EUROGRAPHICS2017
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Topology Optimization for Computational Fabrication

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DTU

Topology optimization: Basics tools and methods

by Niels Aage

@Eurographics 2017

Mechanical Engineering
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Technical University of Denmark (DTU)

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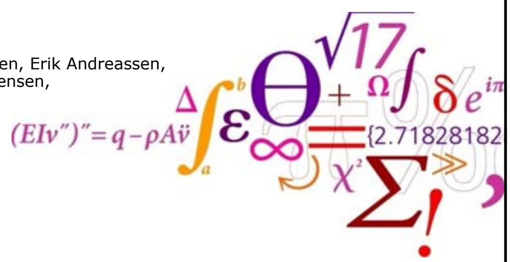

DTU Mekanik
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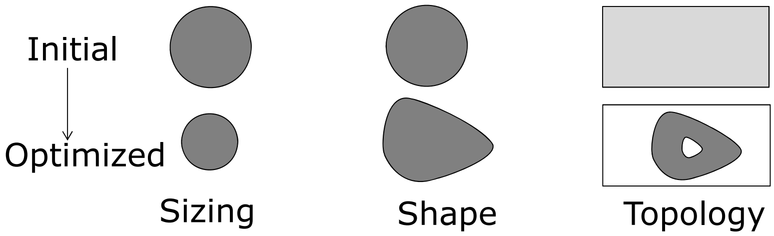
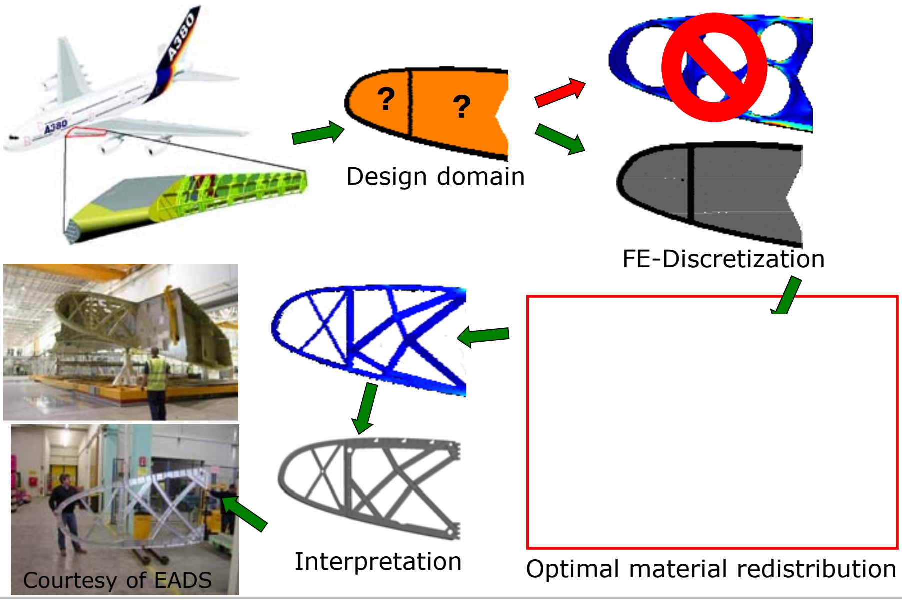
Classes of structural optimization

Classes of structural optimization methods:

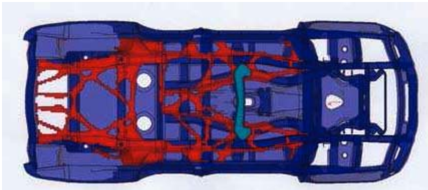


Topology Optimization in Aerospace

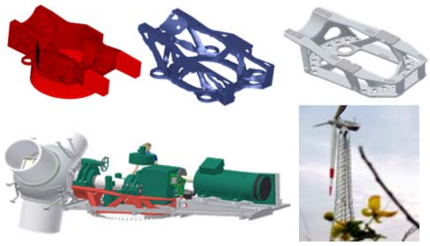
Bendsøe and Kikuchi (1988)



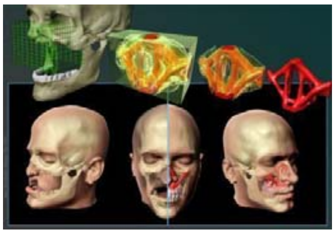
Topology Optimization Applications



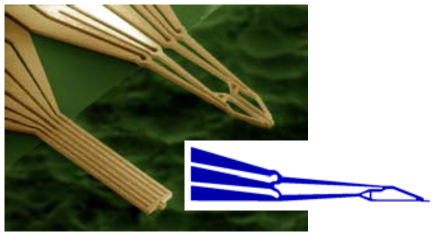
Automotive industry (Fabian Duddeck)



Wind turbines (SUZLON and FE-Design GmbH)

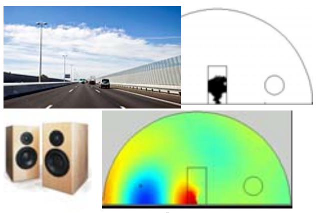


Reconstructive surgery (Paulino/Sinn-Hanlon)

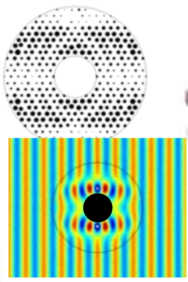


Micromachines (DTU Nanotech)

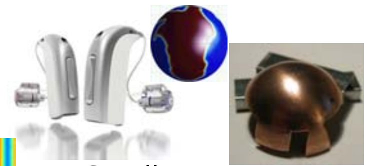
Topology Optimization Applications



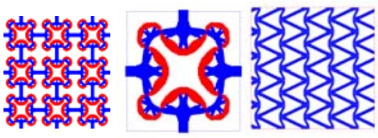
Acoustics



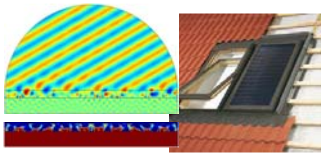
Cloaking



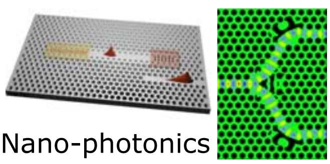
Small antennas



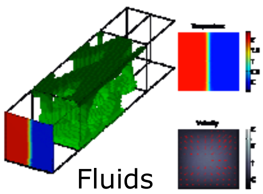
Extreme materials



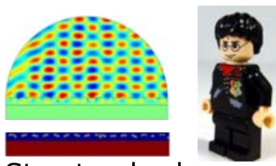
Energy harvesting



Nano-photonics



Fluids



Structural colours

Before we get startet …

- TopOpt falls into the catagory of PDE constrained optimization:

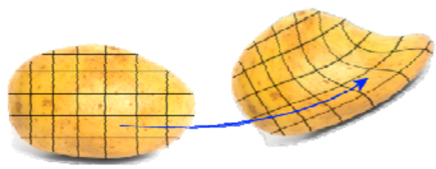
$$
\begin{aligned}
&\min J(y,u)\\
&\text{s.t. } c(y,u)=0,\\
&\qquad g(y,u)=0,\\
&\qquad h(y,u)\in -K\\
&\qquad y\in \mathcal{Y}_{ad}, u\in \mathcal{U}_{ad}.
\end{aligned}
$$

u: state variables
y: control/design variables
J: Objective function
c: PDE
g: equality constraints
h: Inequality constraints
&
Y_{ad},U_{ad}: admissible sets

- PDE – Partial Differential Equation:
  Often arise from conservation laws in physics.

Basic continuum mechanics

It starts with observations…

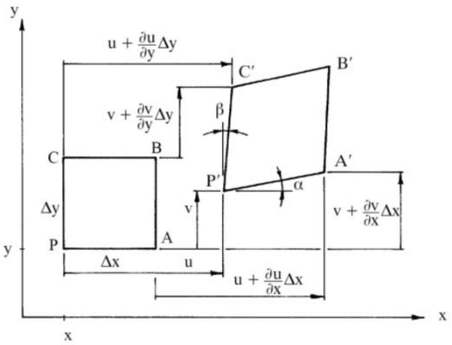


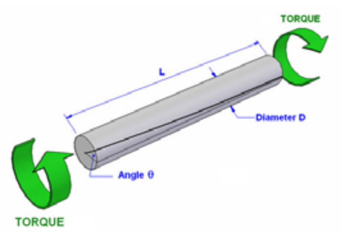
- **Deformations** (displacement)
  - Vector function that maps a material point into its new coordinate, i.e.

$$\boldsymbol{u} = [u(x,y,z), v(x,y,z), w(x,y,z)]^T$$

Basic continuum mechanics

- **Strains** (measurable) - relative deformation





$$\epsilon_x = \frac{\partial u}{\partial x}, \qquad \epsilon_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\epsilon_y = \frac{\partial v}{\partial y}, \qquad \epsilon_{xz} = \frac{1}{2}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

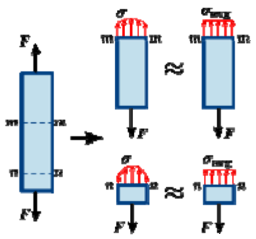
$$\epsilon_z = \frac{\partial w}{\partial z}, \qquad \epsilon_{yz} = \frac{1}{2}\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

- Def.: $\epsilon := \frac{\Delta L}{L}$ - general: (Linear!)

(elongations - rotations)

Basic continuum mechanics

- **Stresses** (NOT measurable):



Important – the stress depends on the point (position) AND the orientation of cut-surface.

