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Tutorial TH1: More than RGB: Spectral Trends in Color Reproduction

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More than RGB: Spectral Trends in Color Reproduction

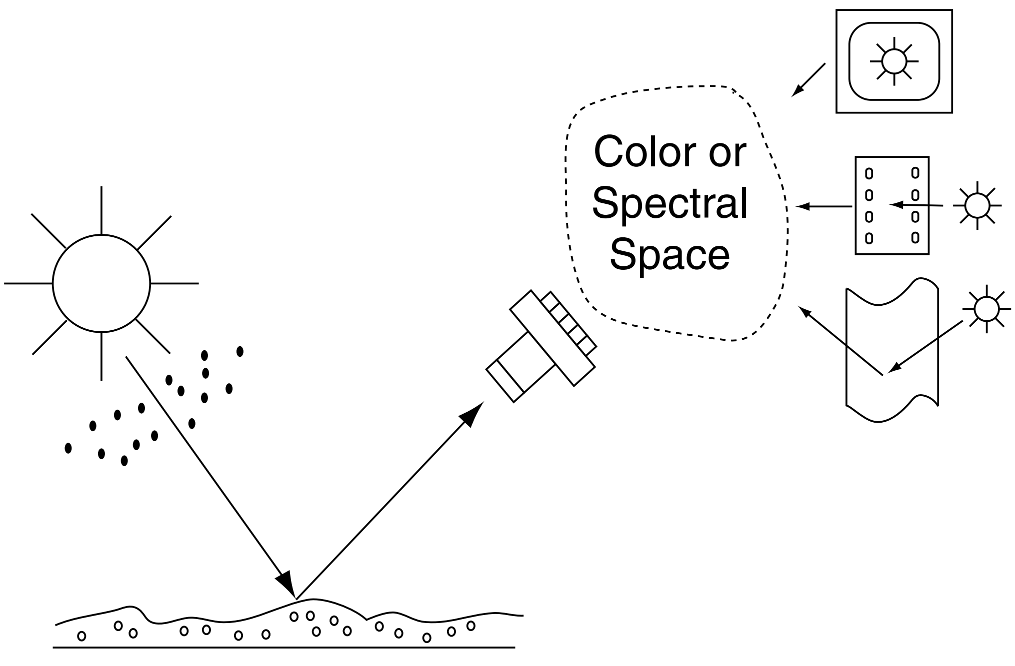
Ian E. Bell and Gladimir V. G. Baranoski

Abstract

Early rendering algorithms relied exclusively on three-dimensional spaces for color computation, such as RGB and CIE XYZ. Recent rendering advances use full spectral information for illuminants and surfaces, resulting in much greater accuracy and realism. These expensive computations can be wasted, however, if *ad hoc* methods are used to adjust the final image on the monitor, in film, or in print. Inefficiency and inaccuracy can be avoided with some knowledge of device gamuts and color reproduction algorithms. This course follows spectral data through the graphics pipeline, examining issues of rendering, color science, perception, gamut mapping, and color management. We conclude with a discussion of trends and open problems in managing spectral data for accurate color reproduction. Participants will learn not only the theoretical background of color and spectral reproduction, but practical guidelines often omitted in technical papers.

I. ILLUMINATION AND REFLECTION

1. Introduction



Multispectral Management

- spectral data from
  - measurement
  - models
  - simulation
- directed to output devices
  - monitor
  - printer
  - film recorder

Adapting Color Management

- conventional color management tools
  - device profiles
  - color management system
  - gamut mapping
- must adapt approach for high-dimensional spectral data
- follow spectral data through rendering pipeline

I. ILLUMINATION AND REFLECTION

1. Introduction
2. Illumination
   (a) Light as radiation
   (b) Spectral power distributions
   (c) Spectrophotometry
   (d) Additive color
3. Reflection and Transmission
   (a) Surface geometry
   (b) Bidirectional reflection distribution functions (BRDFs)
   (c) Illumination equations
   (d) Subtractive color
   (e) Linear models of spectra

II. SPECTRA AND PERCEPTION

1. Applications: Rendering with Spectral Data
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      ii. Aurorae and nebulae
   (b) Biologically-based rendering
      i. Plants
      ii. Human skin
   (c) Art and archival imaging
2. Perceptual Response and Color
   (a) Early color experiments
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   (e) Luminous efficiency and metamerism

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   (b) The virtual camera versus the digital camera
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   (d) Principal components analysis (PCA)
   (e) Linear reflectance spaces
2. Device Characterization and Gamuts
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   (b) Characterization by model
   (c) Device gamuts
   (d) Spectral gamuts

IV. GAMUT MAPPING AND COLOR MANAGEMENT

1. Gamut Mapping
   (a) Characterization by Measurement
      i. Look-up tables with interpolation
      ii. Sequential linear interpolation
      iii. Regression
      iv. Other methods
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      i. Black- and white-point mapping
      ii. Out-of-gamut projection
      iii. Gamut compression
      iv. Geometric methods
2. Challenges in Multispectral Management
   (a) Color management and ICC standards
   (b) Working with spectral and high-dimensional data
   (c) Open problems
3. Discussion

2. Illumination

Light as Radiation

- Dual nature of the light:
  - stream of particles (photons)
  - wave
- Studies on the nature of light based on:
  - wave optics
    - polarization, interference and diffraction phenomena
  - geometrical or ray optics
    - particle-based transport theory
  - quantum optics
    - photon considered as a small, physically localized wave packet

- Light processes:
  - reflection: process in which light at a specific wavelength incident on a material is propagated outward by the material without a change in wavelength
  - transmission: process in which light at a specific wavelength incident on the interface between materials passes through it without a change in wavelength
  - absorption: process by which the light incident on a material is converted to another form of energy, usually to heat
  - fluorescence: process by which light of one wavelength is reradiated, or propagated, at another (usually longer)

- Spectral Radiometric Quantities
  - Radiant energy: Q (J)
    - How much?
  - Radiant power or flux: $\Phi = \frac{dQ}{dt}$ (W)
    - What rate?
  - Radiant intensity: $I = \frac{d\Phi}{d\vec{\omega}}$ (W/sr)
    - What rate in what direction?
- We need a radiometric quantity that does not depend on the size of the object being viewed or on the distance to the viewer

- Radiance

$$L(x,\psi,\lambda) = \frac{dI(x,\psi,\lambda)}{dA\,cos\theta} = \frac{d^2\Phi(x,\psi,\lambda)}{d\vec{\omega}\,dA\,cos\theta} \qquad (\frac{W}{m^2\,sr})$$

where:

$dI(x,\psi,\lambda)$	=	radiant intensity at x and in a direction ψ,
$d\Phi(x,\psi,\lambda)$	=	radiant power at x and in a direction ψ,
θ	=	angle between the normal and the direction ψ,
dA	=	differential area surrounding x,
$d\vec{\omega}$	=	differential solid angle at which $d\Phi$ arrives at x.

