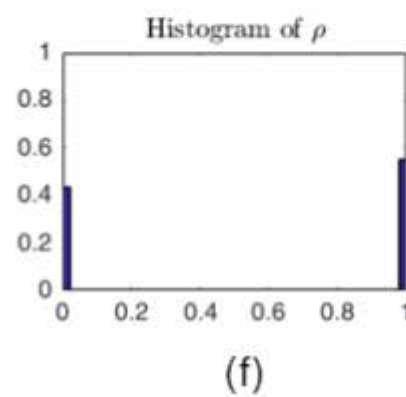
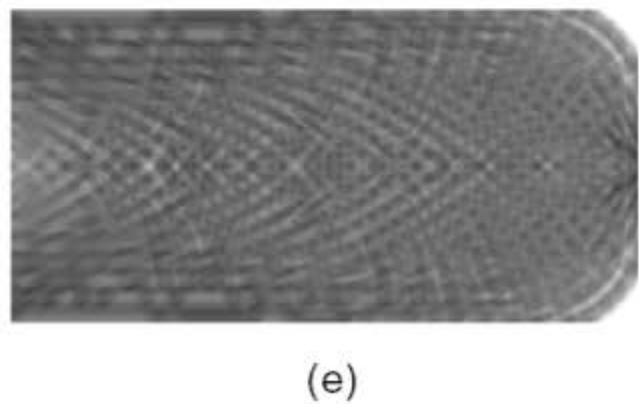
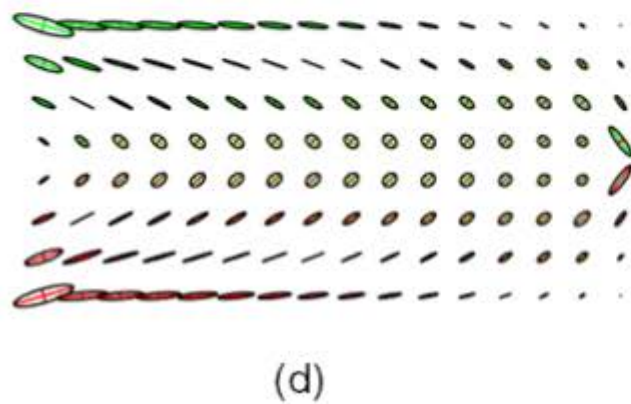
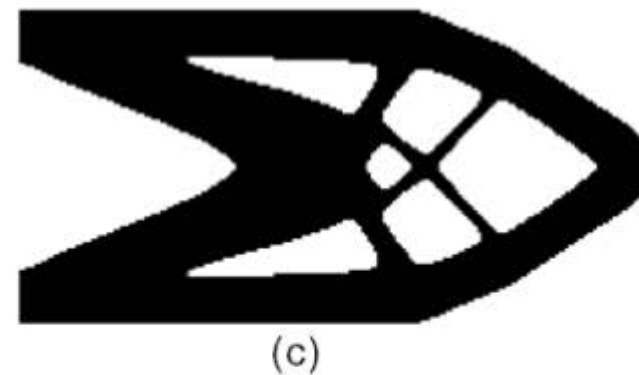
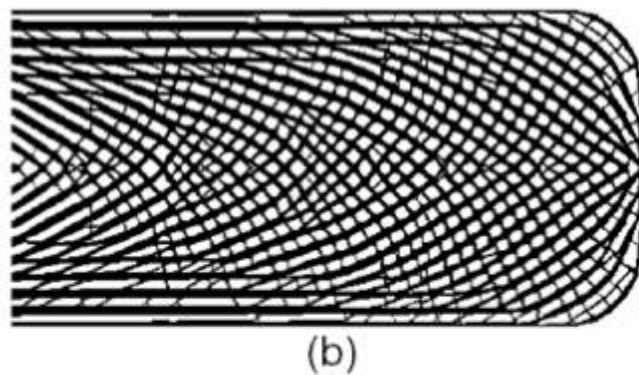
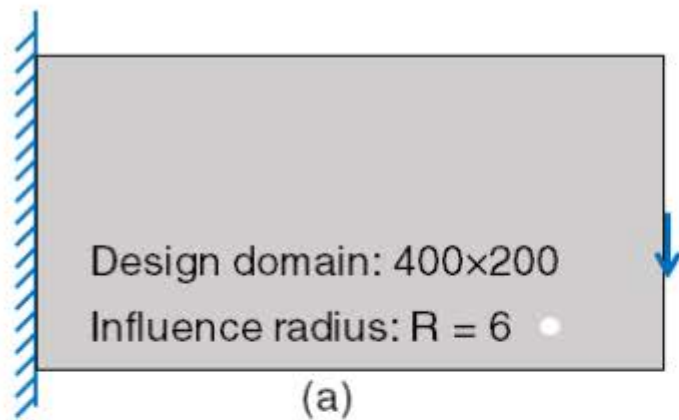
Bone-like Infill in 2D