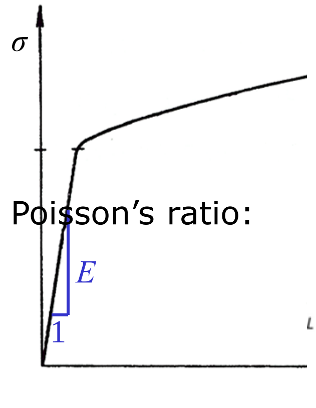
- Def.: $\sigma_{avg} := \frac{F}{A}$ or $\sigma = \lim_{A \to 0} \frac{F}{A}$

- General stress state: (similar to strains)

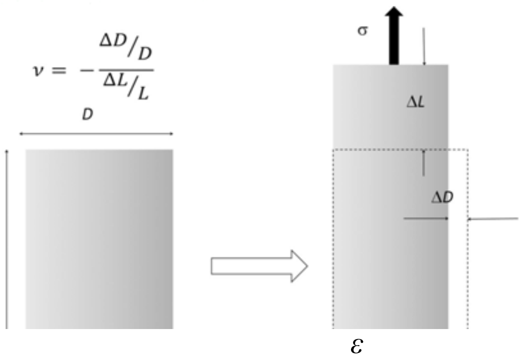
$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_x & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_y & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_z \end{bmatrix}$$

Basic continuum mechanics

- Hooke's law – linear, isotropic materials:
  Just two independent material parameters
- Stiffness: $\sigma = E\epsilon$ (E in [Pa])

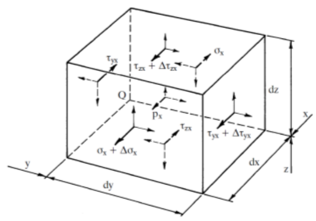


- Poisson's ratio:



Basic continuum mechanics and FEM

Governing equations (using Newton's 2nd law)



The linear system of partial differential equations:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} + p_x = 0$$

$$\frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} + p_y = 0$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_z}{\partial z} + p_z = 0$$

or

$$(\lambda + \mu)\nabla(\nabla \cdot \boldsymbol{u}) + \mu^2 \nabla^2 \boldsymbol{u} + \boldsymbol{p} = \boldsymbol{0}$$

$$\lambda = \frac{E\nu}{(1+\nu)(1-2\nu)}$$

$$\mu = \frac{E}{2(1+\nu)}$$

Constitutive parameters and TopOpt

- Essential since it allows us to interpolate, e.g. stiffness, density, conductivity, …

$$E(\rho) = E_{\min} + \rho^p (E_{\max} - E_{\min})$$

Different problems need different interpolations

- Principle of virtual work

$$\int_\Omega \delta\boldsymbol{\epsilon}^T \boldsymbol{E}(\rho)\boldsymbol{\epsilon}\mathrm{d}\Omega - \int_\Omega \delta\boldsymbol{u}^T \boldsymbol{P}\mathrm{d}\Omega + \int_{\Gamma_T} \delta\boldsymbol{u}^T \boldsymbol{t}\mathrm{d}\Gamma_T = 0$$

- The finite element method (FEM)

$$\boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{U} = \boldsymbol{F}$$

Important mechanical quantities

- The von Mises stress (or equivalent tensile stress):

$\sigma_{vM} = \sqrt{3J_2}$ or

$$\sigma_{vM}^2 = \frac{1}{2}\left[(\sigma_x - \sigma_y)^2 + (\sigma_y - \sigma_z)^2 + (\sigma_z - \sigma_x)^2 + 6(\sigma_{xy}^2 + \sigma_{xz}^2 + \sigma_{yz}^2)\right]$$

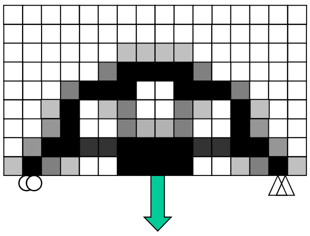
- The strain energy and compliance:

$U = \frac{1}{2}\int_\Omega \boldsymbol{\sigma}^T \boldsymbol{\epsilon}\mathrm{d}\Omega$ and $C = \boldsymbol{u}^T\boldsymbol{F} = \boldsymbol{u}^T\boldsymbol{K}\boldsymbol{u}$

- Stiffness vs compliance: $E = \frac{\partial \sigma}{\partial \epsilon}$ vs $C = \frac{\partial \epsilon}{\partial \sigma}$

Discretized SIMP-approach

Bendsøe (1989), Zhou and Rozvany (1991), Mlejnek (1992)



$$\min_{\boldsymbol{\rho}} : \quad \Phi(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho}))$$

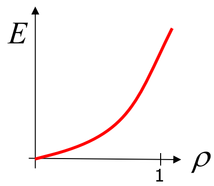
$$s.t. : \quad \sum_{e=1}^{N} v_e \rho_e = \mathbf{v}^T \boldsymbol{\rho} \leq V^*$$

$$: \quad g_i(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho})) \leq g_i^*, \quad i = 1, ..., M$$

$$: \quad \mathbf{0} \leq \boldsymbol{\rho} \leq \mathbf{1}$$

$$(: \quad \mathbf{K}(\boldsymbol{\rho})\mathbf{U} = \mathbf{F} \ )$$

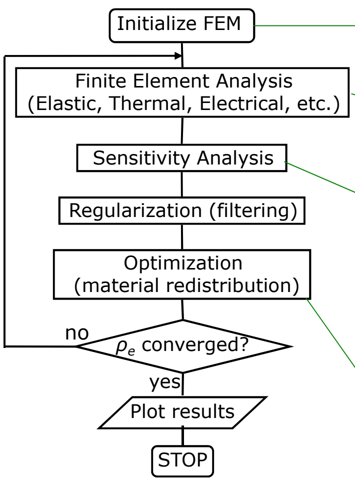
Stiffness interpolation:

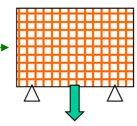


$$E(\rho_e) = E_1 + \rho_e^p (E_2 - E_1)$$

$$p > 1$$

The Topology Optimization Process





$\boldsymbol{KU} = \boldsymbol{F}$ or $\boldsymbol{R}(\boldsymbol{U}) = 0$

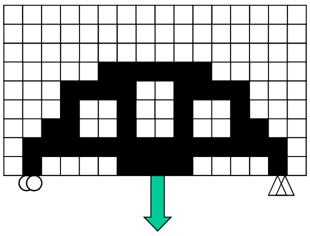
Sensitivity analysis by adjoint method

$$\frac{d\Phi}{d\rho_e} = \frac{\partial \Phi}{\partial \rho_e} + \boldsymbol{\lambda}^T \left(\frac{\partial \boldsymbol{K}}{\partial \rho_e} \boldsymbol{U} + \frac{\partial \boldsymbol{F}}{\partial \rho_e} \right)$$

$$\boldsymbol{K}^T \boldsymbol{\lambda} = -\frac{\partial \Phi}{\partial \boldsymbol{U}}$$

Mathematical Programming,
Method of Moving Asymptotes (MMA)
by Svanberg (1987)

Why gradient based methods ?



$$\min_{\boldsymbol{\rho}} : \quad \Phi(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho}))$$
$$s.t. : \quad \sum_{e=1}^{N} v_e \rho_e = \mathbf{v}^T \boldsymbol{\rho} \leq V^*$$
$$: \quad g_i(\boldsymbol{\rho}, \mathbf{U}(\boldsymbol{\rho})) \leq g_i^*, \quad i = 1, ..., M$$
$$: \quad \rho_e = \begin{cases} 0 \text{ (void)} \\ 1 \text{ (material)} \end{cases}, \quad e = 1, ..., N$$
$$: \quad \mathbf{K}(\boldsymbol{\rho})\mathbf{U} = \mathbf{F}$$

0/1 Integer problem

- Combinations: $\frac{N!}{(N-M)!M!}$

N=10, M=5 => 252
N=20, M=10 => 185.000
N=40, M=20 => $1.4 \cdot 10^9$
N=100, M=50 => 10^{29}

Adjoint method for sensitivities - discrete

- A general function and a general residual:

$$\Phi = \Phi(\boldsymbol{\rho}, \boldsymbol{u}(\boldsymbol{\rho})), \quad \boldsymbol{R}(\boldsymbol{\rho}, \boldsymbol{u}(\boldsymbol{\rho})) = \mathbf{0}$$

- Step 1: differentiate using the chainrule

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\rho_e} = \frac{\partial \Phi}{\partial \rho_e} + \frac{\partial \Phi}{\partial \boldsymbol{u}} \frac{\partial \boldsymbol{u}}{\partial \rho_e} \qquad \frac{\mathrm{d}\boldsymbol{R}}{\mathrm{d}\rho_e} = \frac{\partial \boldsymbol{R}}{\partial \rho_e} + \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}} \frac{\partial \boldsymbol{u}}{\partial \rho_e} = \mathbf{0}$$

- Problem term – must be eliminated!

- Use the residual eqs.: $\frac{\partial \boldsymbol{u}}{\partial \rho_e} = -\left(\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}}\right)^{-1} \frac{\partial \boldsymbol{R}}{\partial \rho_e}$

Adjoint method for sensitivities - discrete

- Step 2: Insert trouble term into derivative

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\rho_e} = \frac{\partial \Phi}{\partial \rho_e} + \underbrace{\frac{\partial \Phi}{\partial \boldsymbol{u}}\left(-\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}}\right)^{-1}}_{\boldsymbol{\lambda}^T} \frac{\partial \boldsymbol{R}}{\partial \rho_e}$$

- Step 3: Adjoint problem

$$\boldsymbol{\lambda}^T = -\frac{\partial \Phi}{\partial \boldsymbol{u}}\left(\frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}}\right)^{-1} \Rightarrow \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}}^T \boldsymbol{\lambda} = -\frac{\partial \Phi}{\partial \boldsymbol{u}}$$

- Final sensitivity

$$\frac{\mathrm{d}\Phi}{\mathrm{d}\rho_e} = \frac{\partial \Phi}{\partial \rho_e} + \boldsymbol{\lambda}^T \frac{\partial \boldsymbol{R}}{\partial \rho_e}$$

Adjoint method for sensitivities - discrete

- Example problem – Linear compliance

$$\Phi = \boldsymbol{F}^T \boldsymbol{u} = \boldsymbol{u}^T \boldsymbol{K} \boldsymbol{u}, \quad \boldsymbol{R} = \boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{u} - \boldsymbol{F} = \boldsymbol{0}$$