- Measurement of appearance: group of measurements necessary to characterize both color and surface finish of an object
  - **Spectral** distribution of the propagated light:
    - reflectance, transmittance and absorptance
  - **Spatial** distribution of the propagated light:
    - bidirectional surface-scattering distribution function (BSSDF, or simply BDF), which can be decomposed into:
      - bidirectional reflectance-distribution function (BRDF)
      - bidirectional transmittance-distribution function (BTDF)

Spectral Power Distribution

- Spectral reflectance:
  - ratio of the reflected to the incident radiant flux (power) for a given wavelength

$$\rho(\lambda) = \frac{\Phi^r(\lambda)}{\Phi^i(\lambda)}$$

  - due to conservation of energy always lies between 0 and 1
  - dimensionless

- Spectral transmittance:
  - ratio of the transmitted to the incident radiant flux for a given wavelength

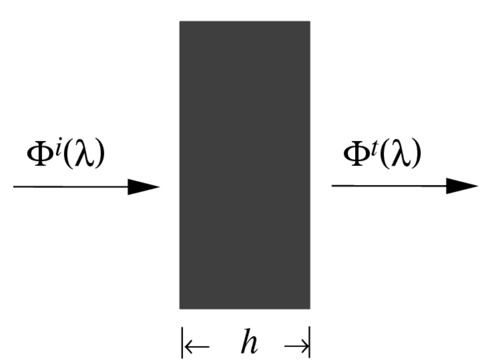
$$\tau(\lambda) = \frac{\Phi^t(\lambda)}{\Phi^i(\lambda)}$$

  - due to conservation of energy always lies between 0 and 1
  - dimensionless
  - reflections at the surface as well as absorption within the material operate to reduce the transmittance

- Bouguer's law (or Lambert's law of absorption):

$$\tau(\lambda) = \frac{\Phi_t(\lambda)}{\Phi_i(\lambda)} = e^{-a(\lambda)\, h}$$

where:
$a(\lambda)$ = absorption coefficient of the medium at wavelength λ,
h = thickness of the medium.



- Beer's law: states that for a dye solution, the absorption coefficient of the solution is directly proportional to its concentration
- Combining Bouguer's law and Beer's law:

$$\tau(\lambda) = e^{-a(\lambda)\; c\; h}$$

where:

$a(\lambda)$	=	absorption coefficient of the medium at wavelength λ,
c	=	concentration of the solution,
h	=	thickness of the medium.

- Spectral absorptance:
  - ratio of the absorbed to the incident radiant flux for a given wavelength

$$\alpha(\lambda) = \frac{\Phi^a(\lambda)}{\Phi^i(\lambda)}$$

  - dimensionless
  - due to conservation of energy, for any given material the following relationship holds:

$$\rho(\lambda) + \tau(\lambda) + \alpha(\lambda) = 1$$

- Spectral reflectance factor:
  - ratio of the reflected flux from a surface to the flux that would have been reflected by a perfectly diffuse surface in the same circumstances

$$R(\lambda) = \frac{\Phi^r(\lambda)}{\Phi_{pd}(\lambda)}$$

  - due to conservation of energy always lies between 0 and 1
  - dimensionless

- Reflectance, transmittance and reflectance factor depend on:
  - the incident and propagation solid angles
    - directional ($d\vec{\omega}$)
    - conical (Γ)
    - hemispherical (Ω)
  - the BDF of the surface
  - polarization

- Types of reflectance, transmittance and reflectance factor according to the incident and propagation solid angles:

	$d\vec{\omega}_i$	Γ_i	Ω_i
$d\vec{\omega}$	Bidirectional	Directional-conical	Directional-hemispherical
Γ	Conical-directional	Biconical	Conical-hemispherical
Ω	Hemispherical-directional	Hemispherical-conical	Bihemispherical

- Directional hemispherical reflectance given by:

$$\rho(x, \psi_i, 2\pi, \lambda) = \int_{outgoing\ \psi} f_r(x, \psi_i, \psi, \lambda) \cos\theta d\vec{\omega}$$

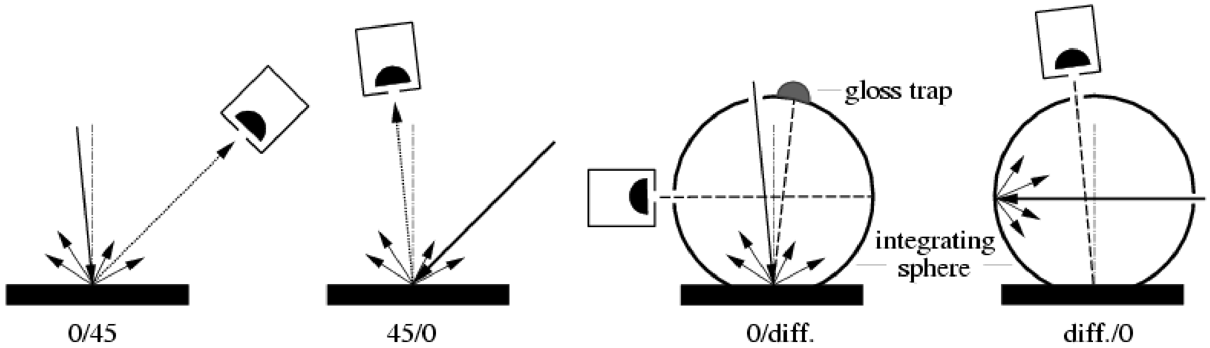
where:

$f_r(x, \psi_i, \psi, \lambda)$	=	BRDF of the surface at x,
θ	=	angle between the normal and the outgoing direction ψ,
$d\vec{\omega}$	=	differential solid angle at which the radiance is reflected.