Cross-section of a human femur

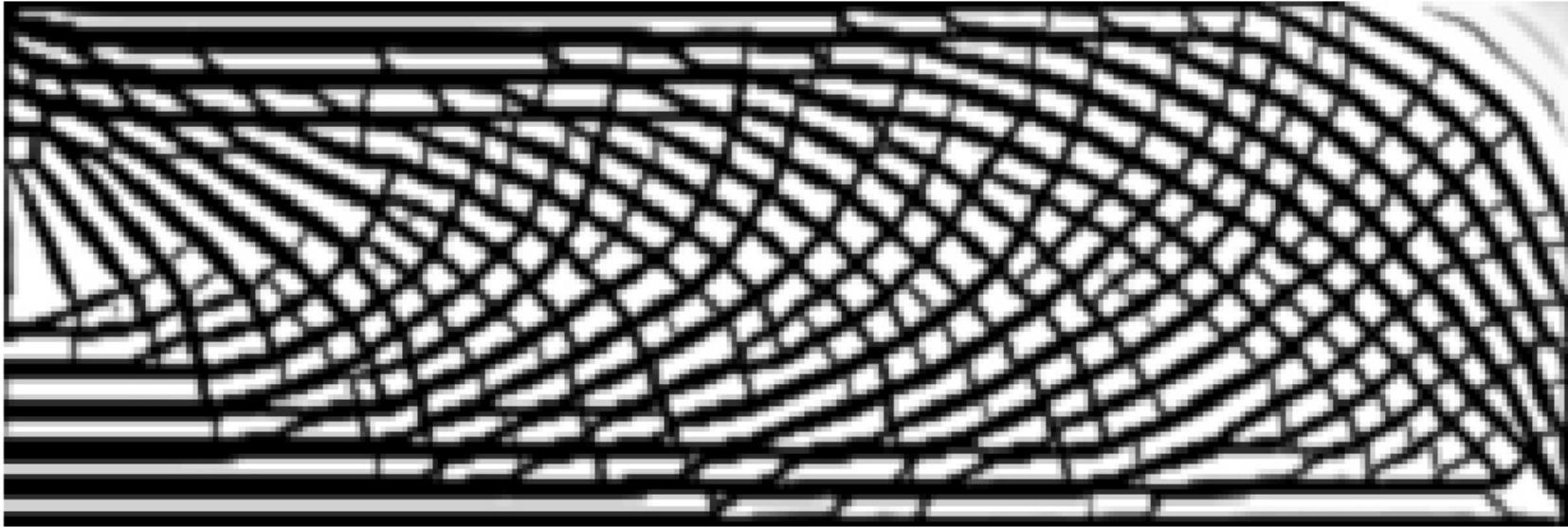


A Test Example



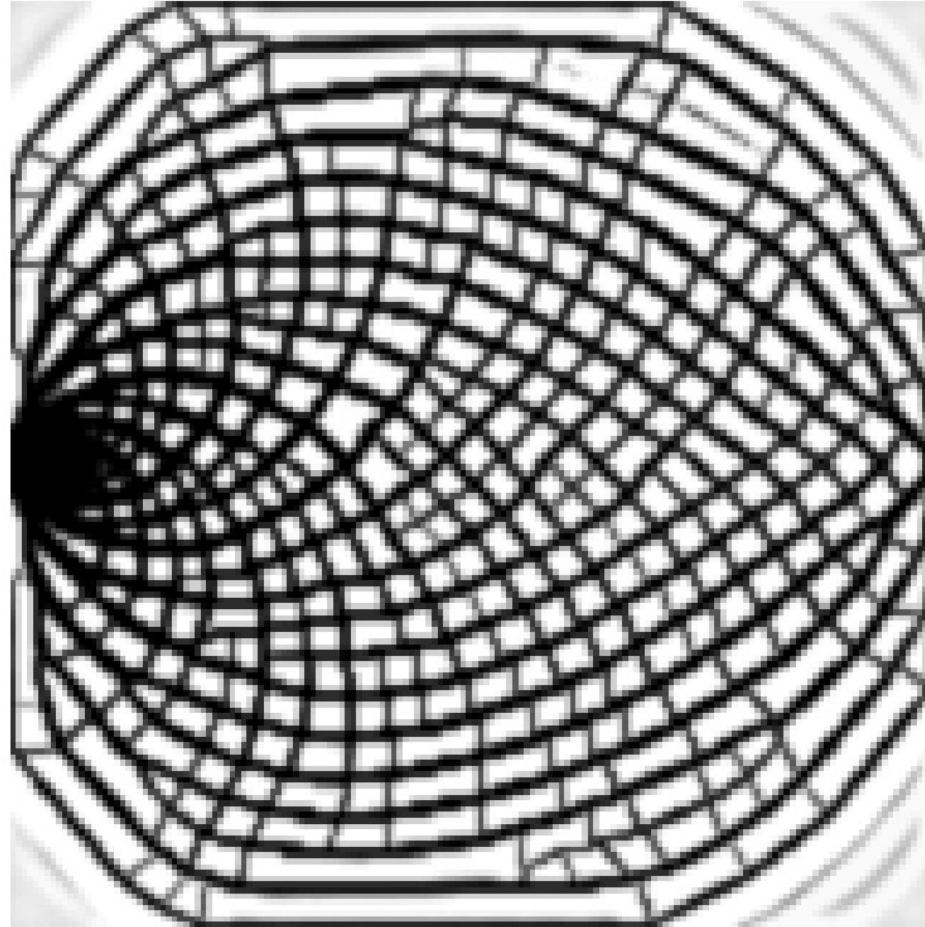
Result: 2D Animation

xPhys



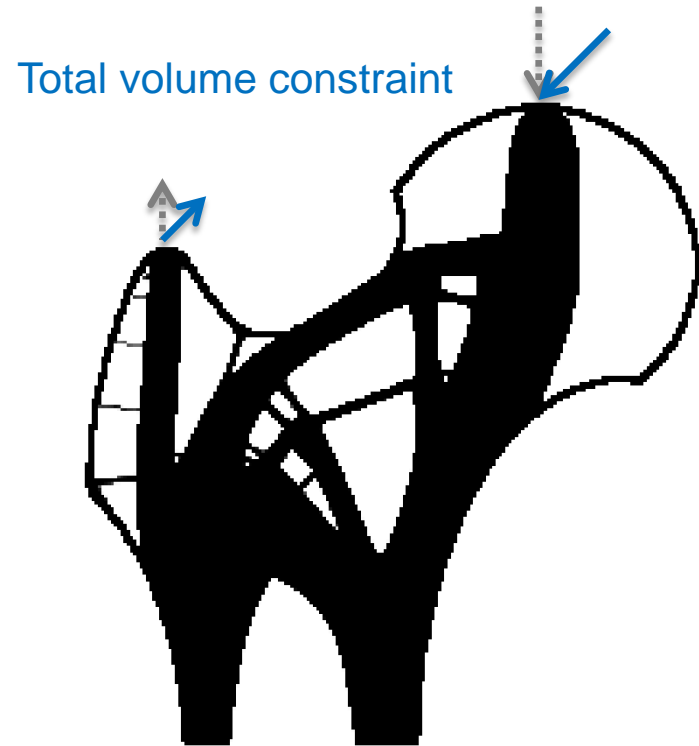
Result: 2D Animation

xPhys

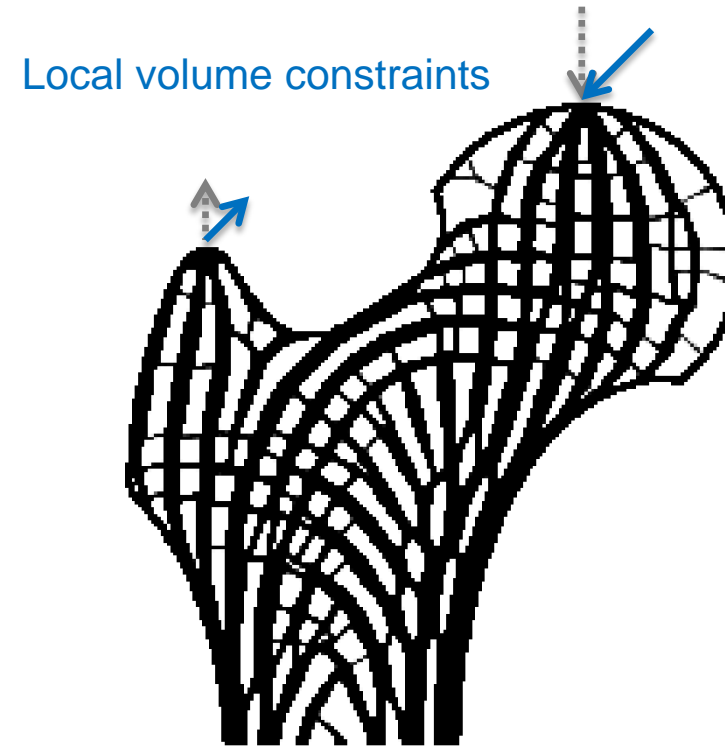


Robustness wrt. Force Variations

- Bone-like structures are significantly stiffer (126%) in case of force variations



$$c = 30.54$$
$$c' = 45.83$$

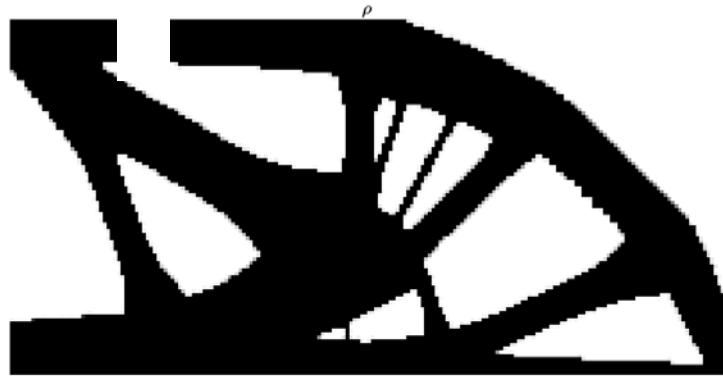


$$c = 36.72$$
$$c' = 36.23$$

Robustness wrt. Material Deficiency

- Bone-like structures are significantly stiffer (180%) in case of **material deficiency**