- The 4 required terms become

$$\frac{\partial \Phi}{\partial \rho_e} = \boldsymbol{u}^T \frac{\partial \boldsymbol{K}}{\partial \rho_e} \boldsymbol{u} \qquad \frac{\partial \Phi}{\partial \boldsymbol{u}} = 2\boldsymbol{F}$$

$$\frac{\partial \boldsymbol{R}}{\partial \rho_e} = \frac{\partial \boldsymbol{K}}{\partial \rho_e} \boldsymbol{u} \qquad \frac{\partial \boldsymbol{R}}{\partial \boldsymbol{u}} = \boldsymbol{K} = \boldsymbol{K}^T$$

- The adjoint becomes (so-called self-adjoint!):

$$\boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{\lambda} = -2\boldsymbol{F} \Rightarrow \boldsymbol{\lambda} = -2\boldsymbol{u}$$

Adjoint method for sensitivities - discrete

- Example problem – Linear compliance

$$\Phi = \boldsymbol{F}^T \boldsymbol{u} = \boldsymbol{u}^T \boldsymbol{K} \boldsymbol{u}, \quad \boldsymbol{R} = \boldsymbol{K}(\boldsymbol{\rho})\boldsymbol{u} - \boldsymbol{F} = \boldsymbol{0}$$

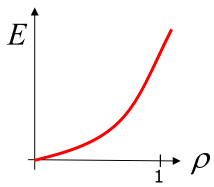
- The sensitivity now reads

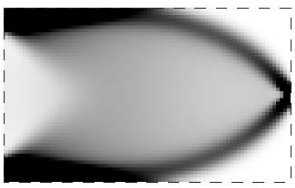
$$\frac{\mathrm{d}\Phi}{\mathrm{d}\rho_e} = \boldsymbol{u}^T \frac{\partial \boldsymbol{K}}{\partial \rho_e} \boldsymbol{u} - 2\boldsymbol{u}^T \frac{\partial \boldsymbol{K}}{\partial \rho_e} \boldsymbol{u} = -\boldsymbol{u}^T \frac{\partial \boldsymbol{K}}{\partial \rho_e} \boldsymbol{u}$$

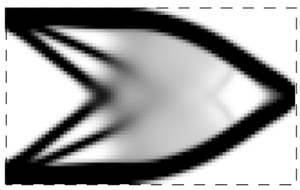
with: $\frac{\partial \boldsymbol{K}}{\partial \rho_e} = p\rho^{p-1}(E_{\max} - E_{\min})\boldsymbol{K}_0$

- Note: this is a negative scaled strain energy

SIMP (Simplified Isotropic Material with Penalization)

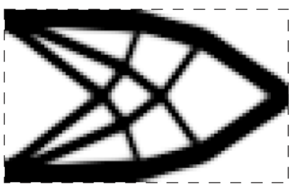


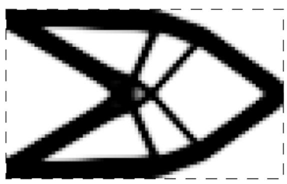

Voigt (p=1)


p=1.5

$$E(\rho_e) = \rho_e^p E_0$$

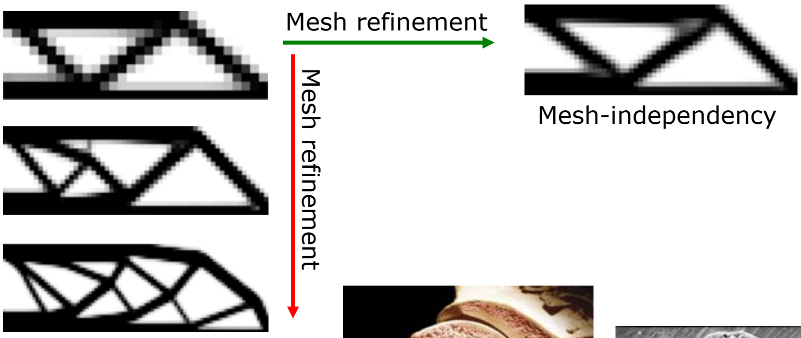
$$p \geq 1$$

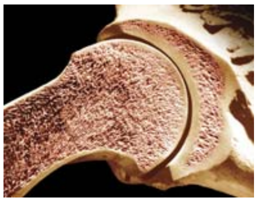

p=2

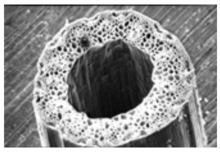

p=3

Physical motivation for SIMP in Bendsøe and Sigmund, *AAM,* **1999**, *69*, 635-654

Mesh-dependence



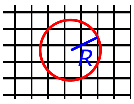




Regularization by sensitivity filtering

Neighborhood:

$$N_e = \{ i \mid ||\mathbf{x}_i - \mathbf{x}_e|| \leq R \}$$



Checkerboards

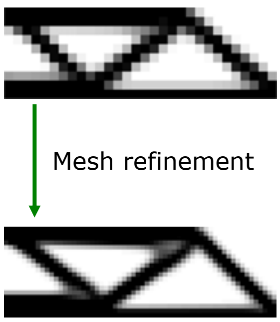
Density filtering: (Bruns/Bourdin 2001)

$$E_e(\boldsymbol{\rho}) = \tilde{\rho}_e^p E_0, \quad \tilde{\rho}_e = \frac{\sum_{i \in N_e} H(\mathbf{x}_i)\rho_i}{\sum_{i \in N_e} H(\mathbf{x}_i)}$$

PDE-based filtering: (Lazarov&Sigmund, 2011)

$$\hat{w} - r^2 \hat{w}_{,mm} = \overline{w}$$

$$\overline{w} = \rho \frac{\partial \Phi}{\partial \rho} \; (= -p\rho^p C^0_{ijkl}\varepsilon_{ij}\varepsilon_{kl} = -SED)$$



Alternative regularizations

Tikhonov / phase-field regularization

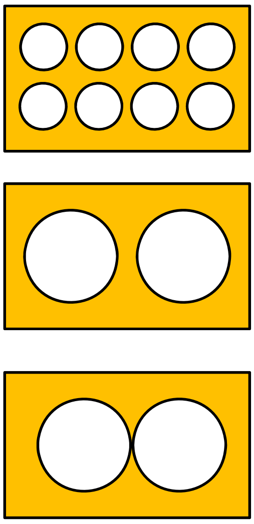
$$\tilde{\Phi}(\boldsymbol{\rho}) = \Phi(\boldsymbol{\rho}) + \int_\Omega \left(\frac{1}{\varepsilon}\rho(1-\rho) + \varepsilon||\nabla\rho||^2\right) dV$$

Niels Aage, Mechanical Engineering, Solid Mechanics Technical University of Denmark

Global regularization schemes

Perimeter control

$$TV = \int_\Omega ||\nabla\rho|| d\Omega \leq P^*$$



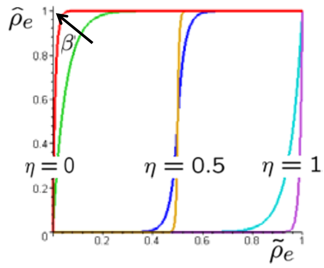
Niels Aage, Mechanical Engineering, Solid Mechanics Technical University of Denmark

Heaviside projection methods

$\boldsymbol{\rho} \rightarrow \tilde{\boldsymbol{\rho}}(\boldsymbol{\rho}) \rightarrow \hat{\boldsymbol{\rho}}(\tilde{\boldsymbol{\rho}}(\boldsymbol{\rho}))$

Design variables — Density filter — Projection

$$\hat{\rho} = \frac{\tanh(\beta\eta) + \tanh(\beta(\tilde{\rho} - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1-\eta))}$$



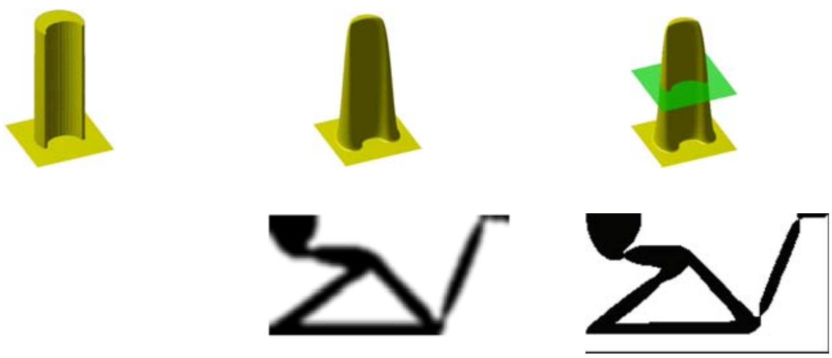
Projection method Guest *et al* (2004)

Design variables — Density filtering — Projection

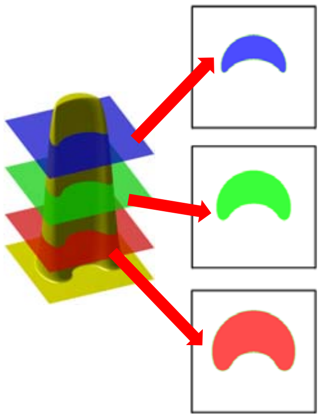
$\boldsymbol{\rho} \rightarrow \tilde{\boldsymbol{\rho}}(\boldsymbol{\rho}) \rightarrow \hat{\boldsymbol{\rho}}(\tilde{\boldsymbol{\rho}}(\boldsymbol{\rho}))$

$-r^2\Delta\tilde{\rho} + \tilde{\rho} = \rho$

$\hat{\rho} = \frac{\tanh(\beta\eta) + \tanh(\beta(\tilde{\rho} - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1-\eta))}$



”Robust” design formulation



$$\min_{\rho} : \max \quad \left(\mathbf{f}(\bar{\tilde{\rho}}^{e}(\rho)), \mathbf{f}(\bar{\tilde{\boldsymbol{\rho}}}^{i}(\rho)), \mathbf{f}(\bar{\tilde{\boldsymbol{\rho}}}^{d}(\rho))\right)$$

$$s.t. : \mathbf{K}(\bar{\tilde{\rho}}^{e})\mathbf{u}^{e} = \mathbf{f}$$

$$: \mathbf{K}(\bar{\tilde{\rho}}^{i})\mathbf{u}^{i} = \mathbf{f}$$

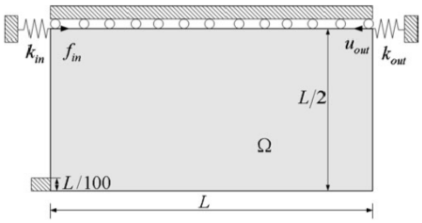
$$: \mathbf{K}(\bar{\tilde{\rho}}^{d})\mathbf{u}^{d} = \mathbf{f}$$

$$: f_v(\rho) = \frac{\sum_i \bar{\tilde{\rho}}_i^{d} v_i}{V} \leq V_d^*$$

$$: 0 \leq \rho \leq 1$$

”Robust” design formulation

- Force inverter – hinges in standard formulation



- Robust formulation - no hinges☺

(d) Eroded design (e) Intermediate design (f) Dilated design

Weapon of choice in TopOpt - MMA

The Method of Moving Asymptotes (Svanberg 1987).