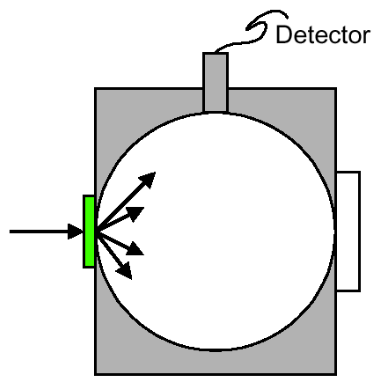
Spectrophotometry

- Instruments for measuring color attributes:
  - Spectrophotometer
    - measure spectral reflectance factor and spectral transmittance
  - Reflectometer (colorimeter)
    - use filter(s) to respond to spectral distributions of light in the same manner as the human eye

- Spectrophotometers: illumination and viewing geometries



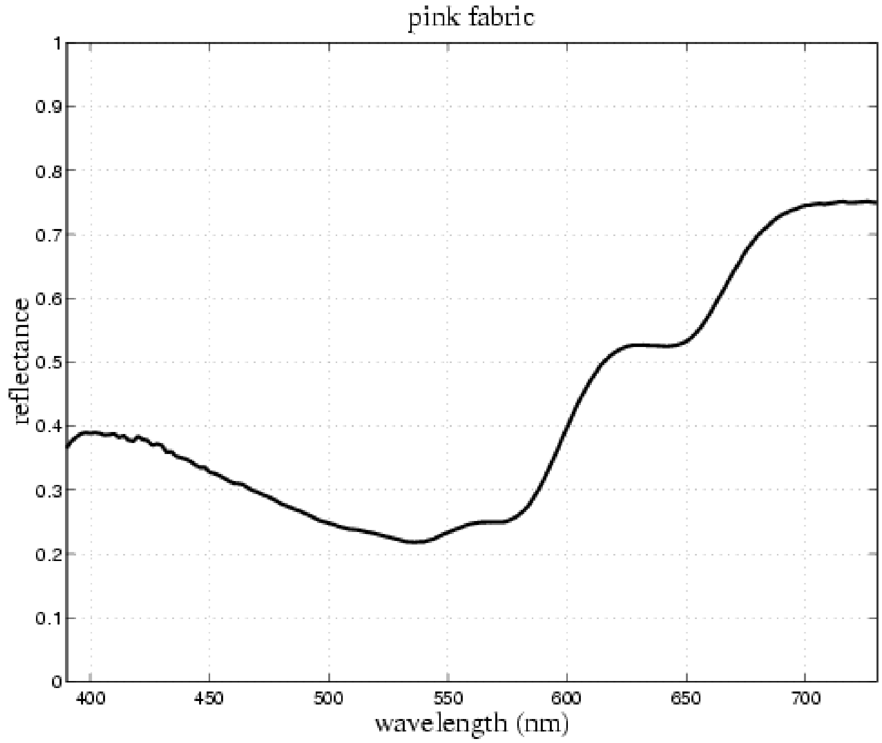
- Types of specimens:
  - opaque, glossy, polarizing, fluorescent and translucent



Integrating Sphere

- Characteristics of spectrophotometers:
  - integrating sphere wall is the standard, *i.e.*, wall reflectance is treated as unity
  - a gloss trap can be used to reduce the influence of the specular component of specimens with mixed reflection
  - precision is estimated by the ability of the device to replicate a measurement for a given specimen under same conditions
    - uncertainty: between 0.007 and 0.005
  - accuracy is estimated by the ability of the device to provide the true reflectance and transmittance of a given specimen

- Typical output of spectrophotometers:



Additive Color

- when the energy from two illuminants are combined, the result is the sum of their spectral power distributions

$$\Phi(\lambda) = \Phi_1(\lambda) + \Phi_2(\lambda)$$

- this is known as *additive mixing*
- modelling this property of light is possible with simple algebra
  - summing functions over wavelength, $\sum_j \Phi_j(\lambda)$
  - summing vectors sampling such functions, $\sum_j \begin{bmatrix} \Phi_j(\lambda_1) \\ \Phi_j(\lambda_2) \\ \vdots \\ \Phi_j(\lambda_n) \end{bmatrix}$

Additive Color

- the effect of additive mixture of light on the human visual system is also additive
- Grassman's Laws (1853) are the basis for *colorimetry* [found in many sources, Wyszecki and Stiles (1982), Hardeberg (2001)]
  1. Three independent variables are necessary and sufficient to psychophysically characterize a color.
  2. The result of an additive mixture of colored light depends only on the psychophysical characterization, and not on the spectral composition of the colors.
  3. If the components of a mixture of color stimuli are moderated with a given factor, the resulting psychophysical color is moderated with the same factor.

Additive Color

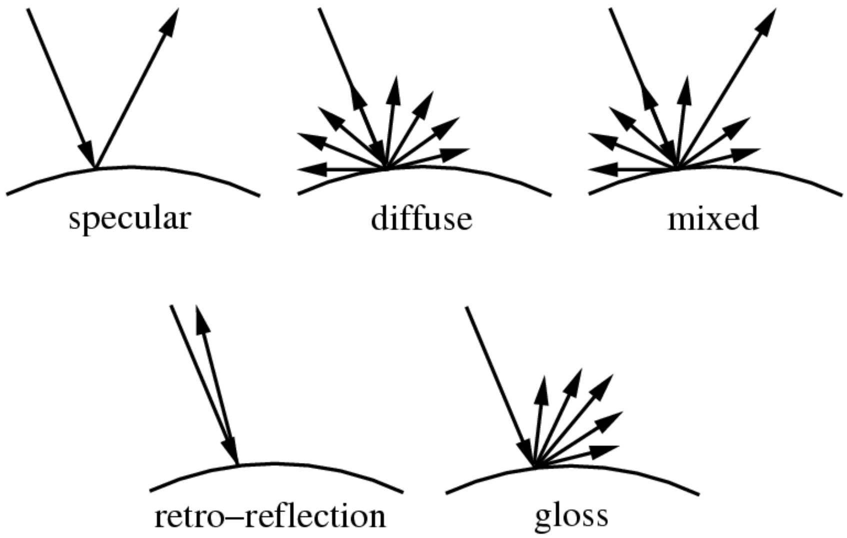
- Grassman's laws show that not only light but color can be treated with a vector space approach
- *tristimulus spaces* are three-dimensional vector spaces with a basis of primaries, such as $\mathbf{r}$, $\mathbf{g}$, $\mathbf{b}$
- a color stimulus $\mathbf{c}$ is then written as a linear combination of the primaries, $\mathbf{c} = R\mathbf{r} + G\mathbf{g} + B\mathbf{b}$
- if spectra are modeled with N-vectors, and color with 3-vectors, matrix algebra can be used to describe the interaction between spectral and color spaces