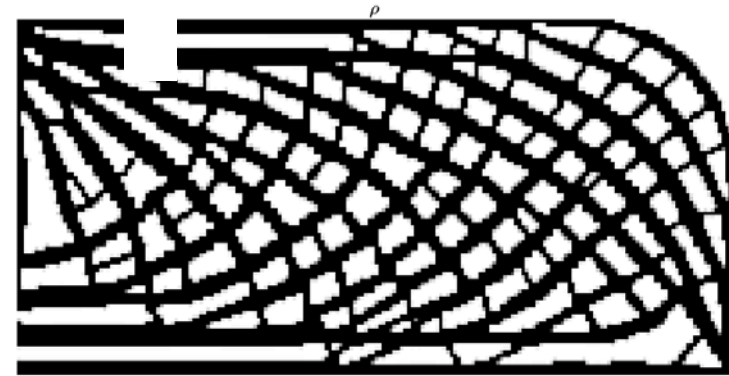
Total volume constraint



$$c = 76.83$$

$$c' = 242.77$$

Local volume constraints



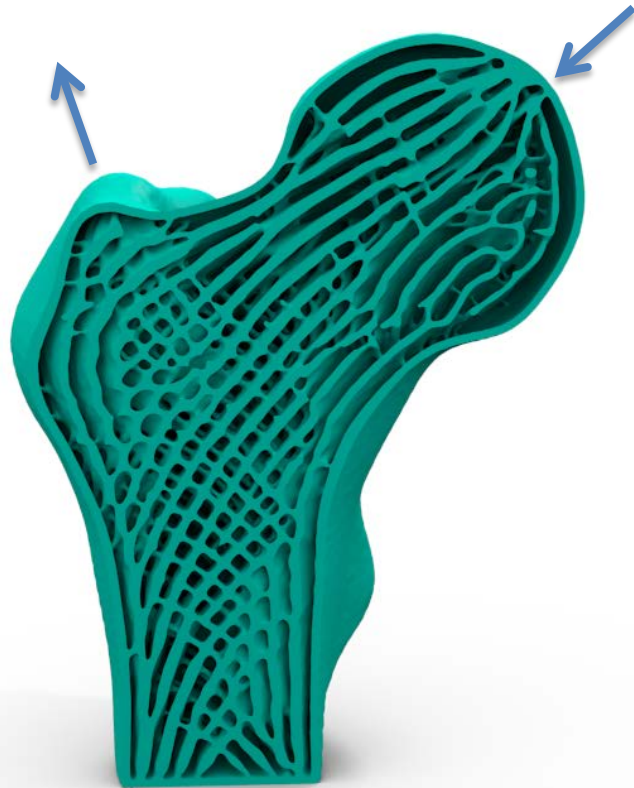
$$c = 93.48$$

$$c' = 134.84$$

Bone-like Infill in 3D



Infill in the bone

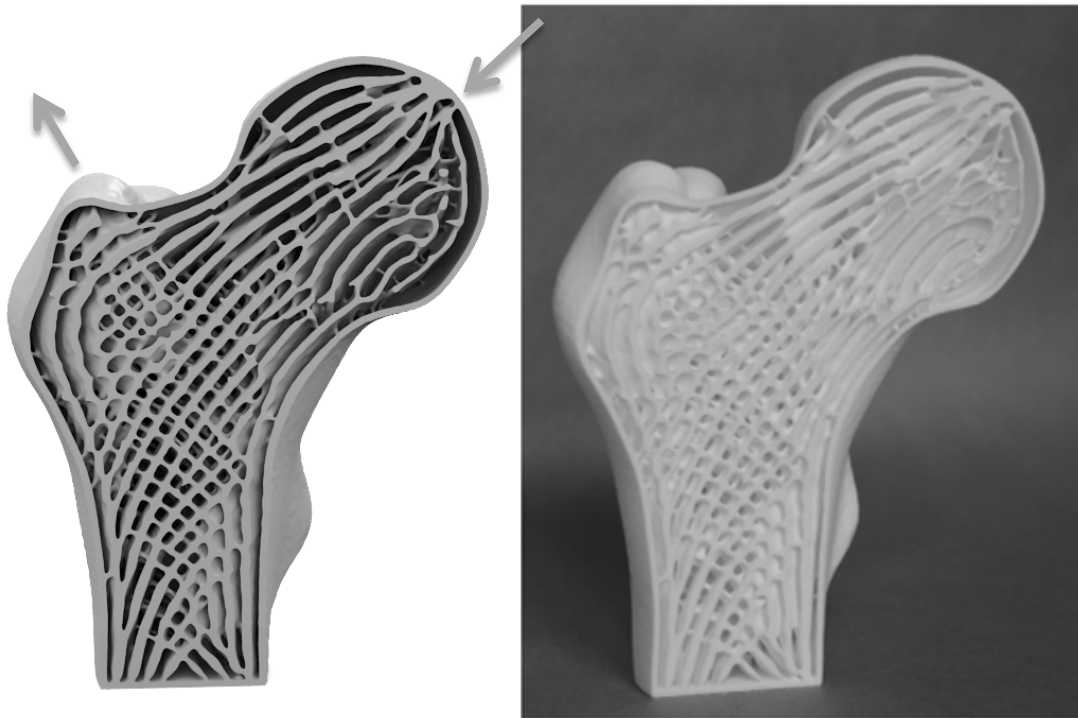


Optimized bone-like infill



Outline

- Geometric feature control by **density filters**
- Geometric feature control by **alternative parameterizations**