- Problem you want to solve / Problem that MMA solves

$$\begin{aligned} \min_{x\in\mathbb{R}^n} \quad & g_0(x) \\ \text{s.t.} \quad & g_i(x) \le 0, \quad i = 1, m \\ & x_{\min} \le x_j \le x_{\max}, \quad j = 1, n \end{aligned}$$

$$\begin{aligned} \min_{x\in\mathbb{R}^n, y\in\mathbb{R}^m, z\in\mathbb{R}} \quad & f_0(x) + z + \tfrac{1}{2}z^2 + \sum_{i=1}^{m}\left(y_i c_i + \tfrac{1}{2}y_i^2\right) \\ \text{s.t.} \quad & f_i(x) - a_i z - y_i \le 0, \quad i = 1, m \\ & \alpha_j \le x_j \le \beta_j, \quad j = 1, n \\ & y_i \ge 0, \quad i = 1, m \\ & z \ge 0 \end{aligned}$$

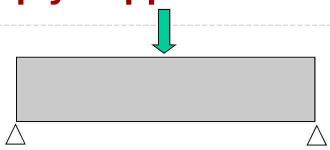
- Using first order convex seperable approximations:

$$f_i(x) = \sum_{j=1}^{n}\left(\frac{p_{ij}}{U_j - x_j} + \frac{q_{ij}}{x_j - L_j}\right) + r_i$$

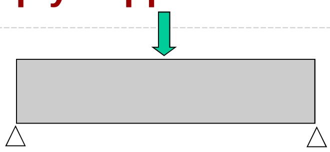
Understanding the principles of TopOpt

Influence of number of load cases
and boundary conditions

TopOpt for a simply supported beam

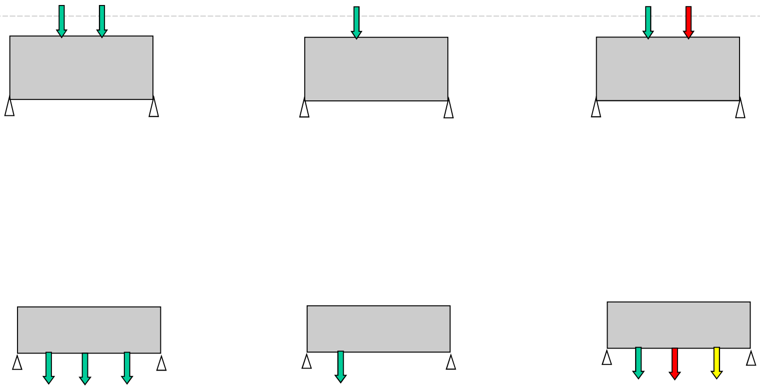


TopOpt for a simply supported beam



Niels Aage, Mechanical Engineering, Solid Mechanics Technical University of Denmark

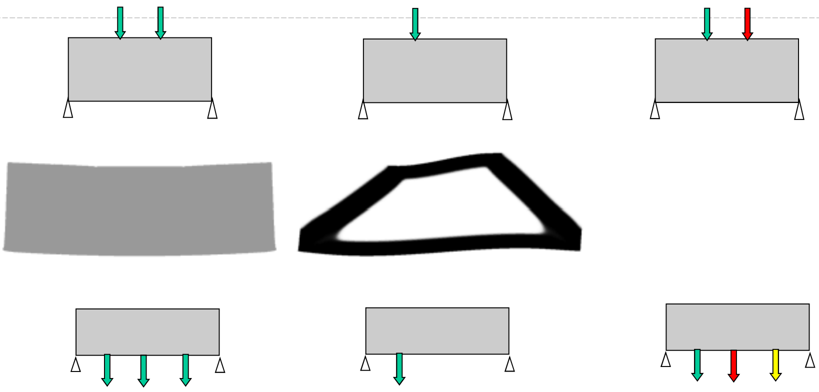
One or more load cases?



Niels Aage, Mechanical Engineering, Solid Mechanics

Technical University of Denmark

One or more load cases?



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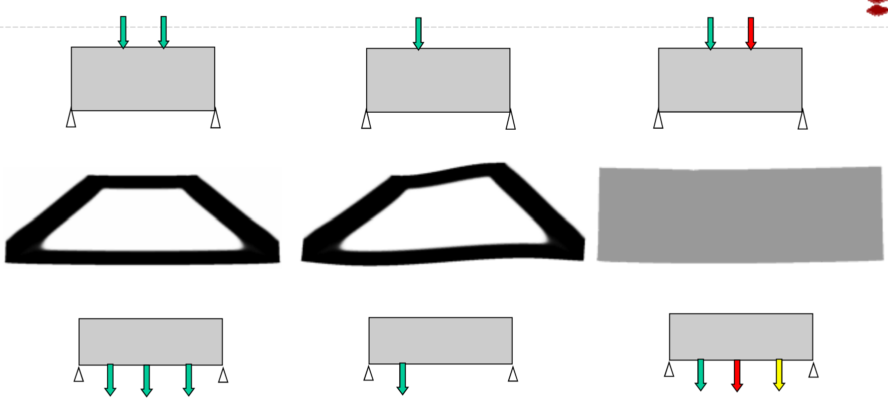
Technical University of Denmark

One or more load cases?



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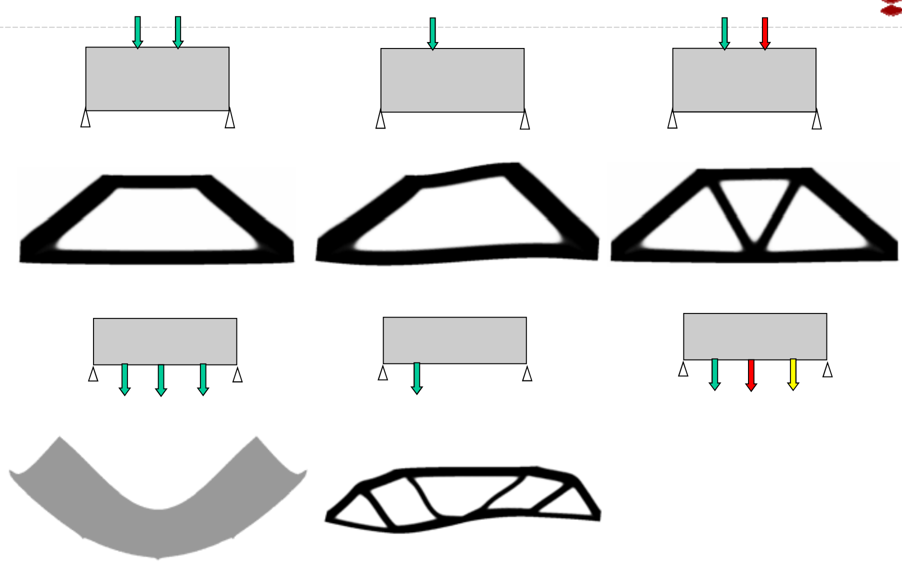
One or more load cases?



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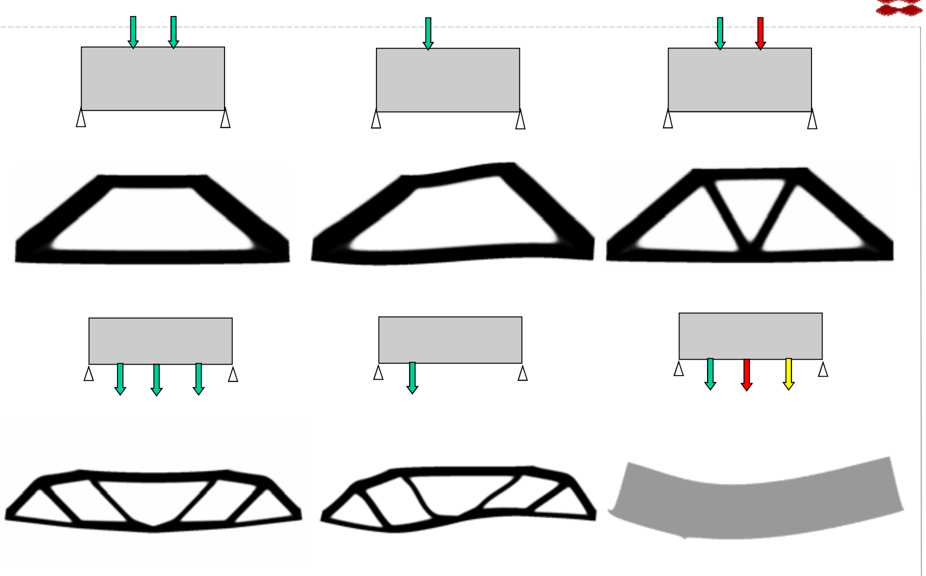
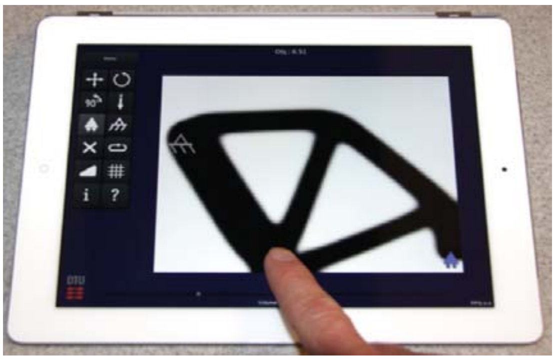
One or more load cases?