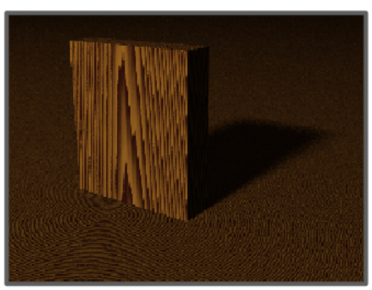
Additive Color

- additive color principles support devices like monitors and televisions
  - small red, green, and blue phosphors, placed close together and excited by electron beams can take the appearance of a wide range of colors
- the same principle underlies electronic billboards
- half-toning in newspaper and magazine images is partly additive
- pointillistic painting techniques (Seurat, Pissarro, Signac)

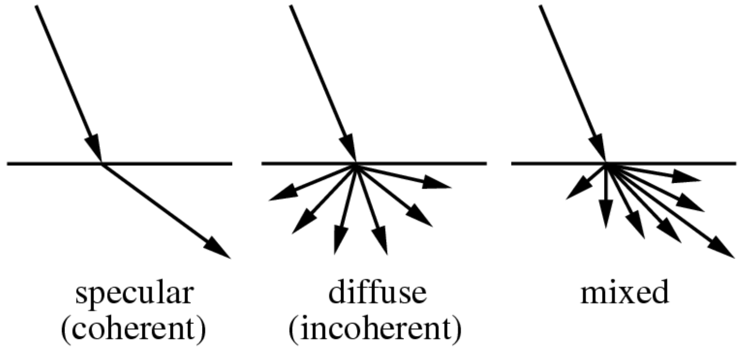
3. Reflection and Transmission

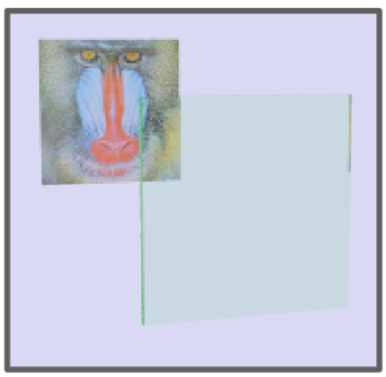
- Mechanisms by which light is reflected by a surface:





- Mechanisms by which light is transmitted by a surface:





Scattering Functions

- BRDF is given by:

$$f_r(x,\psi_i,\psi_o,\lambda) = \frac{dL(x,\psi_{in},\lambda)}{L_i(x,\psi_i,\lambda)\, d\vec{\omega}_i\, cos\theta_i}$$

where:

$dL(x,\psi_o,\lambda)$	=	radiance propagated at x and in a direction ψ_o,
$L_i(x,\psi_i,\lambda)$	=	incident radiance at x and in a direction ψ_i,
θ_i	=	angle between the normal at x_i and the direction ψ_i,
$d\vec{\omega}_i$	=	differential solid angle at which L_i arrives at x.

- SPF (scattering probability function) is given by:

$$s(x, \psi_i, \psi_o, \lambda) = \frac{dI(x, \psi_o, \lambda)}{\rho(x, \psi_i, \lambda) d\Phi(x, \psi_i, \lambda)}$$

where:

$$\begin{array}{lcl} dI(x, \psi_o, \lambda) & = & \text{radiant intensity reflected at } x \text{ and in a direction } \psi_o, \\ \rho(x, \psi_i, \lambda) & = & \text{reflectance of the surface at } x, \\ d\Phi(x, \psi_i, \lambda) & = & \text{radiant flux incident at } x \text{ and in a direction } \psi_i. \end{array}$$

- BRDF and SPF have a simple relationship:

$$s = f_r C cos\theta_o$$

where C is the constant that enforces the unit area constratint for a probability density.

- Energy conservation:

$$\rho(x, \psi_i, 2\pi, \lambda) = \int_{outgoing\ \psi} f_r(x, \psi_i, \psi_o, \lambda) \cos\theta d\vec{\omega} \leq 1, \quad \forall \psi_i$$

- For a perfect diffuse surface:

$$f_r(x, \psi_i, \lambda) = \frac{\rho(x, \lambda)}{\pi}$$

$$s = \frac{cos\theta_o}{\pi}$$

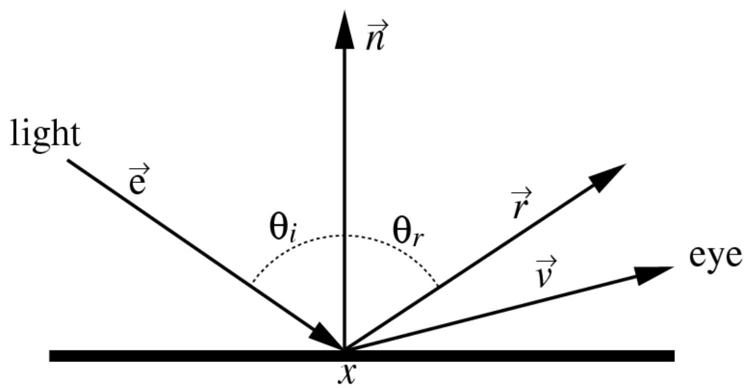
Illumination

- Traditional local illumination models:

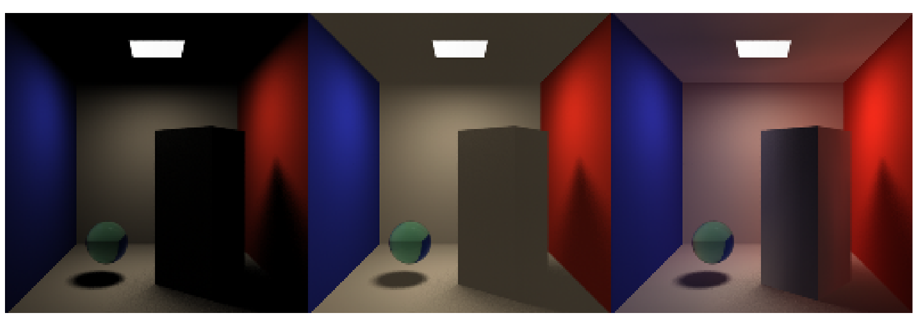
$$I(x, \vec{v}, \lambda) = k_a I_a + k_d(\vec{e}.\vec{n}) I_d + k_s(\vec{r}.\vec{v})^p I_s$$

where:

I_a, I_d, I_s	$=$	ambient, diffuse and specular radiant intensities,
k_a, k_d, k_s	$=$	ambient, diffuse and specular coefficients,
p	$=$	specular exponent.