Bone-inspired infill

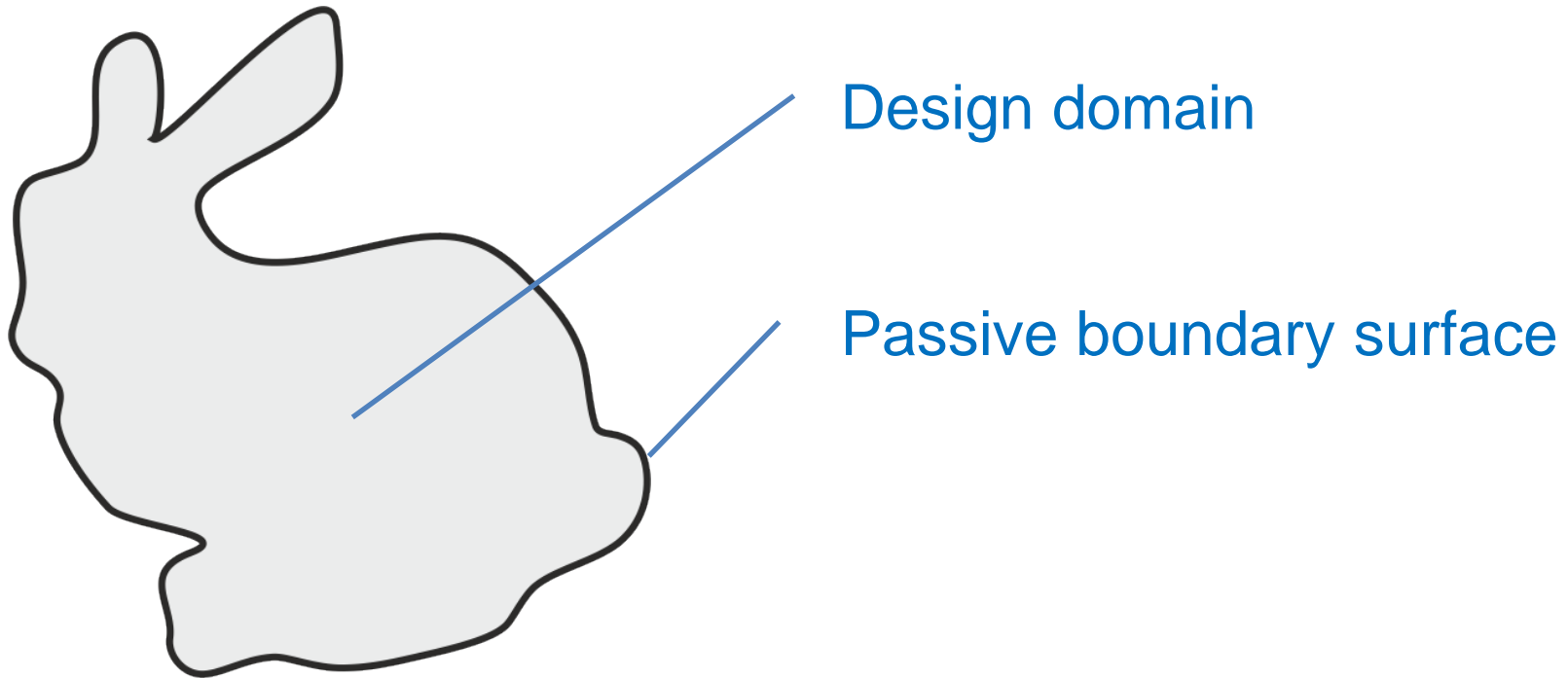


Self-supporting infill



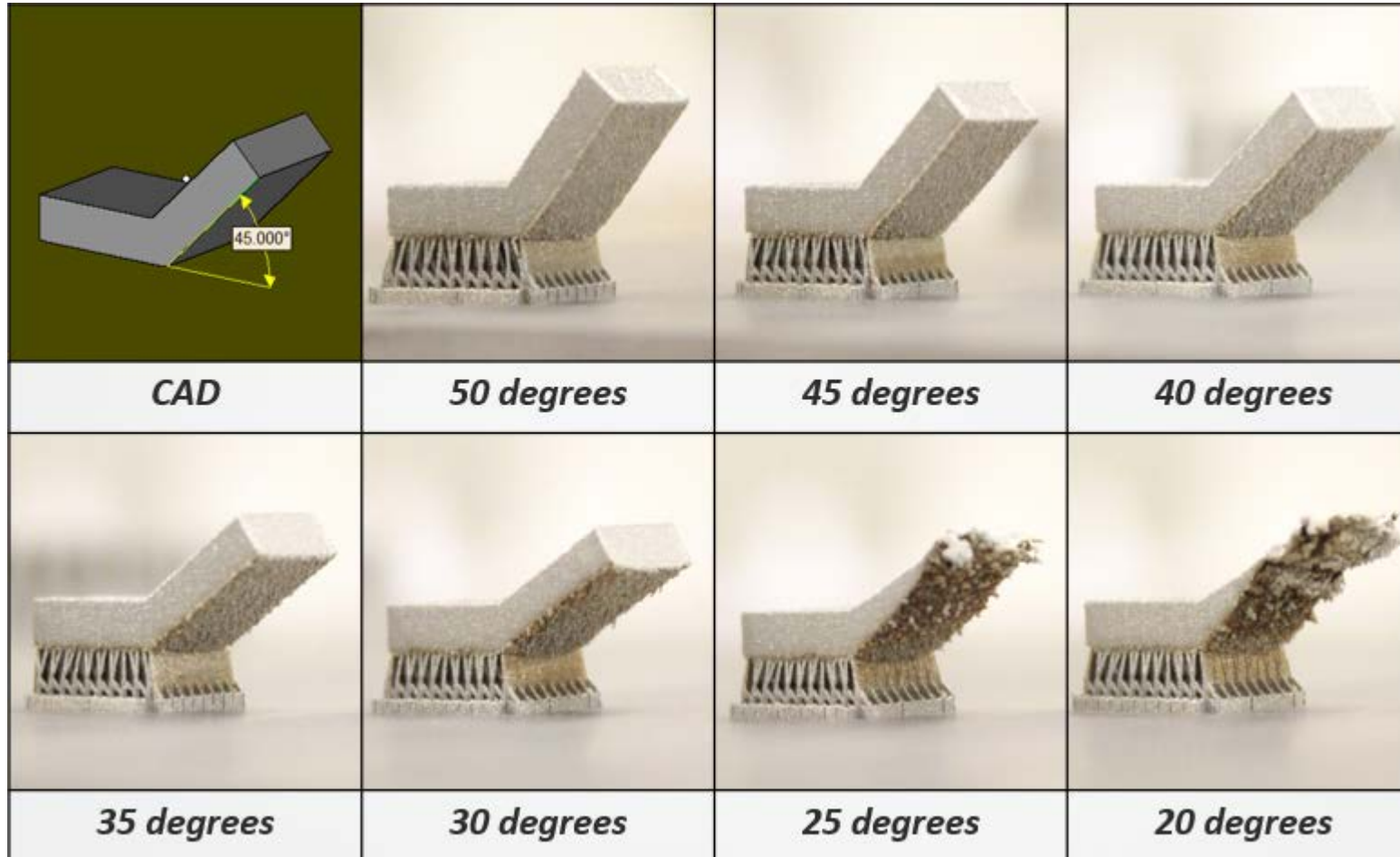
Infill Optimization

- To find the **optimal** material distribution in **the interior** of a given shape