The ”TopOpt App”

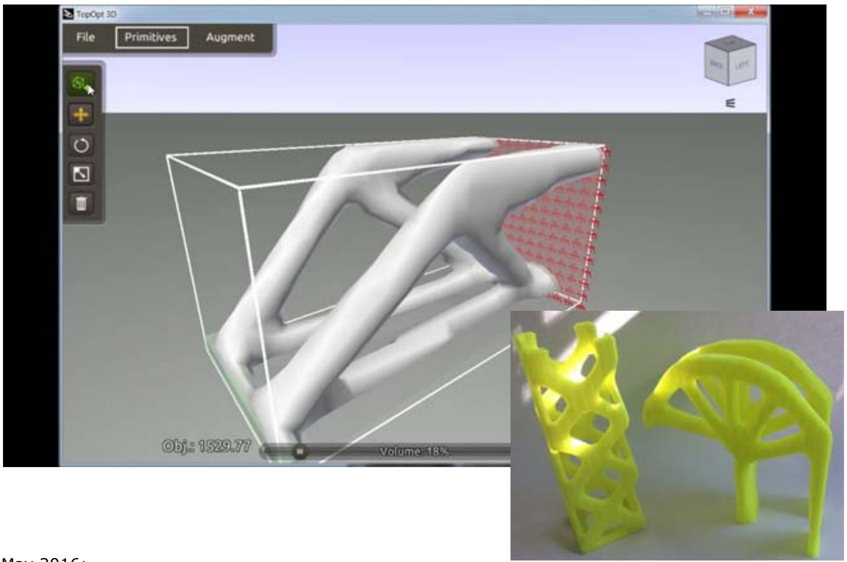


The ”TopOpt App”: AppStore (iOS)
Google Play (Android)
Web-version: www.topopt.dtu.dk

Stats: May 2016:
Android: 5380, iOS: 9450

See **www.topopt.dtu.dk** for more

TopOpt3D App



Stats: May 2016:
iOS: 4100, web: 1500

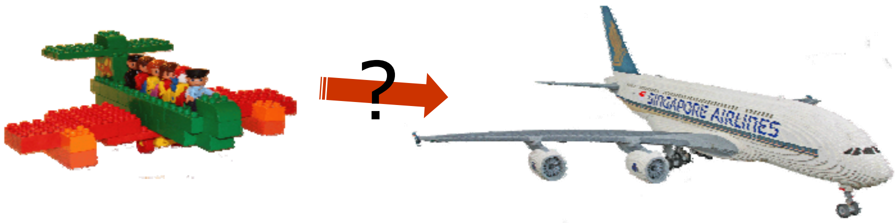
(NB! Only iOS, OSX and PC – see **www.topopt.dtu.dk**)

www.topopt.dtu.dk

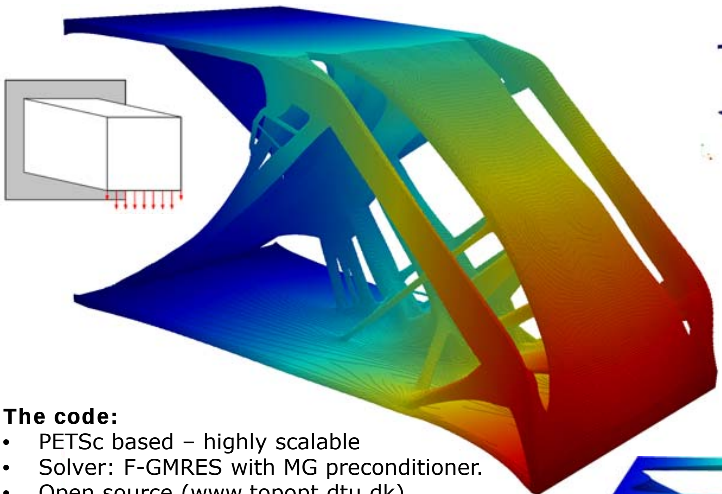
Code refs and image of topopt site

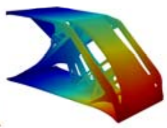
High resolution TopOpt

(overcoming the Duplo problem)



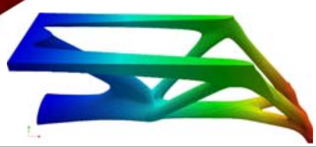
+100M design variables





The code:

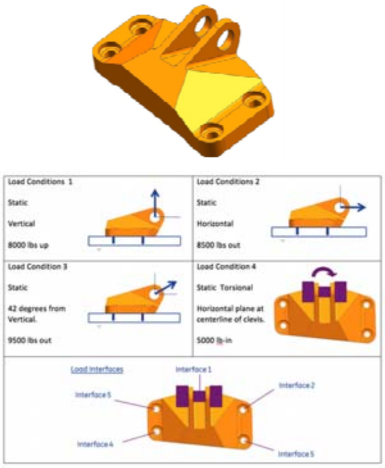
- PETSc based – highly scalable
- Solver: F-GMRES with MG preconditioner.
- Open source (www.topopt.dtu.dk)
- Include: Filters, MMA, IO.
- Comes with minimum compliance example.
- Aage; Andreassen & Lazarov, **2015,** *SMO.*



GrabCAD Challenge 2013 (640 entries)

Minimize weight of additive manufactured jet engine bracket

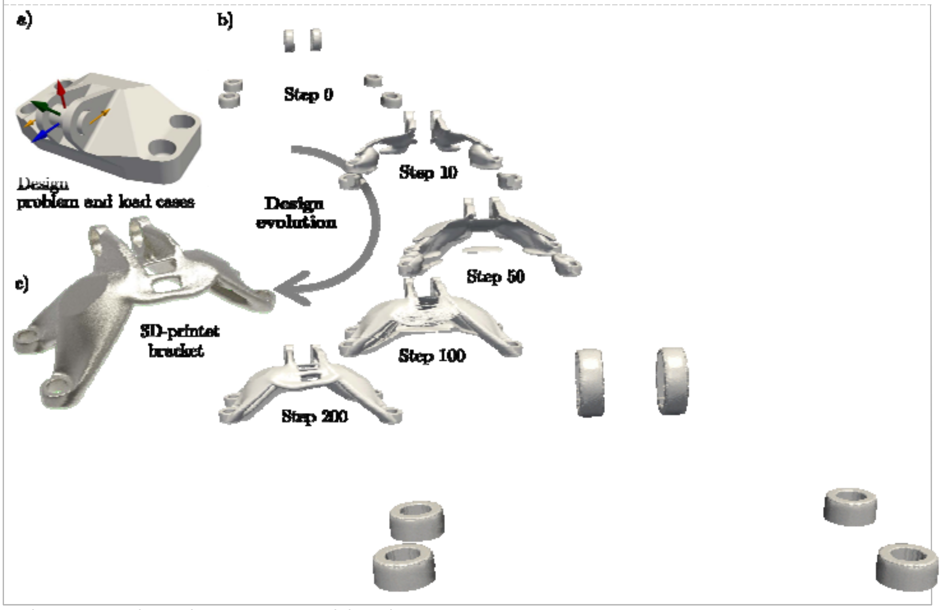
Design problem



Winner – 340 g
16 % volume fraction

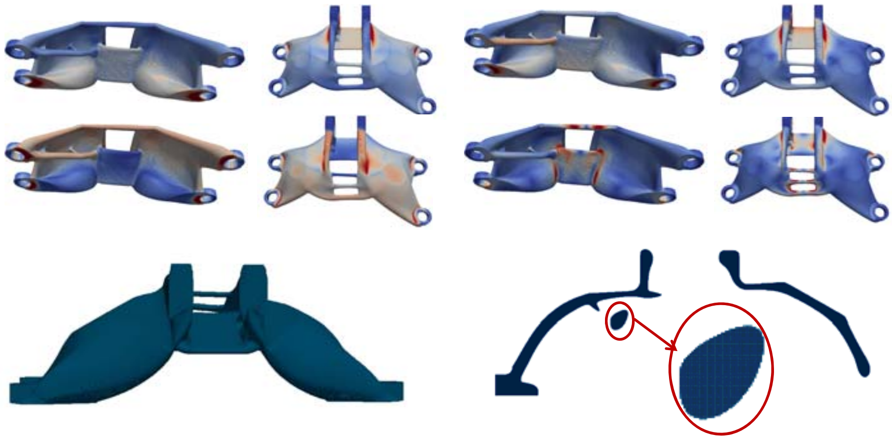
From: GrabCAD.com,
by M. Kurniawan

Design history

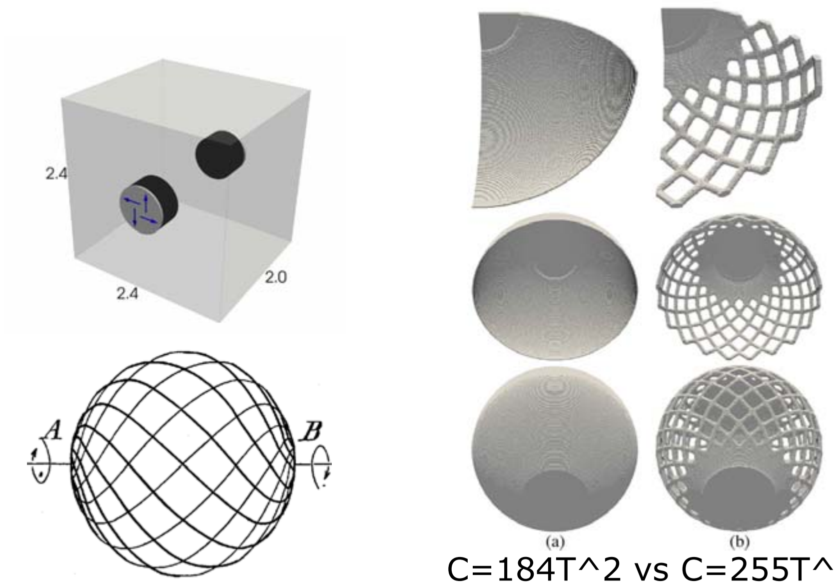


Optimized bracket

- 35M cubic elements (size 0.6mm)
- Result obtained in approximately 12,000 CPU hours
- Target weight 300 g (10% lighter than challenge winner)
- Max. von Mises stress around 700 MPa (yield stress >900 MPa)