- Three major global illumination approaches have been used in rendering to simulate the light transfer mechanisms:
  - ray tracing, radiosity and multipass methods



- Kajiya (1986) unified the discussion of global illumination methods with the rendering equation:

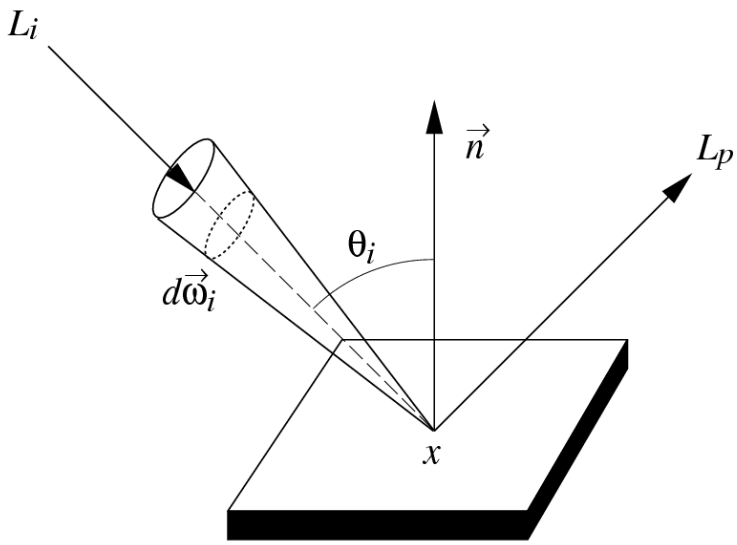
$$\underbrace{L(x, \psi, \lambda)}_{total} = \underbrace{L_e(x, \psi, \lambda)}_{emitted} + \underbrace{L_p(x, \psi, \lambda)}_{propagated}$$

- The propagated radiance can be expressed as:

$$L_p(x,\psi,\lambda) = \int_{incoming\ \psi_i} f(x,\psi,\psi_i,\lambda) L_i(x,\psi_i,\lambda) \cos\theta_i d\vec{\omega}_i$$

where:

$f(x,\psi,\psi_i,\lambda)$ = BDF of the surface at x,
$L_i(x,\psi_i,\lambda)$ = spectral incident radiance at x and in a direction ψ_i.



- Using stochastic ray tracing,

$$\begin{aligned} L(x,\psi,\lambda) = \ & L_e(x,\psi,\lambda) + \\ & \rho(x,\psi) L_e(x',\psi',\lambda) + \\ & \rho(x,\psi,\lambda)\rho(x',\psi',\lambda) L_e(x'',\psi'',\lambda) + ... \end{aligned}$$

where:

- x^n is chosen by sending a ray from x^{n-1} in the direction $-\psi^{n-1}$, and
- ψ^{n-1} is chosen according to the surfaces' SPF

- Aspects often omitted in rendering applications:
  - reflectance and transmittance positional dependence
  - reflectance and transmittance angular dependence
- Major limitations of current rendering pipelines include:
  - low resolution spectral sampling due to:
    - lack of data
    - costly algorithms
  - poor color management
  - unpredictable color reproduction

Subtractive Color

- the effect of illuminant $\Phi(\lambda)$ striking a surface with reflectance $\rho(\lambda)$ is to reflect light with energy $\Phi(\lambda)\rho(\lambda)$
- this simple model does not account for fluorescence, and the spatial distribution of the light is handled separately
- the absorption of light energy by the surface is *subtractive*
- translucent filters act in a similar way, absorbing part of the light energy as it passes through them

Subtractive Color

- whereas additive color can be modeled by addition in a vector space, subtractive color is actually multiplicative
- with clear filters, in the simplest case the transmittance $\tau(\lambda)$ multiples the spectral power of the illuminant to give $\Phi(\lambda)\tau(\lambda)$
- it is common in photographic sensitometry to integrate light energy over the visible wavelengths to give a single number for transmittance, $T = \int_{\mathcal{V}} \tau(\lambda) d\lambda$
- an additive approach is possible by taking logarithms:
  - opacity is the inverse of transmittance, $O = 1/T$
  - density is the logarithm of opacity, $D = \log_{10} O = -\log_{10} T$
- film or filters closely stacked can then be modeled by adding their densities
- this simple approach is very useful, but ignores scattering and refraction

Subtractive Color

- reflective surfaces can be thought of as one or more thin film layers on a substrate
- often scattering effects must be considered in the layers, and in the substrate
- considerably more complex to model than additive color
  - characterize by model
  - characterize by measurement

Linear Models of Spectra

- spectral power distributions (SPDs) are continuous functions of wavelength $\Phi(\lambda)$
- measuring instruments like luminance meters have sensitivity $\sigma(\lambda)$, and return a single number $s = \int_{\mathcal{V}} \Phi(\lambda)\sigma(\lambda)d\lambda$
- more convenient to model with vectors that sample the continuous functions, so $\mathbf{s} = \sigma^t\phi$
- spectrophotometers then return multiple samples $s_i = \sigma_i^t\phi$, or in matrix form $\mathbf{s} = \mathbf{S}\phi$

Linear Models of Spectra

- spectrophotometers can sample at 10nm, 5nm, 2nm, or even 1nm intervals
- this can give as many as 300 samples in the visible spectrum
- remote sensing data may have thousands of samples, including the near infrared (NIR) and infrared (IR) regions
- it is often useful to reduce the amount of data by using a basis $\mathbf{B}$, so that individual spectra can be written $\mathbf{s} = \mathbf{Bc}$ for some coefficient vector $\mathbf{c}$
- long vectors of spectral data can then be replace by the potentially shorter coefficient vectors

Linear Models of Spectra

- choosing an appropriate basis **B** is usually carried out with *Principal Component Analysis* (PCA)
- one possible implementation is the *singular value decomposition*, (SVD)
- for restricted sets of spectra, and also for reflectances, the SVD yields a low-dimensional basis that approximates high-dimensional data well
- this approach can be used for
  - compression of the spectral data
  - filtering out noise
  - identifying the main axes of variation
- the best results are found by carefully choosing the spectra on which to perform the SVD

II. SPECTRA AND PERCEPTION
1. Applications

- Spectral distribution of light in Nature:
  - scattering
    - Rayleigh
    - Mie
    - Reflective-refractive
  - absorption
  - emission
- Simulation:
  - physically and biologically-based rendering