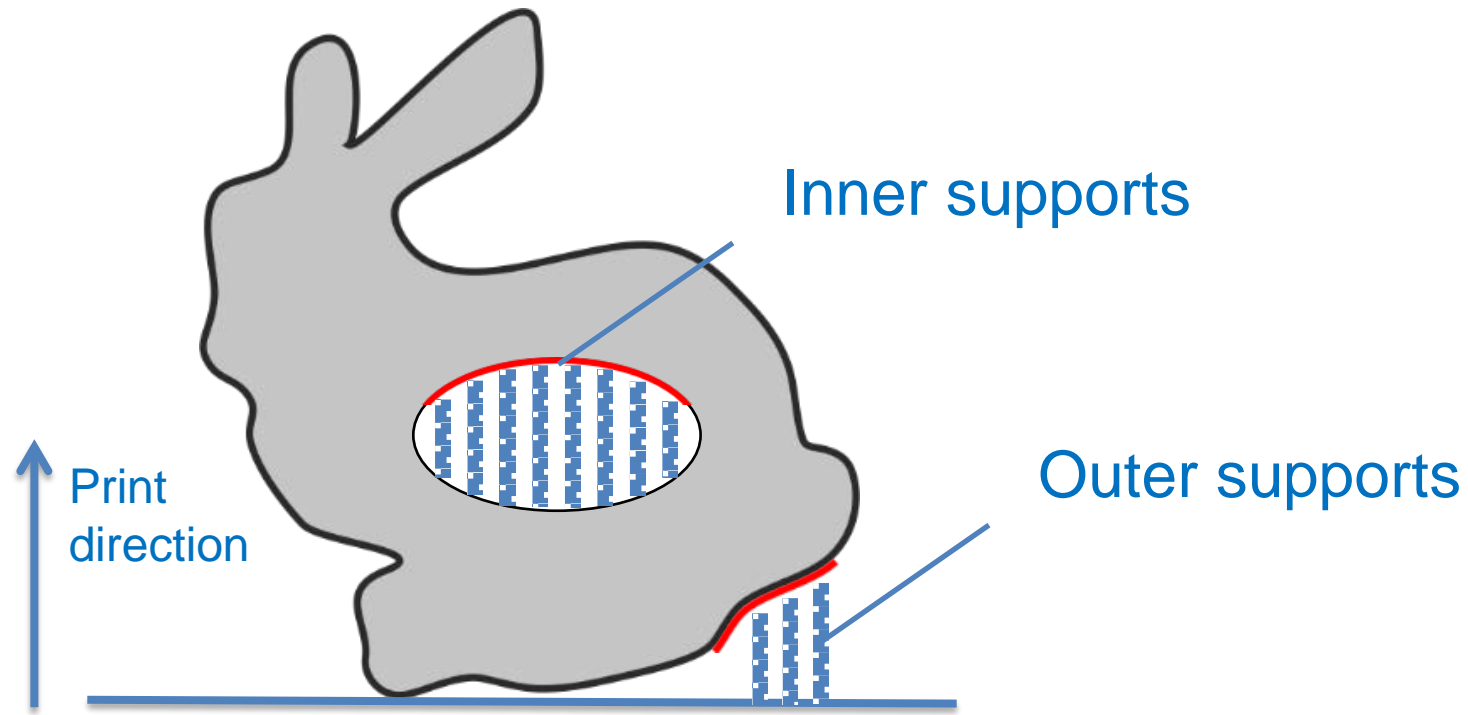
Overhang in Additive Manufacturing

- Support structures are needed beneath overhang surfaces

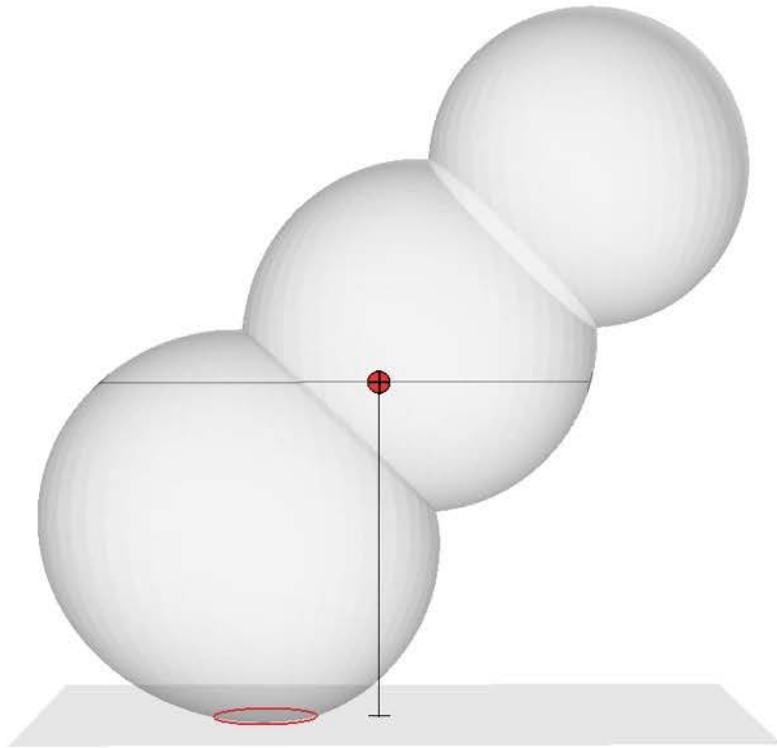


Support Structures in Cavities

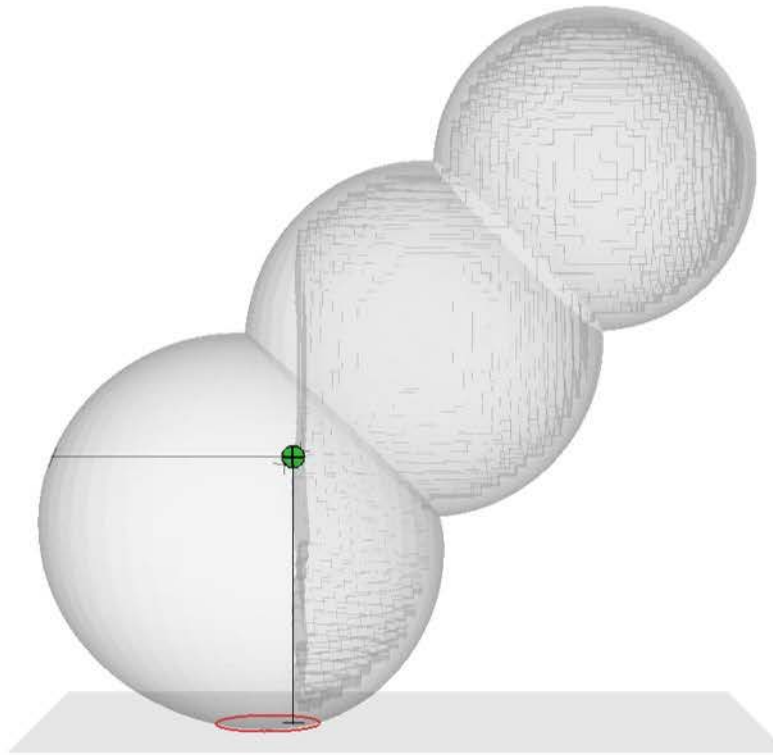
- Post-processing of **inner** supports is problematic