Rediscovering optimality - Michell



C=184T^2 vs C=255T^2

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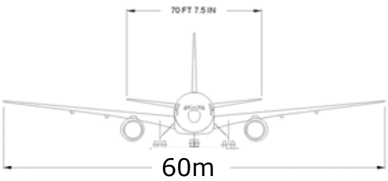
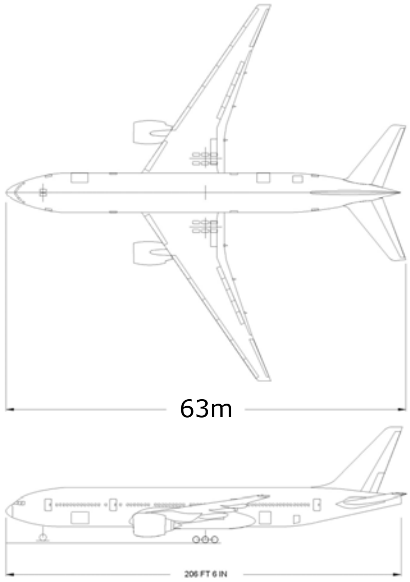
Boing 777 dimensions





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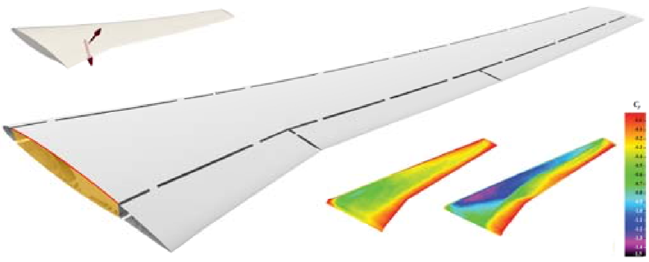
NASA Common Research Model

Geometry and pressure load data from NASA:

Discritized including supports and loads



Mesh with ~**1.1 billion** elements (1216 x 256 x 3456)…
… largest element side **0.8 cm** (wing is ~ 26.5m x 11.5m x 2m)

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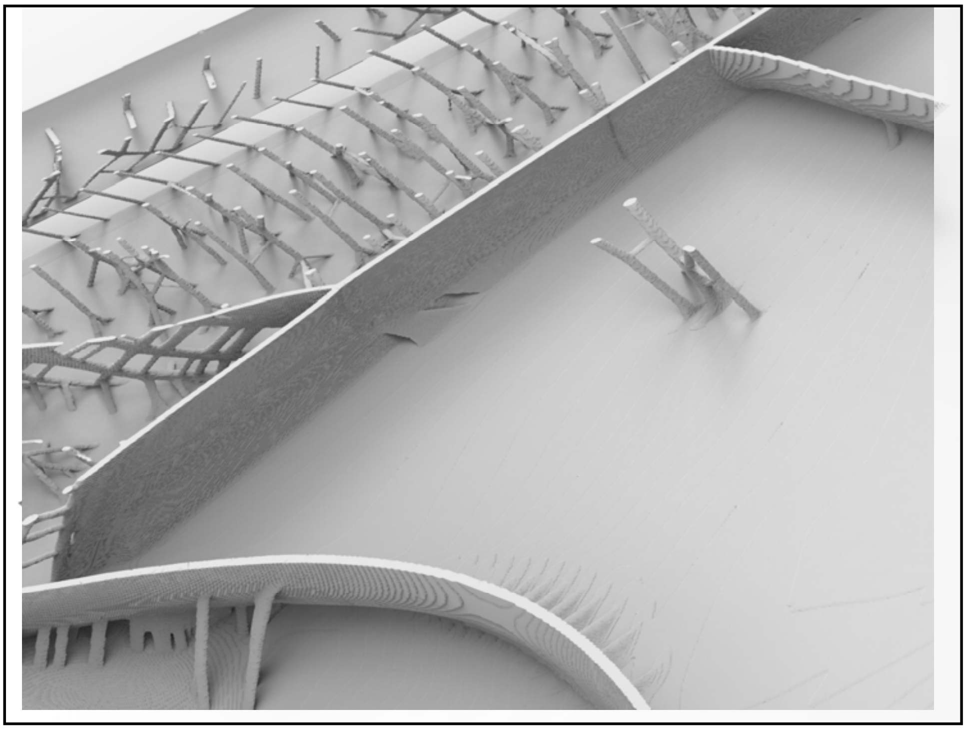
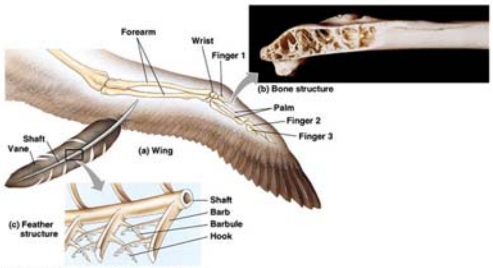
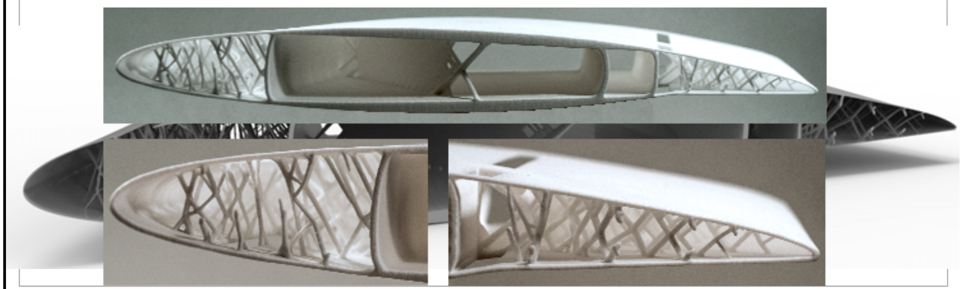


Mimics nature

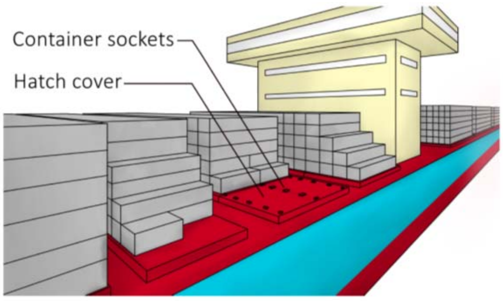
Copyright Natural History Museum, London, UK.





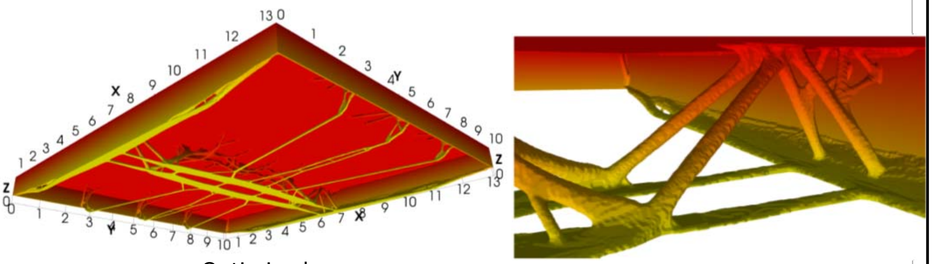
Designing containership components

Study with Mærsk Line with the goal to reduce costs.

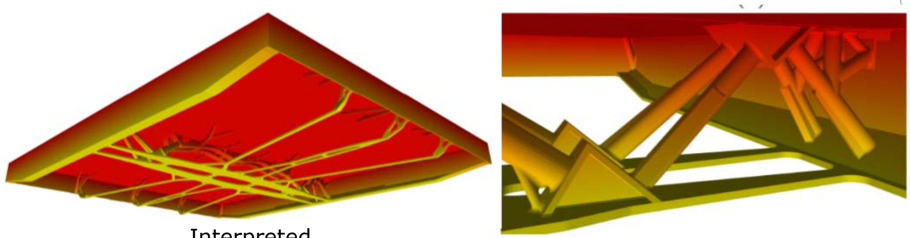


Designing containership components

Parameterizing the optimized design (manually!)