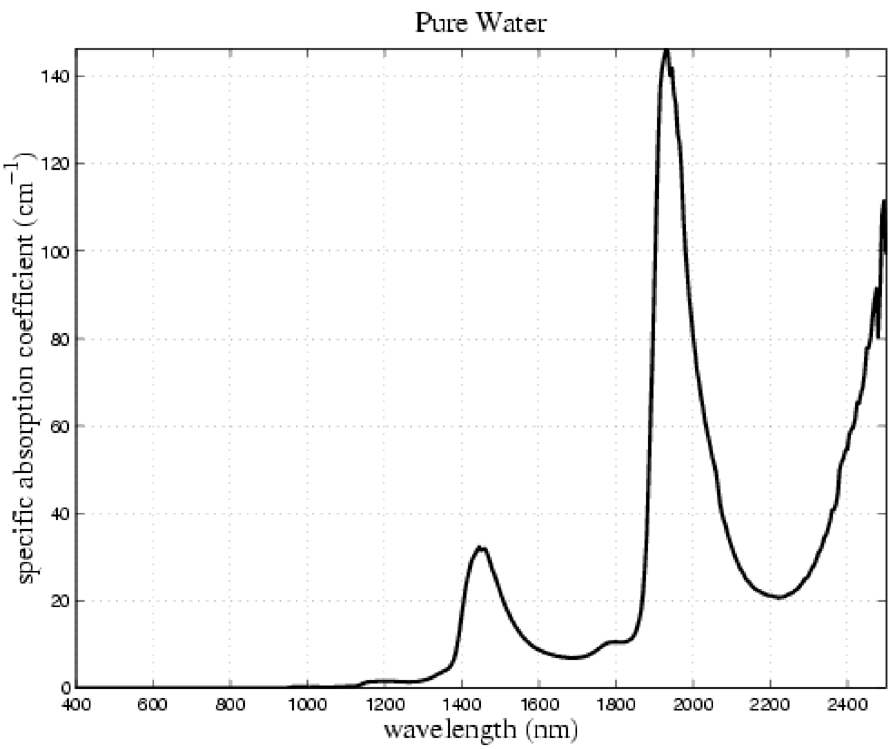
Sky

- Rayleigh scattering
  - gases (O_2, N_2, etc.): inversely proportional to wavelength (light in the blue region is preferentially scattered)
- Mie scattering
  - cloud cover: reflect a proportion of the blue wavelengths, but cause very little change in the long wavelengths ($600 - 800nm$)
  - haze or dust: reduction in the proportion of blue light and an increase in the proportion of red light
- Absorption
  - ozone
  - water

- Daylight
  - diffuse skylight (D-light)
    - high proportion of blue light
  - direct sunlight (I-light)
- Related atmospheric phenomena:
  - purple light: scattering of light from stratospheric dust
    - direct reddened sunlight combines with indirect blue scattered light to produce a purplish hue
  - rainbows: refraction of light by water droplets and ice

Ocean

- water properties
  - refractive index
  - absorption coefficient



- particulate material
  - inorganic: weathering of terrestrial rocks and soils
  - colored dissolve matter (CDOM) or yellow matter
  - organic: bacteria, phytoplankton and zooplankton
    - Example: *Emiliania huxleyi*
      - $\rho = 0.39$ at blue wavelengths (compared to $\rho = 0.02$ to 0.05 for typical ocean waters)
      - milky white or turquoise appearance
  - environmental conditions: incident radiance and bottom conditions