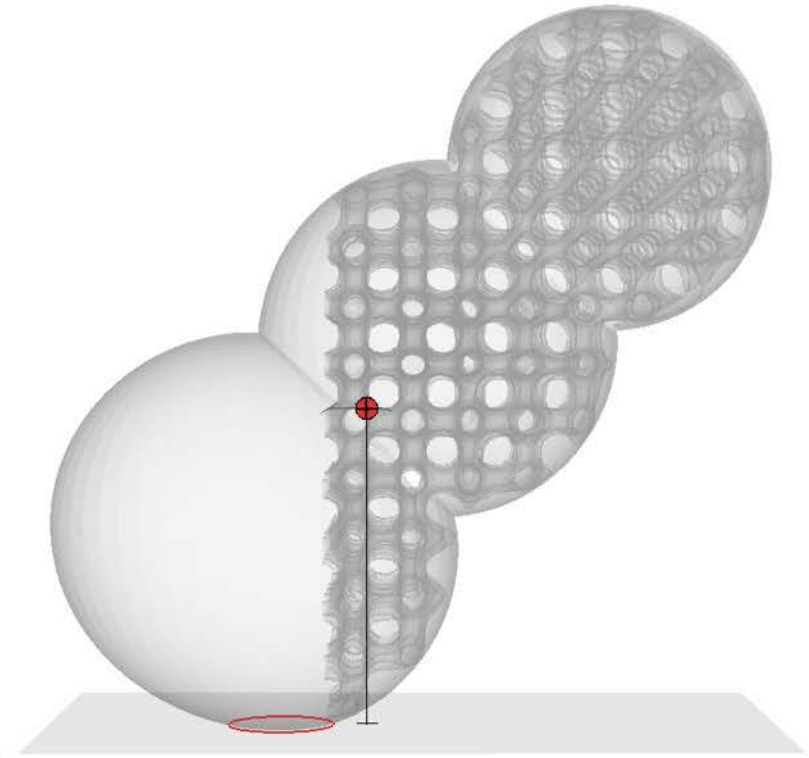
Infill & Optimization Shall Integrate



Solid,
Unbalanced



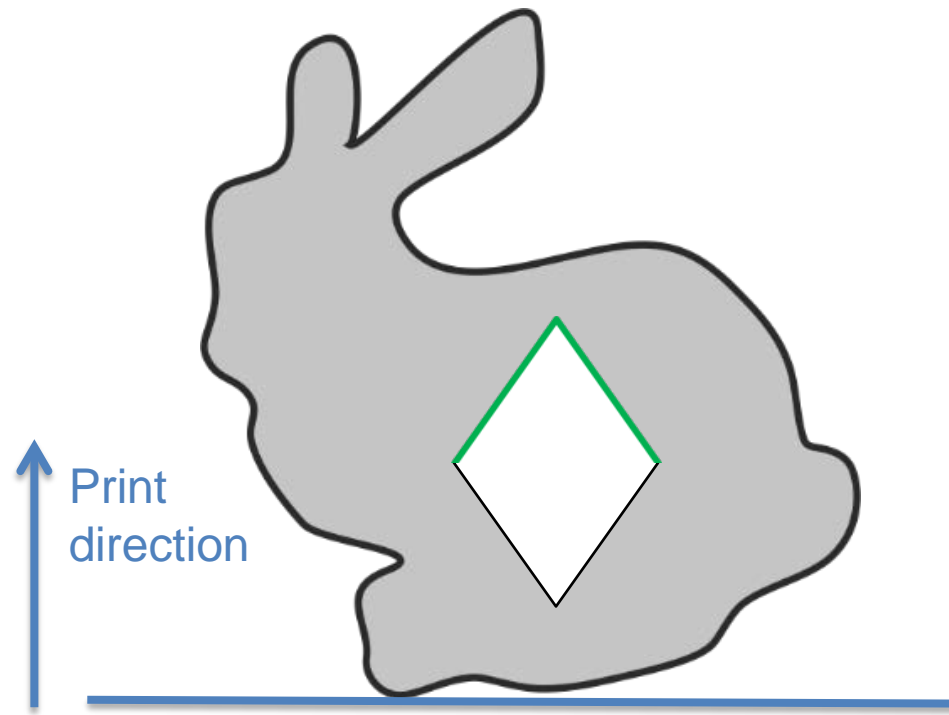
Optimized,
Balanced



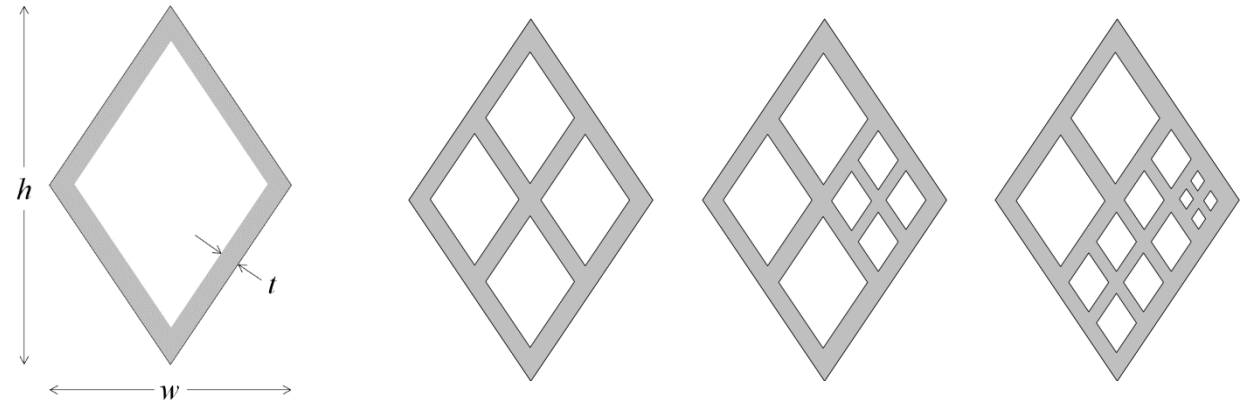
With infill,
Unbalanced

The Idea

- Rhombic cell: to ensure self-supporting
- Adaptive subdivision: as design variable in optimization