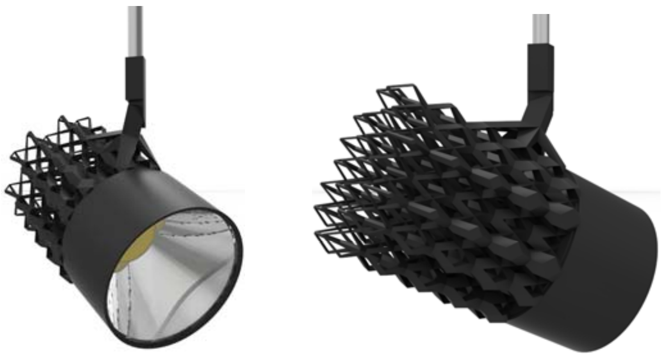
Optimized



Interpreted

Cooling fins for LED lamps

HYPERCOOL – Cool Danish Design



Innovationsfonden

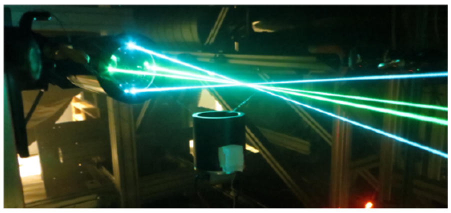
DET FRIE FORSKNINGSRÅD
DANISH COUNCIL FOR INDEPENDENT RESEARCH

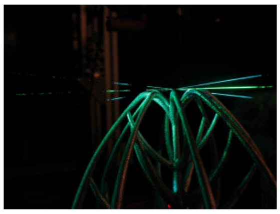
AT • LIGHTING

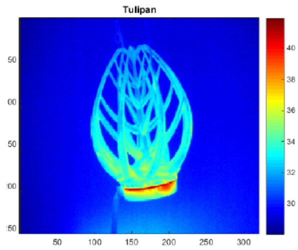
APIOSOFT

DTU Mechanical Engineering
Department of Mechanical Engineering

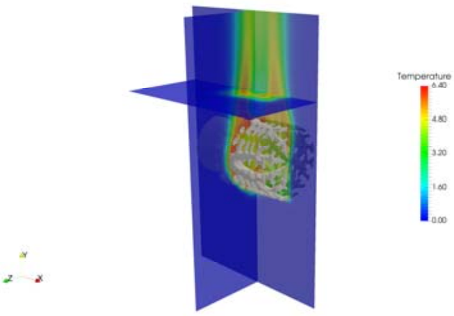
Coolers for LEDs: HyperCool

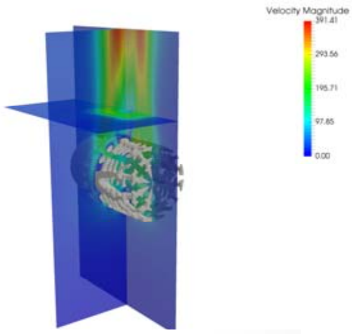




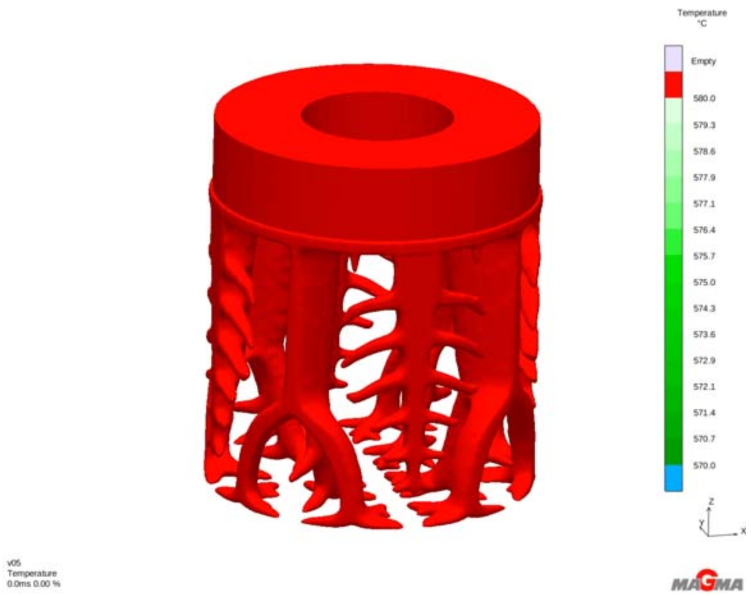


Coolers for LEDs: HyperCool





Optimal casting?



Integration with AM and design of ”shell structures”

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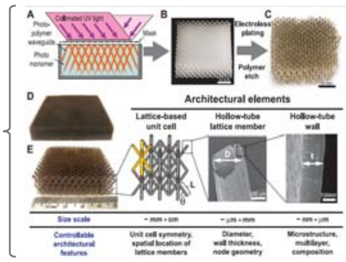
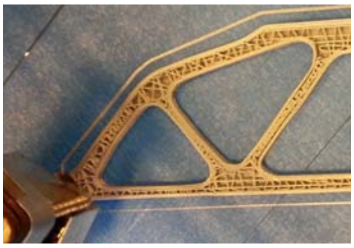
Technical University of Denmark

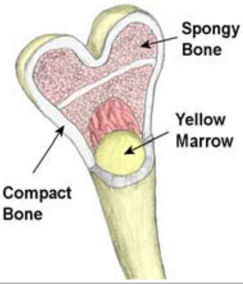
Coating and stiff interface structures



Schaedler et al., Science 334 (6058): 962-965, **2011**



Infill printed by FDM

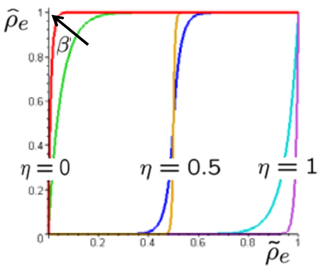


Repeated filtering and projection

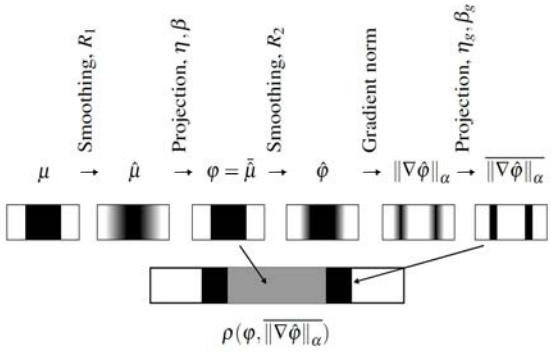
$\rho \rightarrow \tilde{\rho}(\rho) \rightarrow \hat{\rho}(\tilde{\rho}(\rho))$

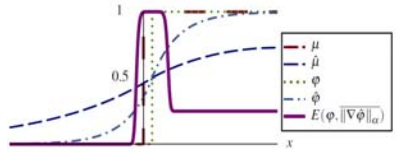
Design variables → Density filter → Projection

$$\hat{\rho} = \frac{\tanh(\beta\eta) + \tanh(\beta(\tilde{\rho} - \eta))}{\tanh(\beta\eta) + \tanh(\beta(1 - \eta))}$$



Material interpolation model

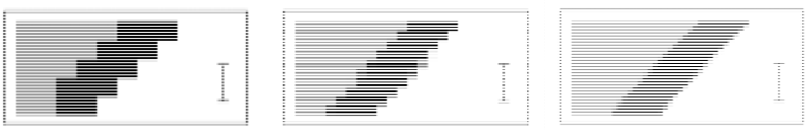




$$E(\varphi, \overline{\|\nabla\hat{\varphi}\|_\alpha}) = E^0 \left[\lambda_E \varphi^p + (1 - \lambda_E \varphi^p)(\overline{\|\nabla\hat{\varphi}\|_\alpha})^p\right]$$

Results and convergence

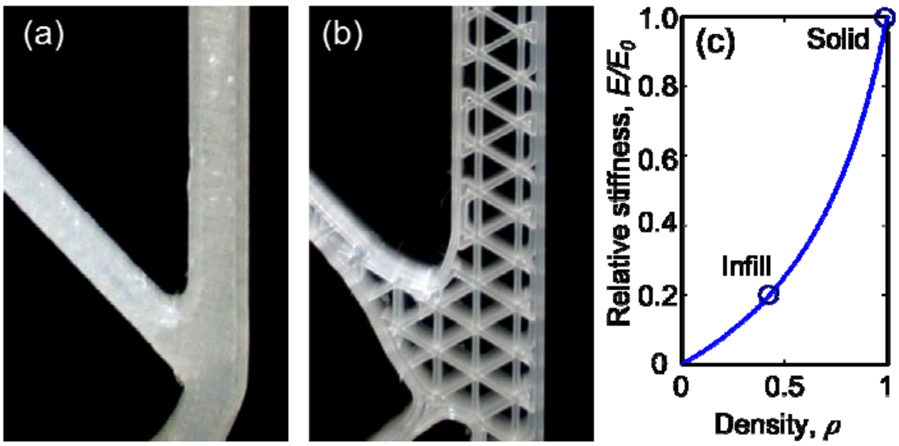
150x50 elements 300x100 elements 600x200 elements



$R_2 = 2.5$ $R_2 = 5.0$ $R_2 = 7.5$

Clausen at al., *CMAME*, **2015**, *290*, 524-541

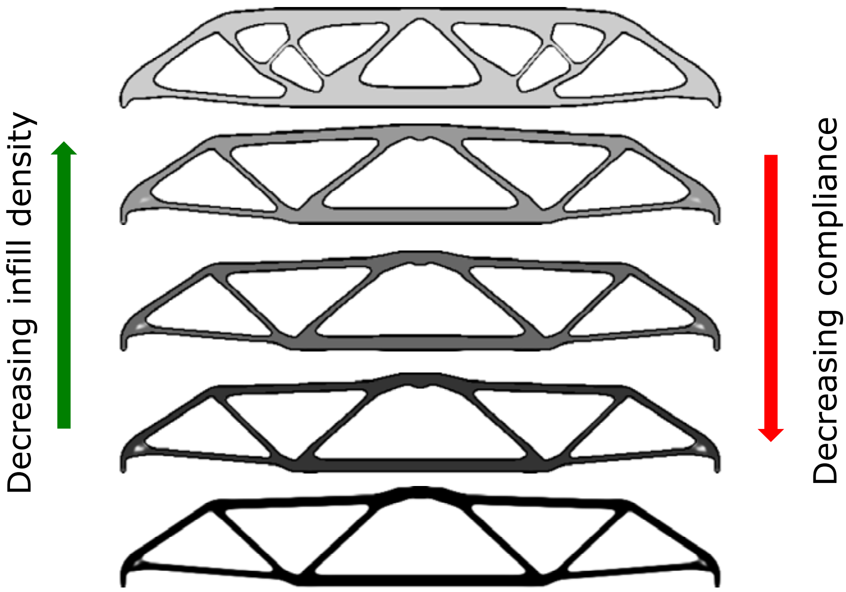
TopOpt formulation for coating and infill



Standard | Coating | Infill properties

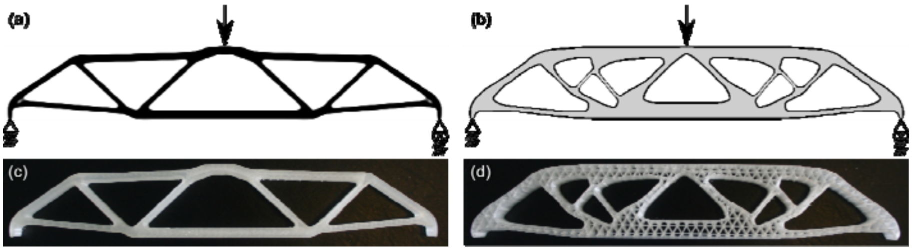
Clausen; Aage & *OS*, *CMAME*, **2015**, *290*, 524-541