Aurorae

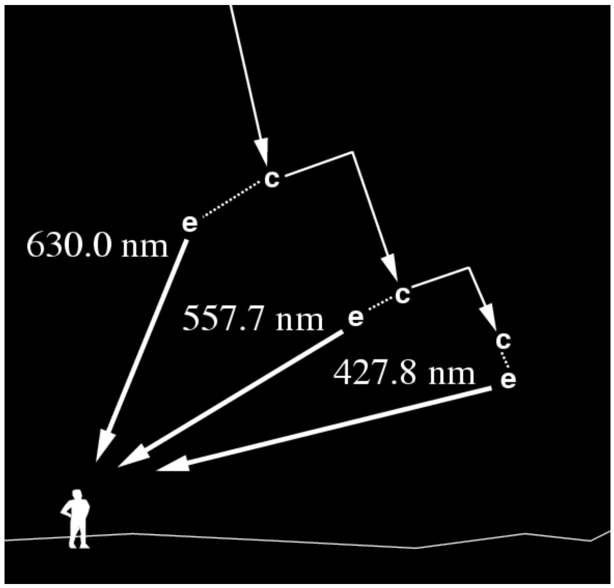
arc

drapery

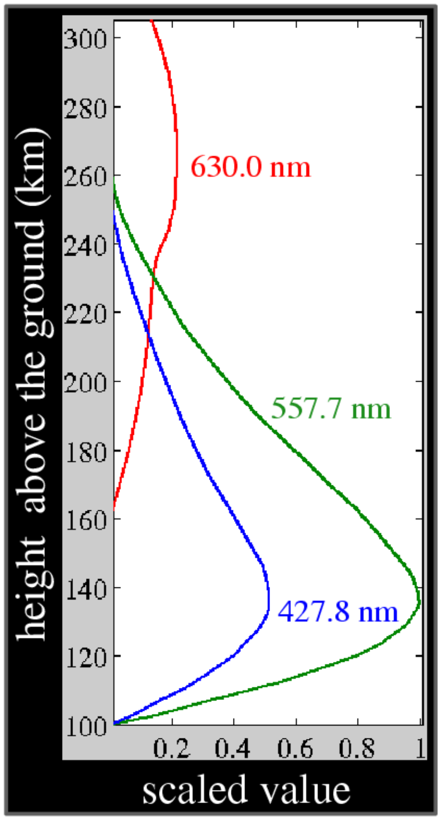
corona

band

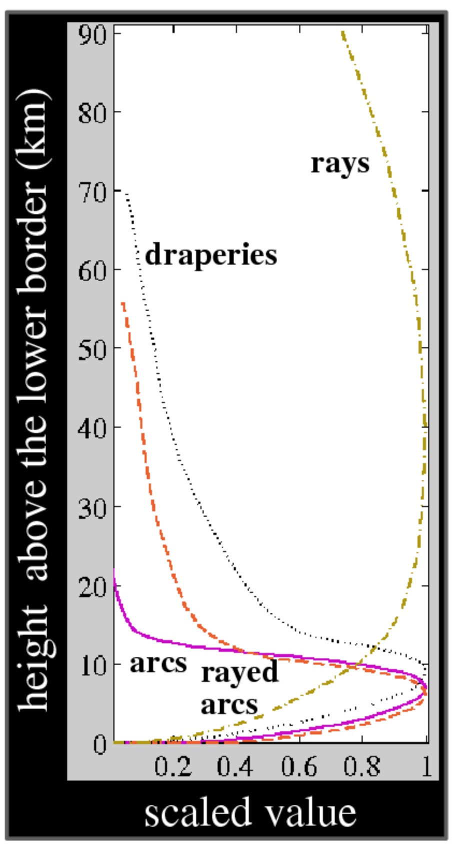
- A. J. Angstrom, ”Spectrum des Nordlichts”, 1869
- Strongest auroral spectral emissions:
  - atomic oxygen green line (delay 0.7s)
  - atomic oxygen red line (delay 110s)
  - ionized nitrogen blue band (delay 0.001s)
- Correlation with height



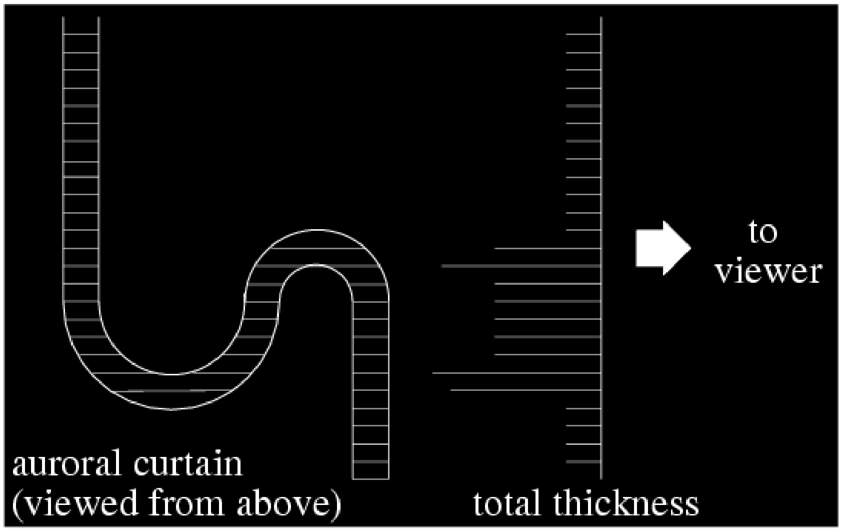
• Typical auroral spectral curves:



• Intensity curves:



- Dynamic range:



- Blur:

without local blur

with local blur

Nebulae

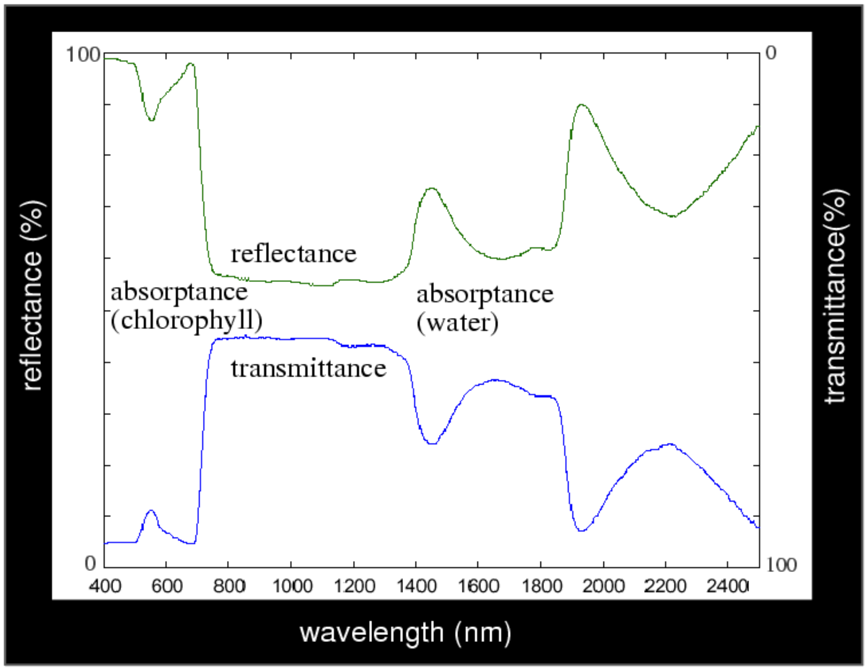
- Clouds of interstellar dust and gas within our own galaxy made visible by their interactions with nearby stars or star remnants.

- Types:
  - Emission nebulae: clouds of high temperature gas ionized by ultraviolet radiation
    - usually red due to the hydrogen spectral line $656nm$
  - Reflection nebulae: clouds of dust reflecting and scattering light from nearby stars
    - usually blue because scattering is more efficient for blue light
  - Planetary nebulae: ejected matter from a low mass star near the end of the star's life
    - green "forbidden" lines of oxygen are stronger in these nebulae
  - Supernova remnants: ejected matter from a high mass star near the end of the star's life
  - Dark nebulae: clouds of dust blocking light

Plants

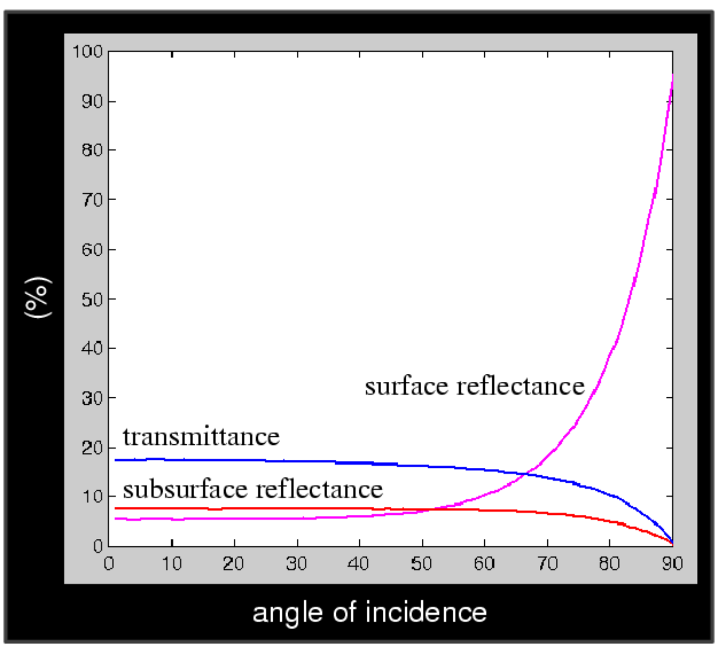
- Surface reflection:
  - affected by the presence of hairs and wax layers
- Subsurface reflection and transmission
  - affected by the internal distribution of tissues
- Absorption
  - affected by the presence of pigments (*e.g.*, chlorophyll), water and dry matter