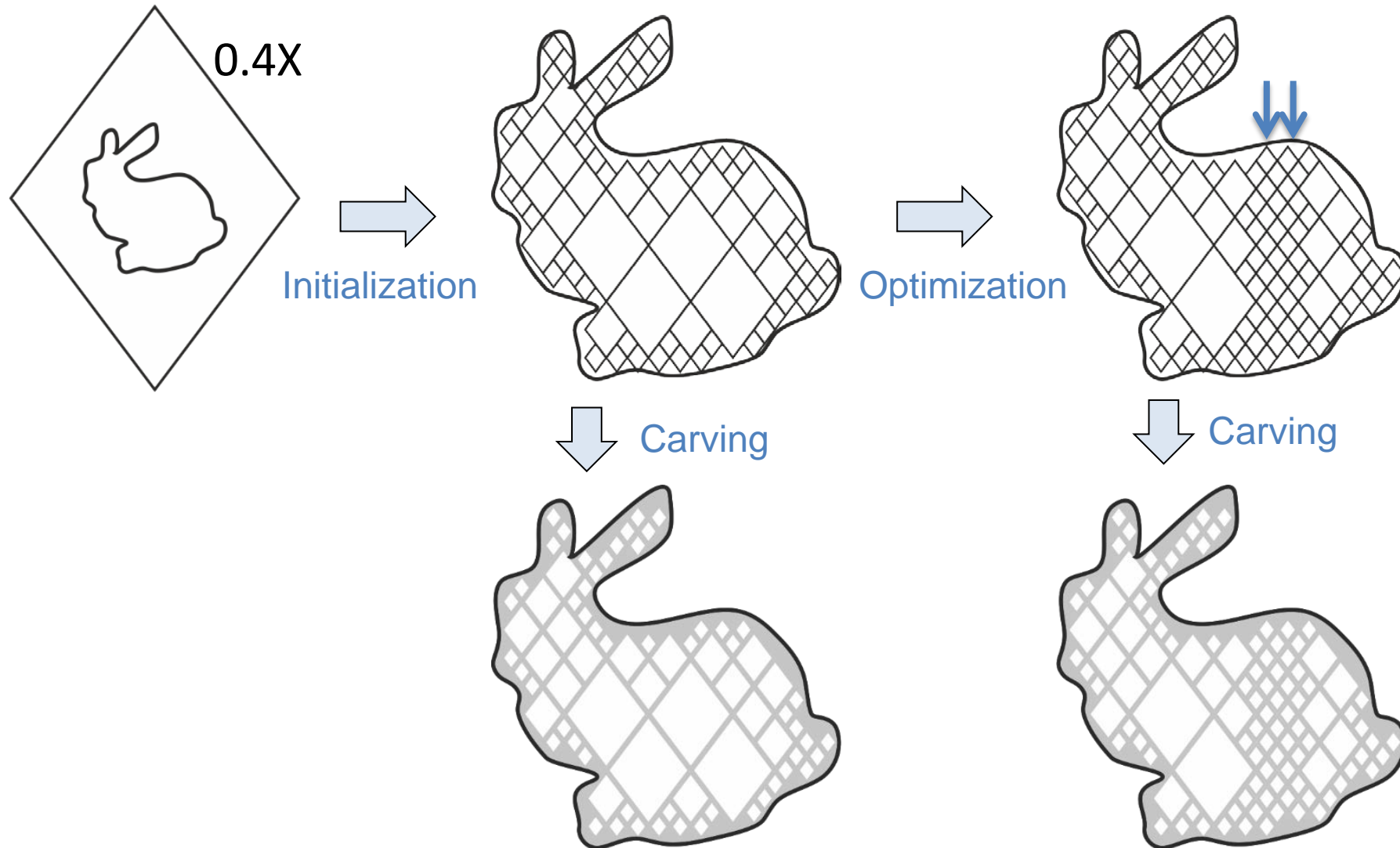


Rhombic cell



Adaptive subdivision

Self-Supporting Rhombic Infill: Workflow



Self-Supporting Rhombic Infill: Subdivision Criteria

- Min: $c = \frac{1}{2} U^T K U$ Subject to: $KU = F; V = \sum_i \rho_i \leq V_0$

Voxel-wise topology optimization

Per-voxel density as variable

$$\rho_i \in \{0.0, 1.0\}, \forall i$$

Per-voxel sensitivity: $G_i = -\frac{\partial c / \partial \rho_i}{\partial V / \partial \rho_i}$

Subdivision-based topology optimization

Per-subdivision as variable

$$\beta_c \in \{0, 1\}, \forall c$$

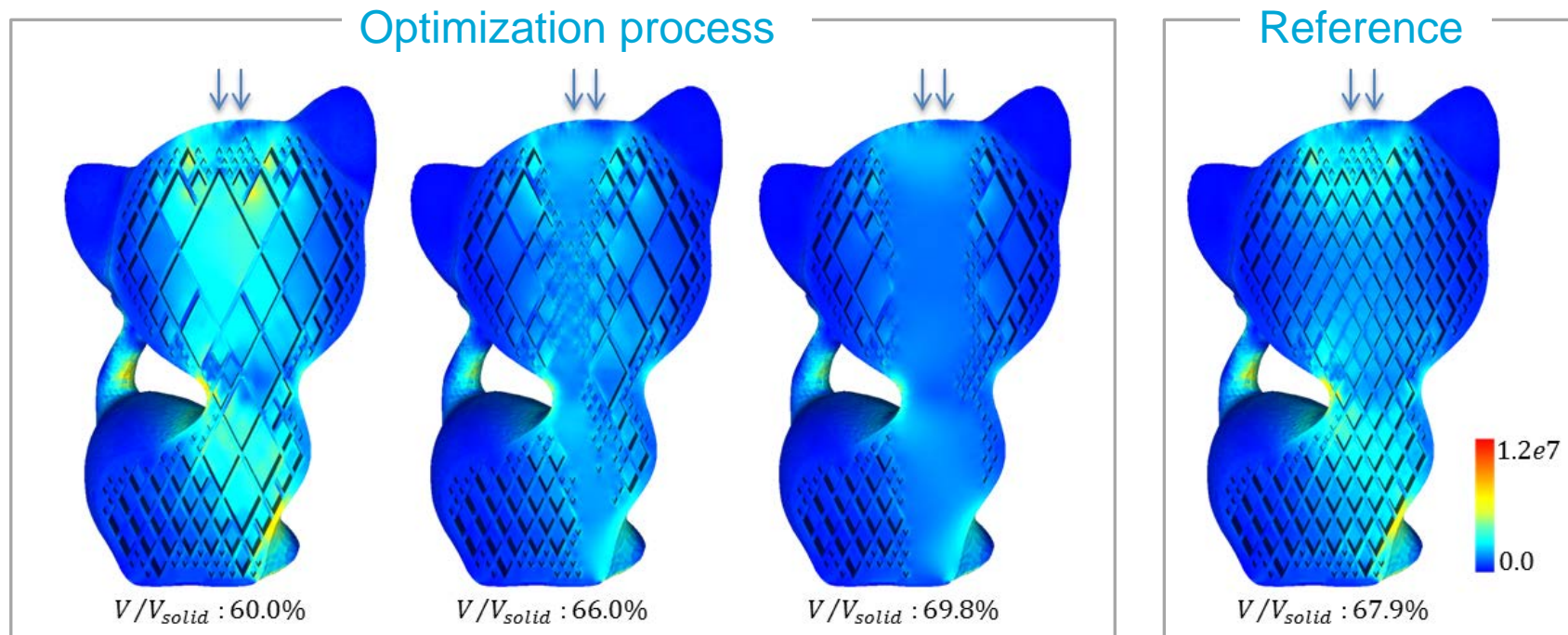
Per-voxel density assigned by subdivision

$$\rho_i(\beta) = \begin{cases} 1.0 & i \text{ covered by walls} \\ 0.0 & \text{otherwise} \end{cases}$$