Dependence on infill stiffness



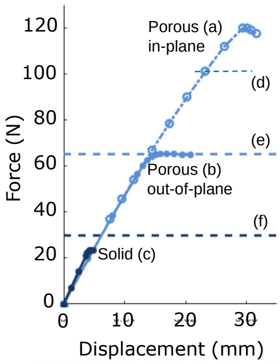
Mechanical tests on MBB beam

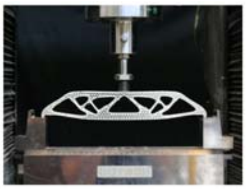
Print material: SEBS (Styrene-Etylene-Butylene-Styrene)

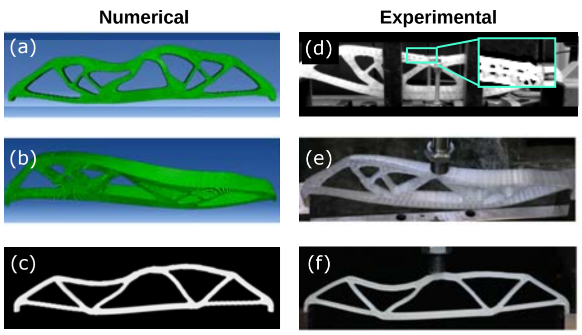


Solid (standard) **Porous (coating)**

Buckling load improved >5 times



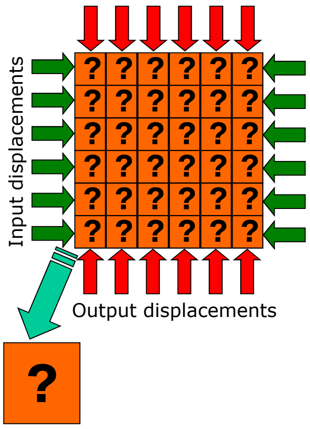


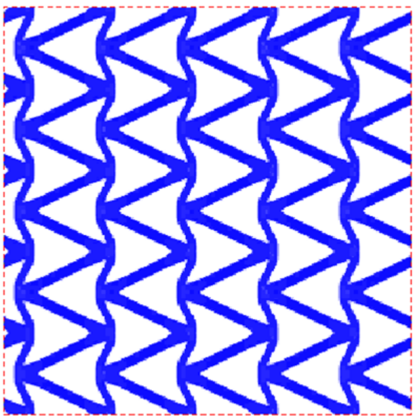


Clausen; Aage & Sigmund, **2016**, *Submitted*

Material design problems

Material with negative Poisson's ratio



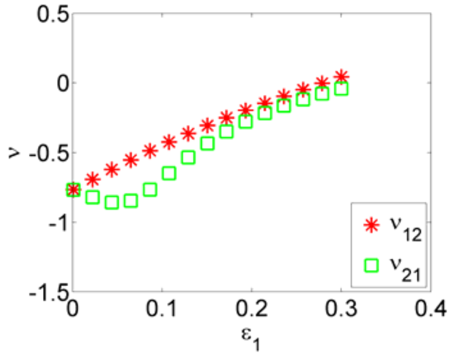


- FE on one cell with periodic B.C.
- Minimize Poisson's ratio
- Constraint on bulk modulus and symmetry

Sigmund (1995)

Non-linear material modelling

$\nu_{12} = -0.766$
$\nu_{21} = -0.770$

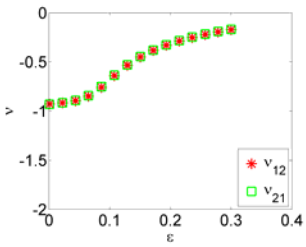


Wang et.al., *JMPS*, **2014**

Negative Poisson's ratio design

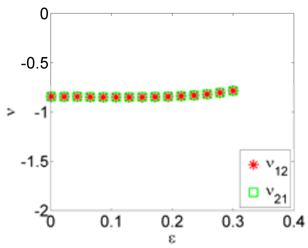
Linear case

$\nu_{12} = -0.931$
$\nu_{21} = -0.929$



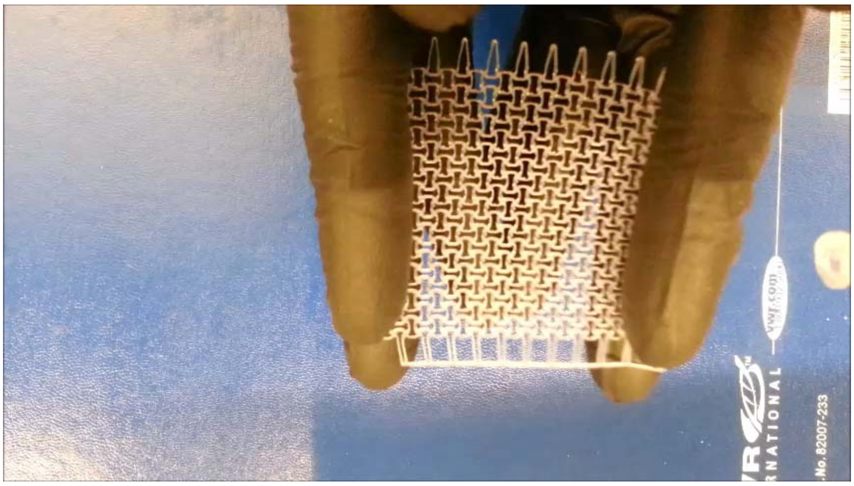
Nonlinear case

$\overline{\nu}_{12} = -0.838$
$\overline{\nu}_{21} = -0.838$



Wang et.al., *JMPS*, **2014**

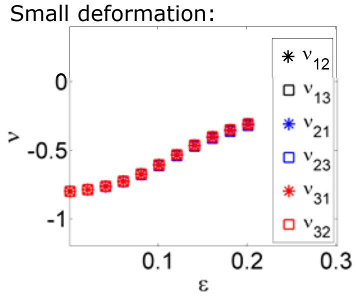
Experimental verifications

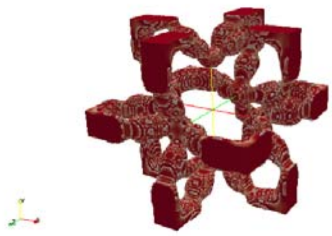


Clausen at al., *Advanced Materials*, **2015**, *27*, 5523-5527

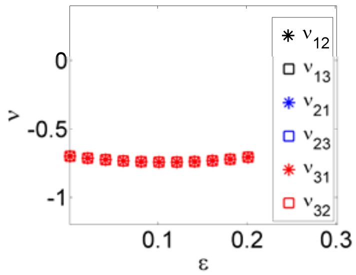
3D Poisson's ratio -0.8

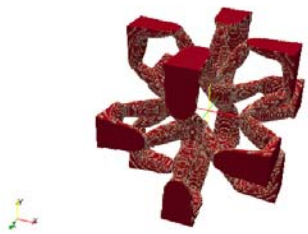
Small deformation:





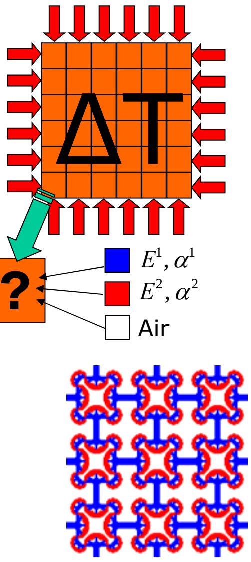
Finite deformation:





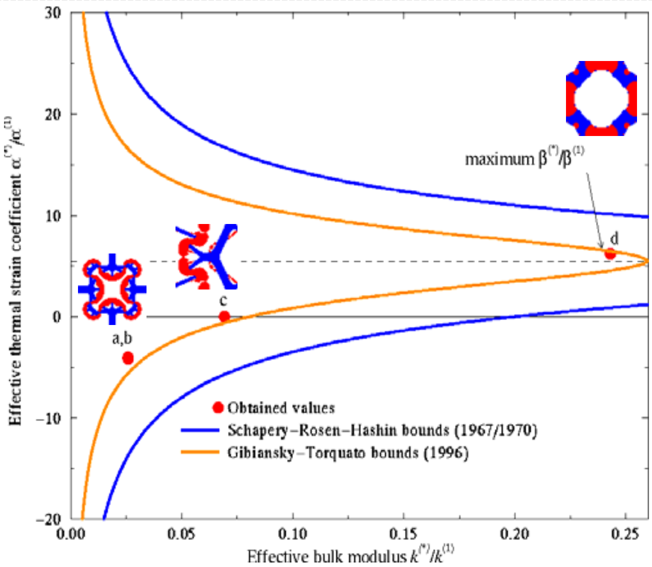
By Fengwen Wang

Negative thermal expansion coefficient



$\alpha^* = -4.02$

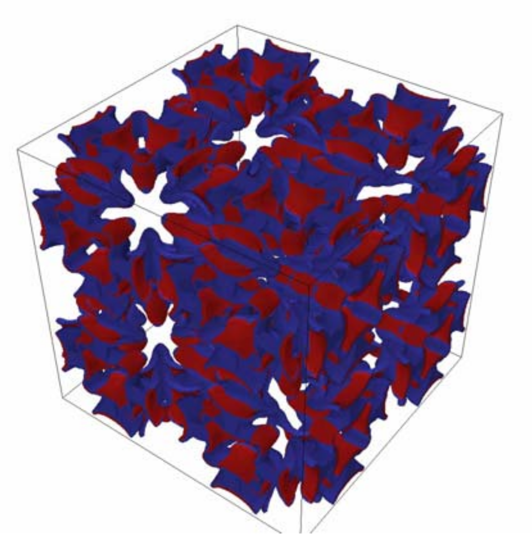
Comparisons with bounds for thermal expansion



Sigmund and Torquato, 1996/1997

Niels Aage, Mechanical Engineering, Solid Mechanics
Technical University of Denmark

3d negative thermal expansion



$$\alpha_{red} = 3.5$$
$$\alpha_{blue} = 1$$
$$E_{red} = 1$$
$$E_{blue} = 3.5$$

$$\nu^H = 0.18$$
$$E^H = 0.0016$$
$$\alpha^H = -5.4$$

Produced by Erik Andreassen

Papers and references

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