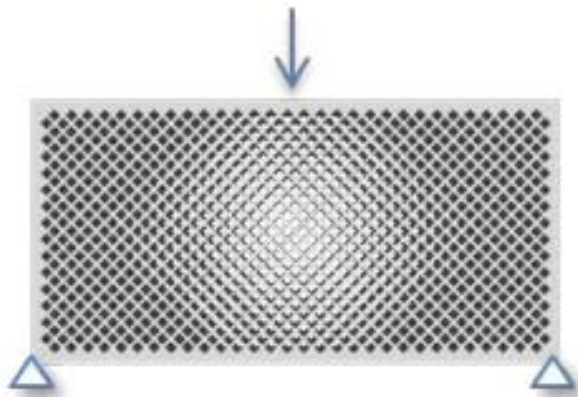
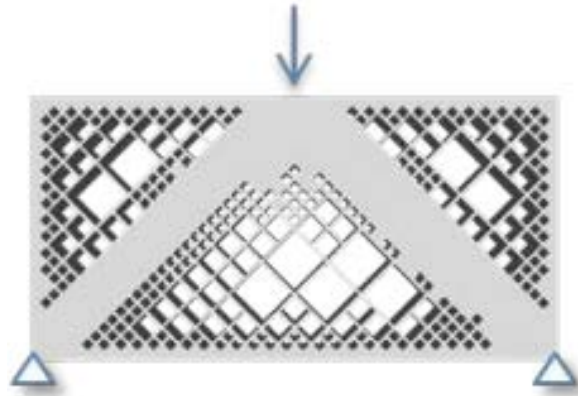
Per-subdivision sensitivity: $G_c = -\frac{\partial c / \partial \beta_c}{\partial V / \partial \beta_c}$

Self-Supporting Rhombic Infill: Results

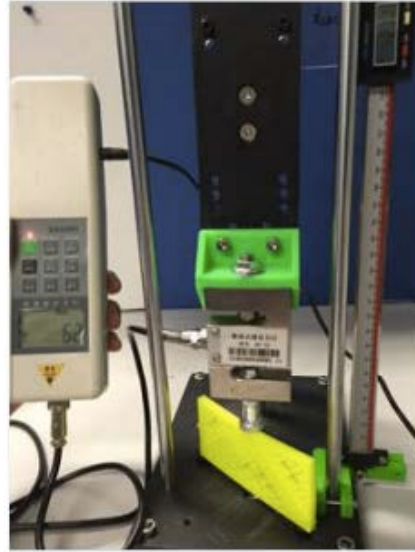
- Optimized mechanical properties, compared to regular infill
- No additional inner supports needed



Mechanical Tests

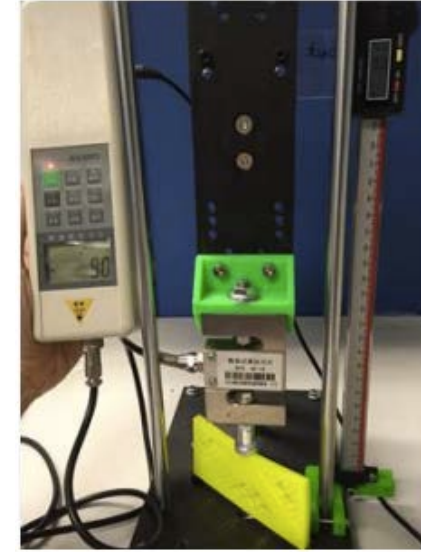


Under same force (62 N)

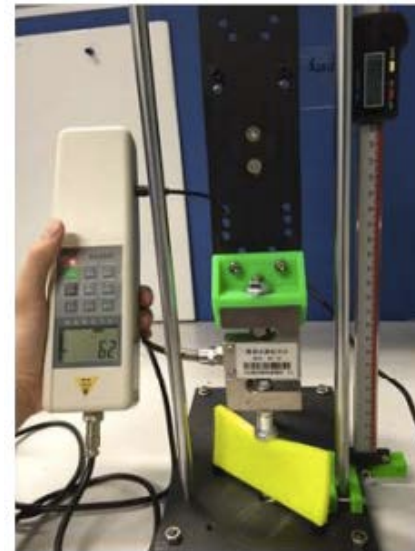


Dis.
2.11 mm

Under same displacement (3.0 mm)



Force
90 N



Dis.
4.08 mm



Force
58 N

Summary

- Geometric feature control by **density filters**
- Geometric feature control by **alternative parameterizations**



Thank you for your attention!

Questions?

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Incomplete references: Density filters

- Guest, James K., Jean H. Prévost, and T. Belytschko. "[Achieving minimum length scale in topology optimization using nodal design variables and projection functions.](#)" International journal for numerical methods in engineering 61, no. 2 (2004): 238-254.
- Wang, Fengwen, Boyan Stefanov Lazarov, and Ole Sigmund. "[On projection methods, convergence and robust formulations in topology optimization.](#)" Structural and Multidisciplinary Optimization 43, no. 6 (2011): 767-784.
- Clausen, Anders, Niels Aage, and Ole Sigmund. "[Topology optimization of coated structures and material interface problems.](#)" Computer Methods in Applied Mechanics and Engineering 290 (2015): 524-541.
- Langelaar, Matthijs. "[An additive manufacturing filter for topology optimization of print-ready designs.](#)" Structural and Multidisciplinary Optimization (2016): 1-13.
- Wu, Jun, Niels Aage, Ruediger Westermann, and Ole Sigmund. "[Infill Optimization for Additive Manufacturing--Approaching Bone-like Porous Structures.](#)" IEEE Transactions on Visualization and Computer Graphics, 2016.

Incomplete references: Alternative parameterizations

- Wang, Weiming, Tuanfeng Y. Wang, Zhouwang Yang, Ligang Liu, Xin Tong, Weihua Tong, Jiansong Deng, Falai Chen, and Xiuping Liu. "[Cost-effective printing of 3D objects with skin-frame structures.](#)" ACM Transactions on Graphics (TOG) 32, no. 6 (2013): 177.
- Lu, Lin, Andrei Sharf, Haisen Zhao, Yuan Wei, Qingnan Fan, Xuelin Chen, Yann Savoye, Changhe Tu, Daniel Cohen-Or, and Baoquan Chen. "[Build-to-last: Strength to weight 3d printed objects.](#)" ACM Transactions on Graphics (TOG) 33, no. 4 (2014): 97.
- Musialski, Przemyslaw, Thomas Auzinger, Michael Birsak, Michael Wimmer, and Leif Kobbelt. "[Reduced-order shape optimization using offset surfaces.](#)" ACM Trans. Graph. 34, no. 4 (2015): 102.
- Wu, Jun, Lou Kramer, and Rüdiger Westermann. "[Shape interior modeling and mass property optimization using ray-reps.](#)" Computers & Graphics 58 (2016): 66-72.
- Wu, Jun, Charlie CL Wang, Xiaoting Zhang, and Rüdiger Westermann. "[Self-supporting rhombic infill structures for additive manufacturing.](#)" Computer-Aided Design 80 (2016): 32-42.

Topology Optimization

Minimize: $c = \frac{1}{2} U^T K U$

Subject to: $KU = F$

$$\rho_i \in [0,1], \forall i$$

$$\sum_i \rho_i \leq V_0$$