- Typical spectral curves for plant leaves:

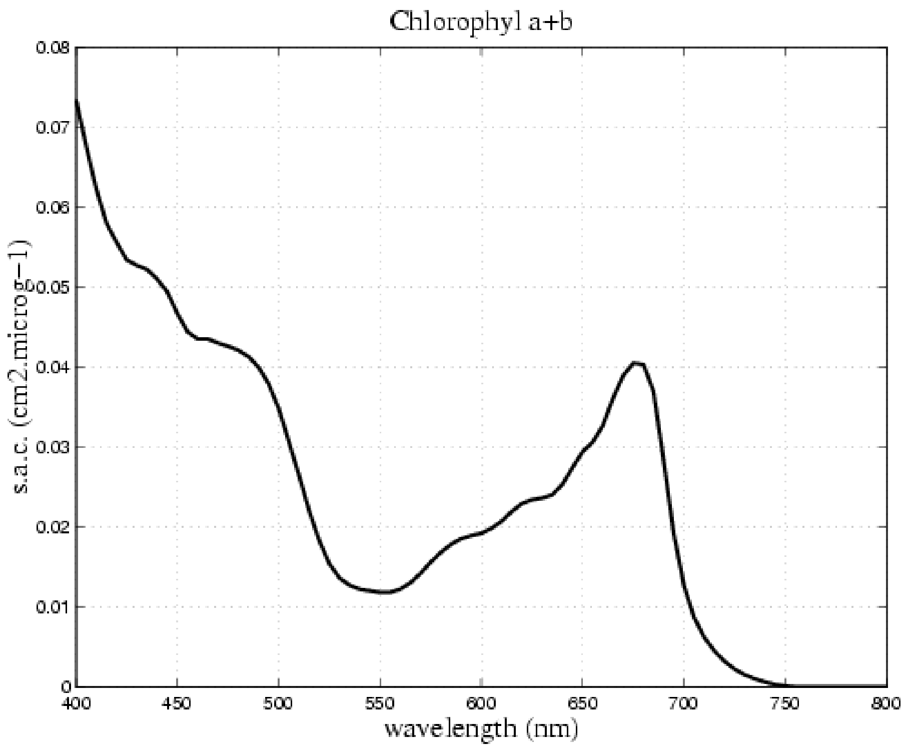


- Angular dependency:

reflectance and transmittance at λ = 550nm



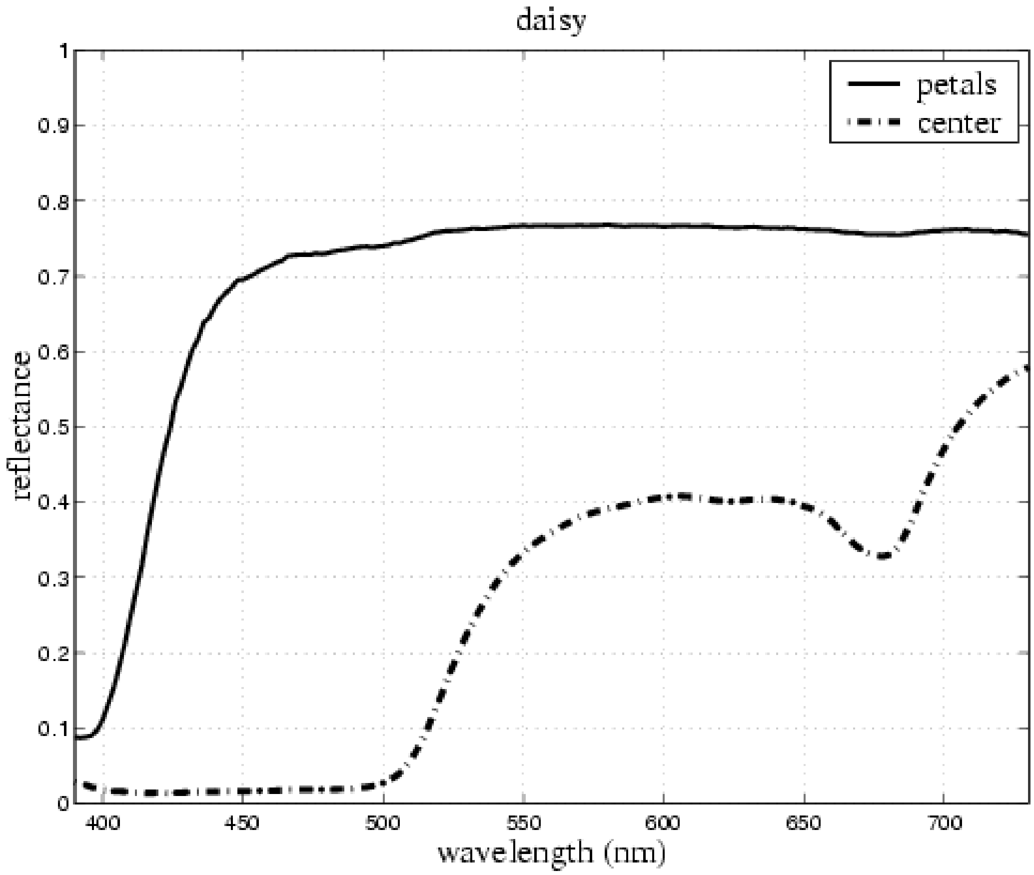
- Specific absorption coefficient (s.a.c.) for plant pigments:
  - are independent of plant species
  - may be adjusted according to the pigment concentration



- Other pigments found in plant leaves:
  - carotenoids
    - are usually red, orange, yellow or brown
    - are associated with chlorophyll in the chloroplasts
    - their yellow colors are evident in many autumn leaves

  - xanthopylls (yellow pigments)
  - anthocyanins (red and purple pigments)
  - tannins (brown pigments)
  - problems to determine their concentration

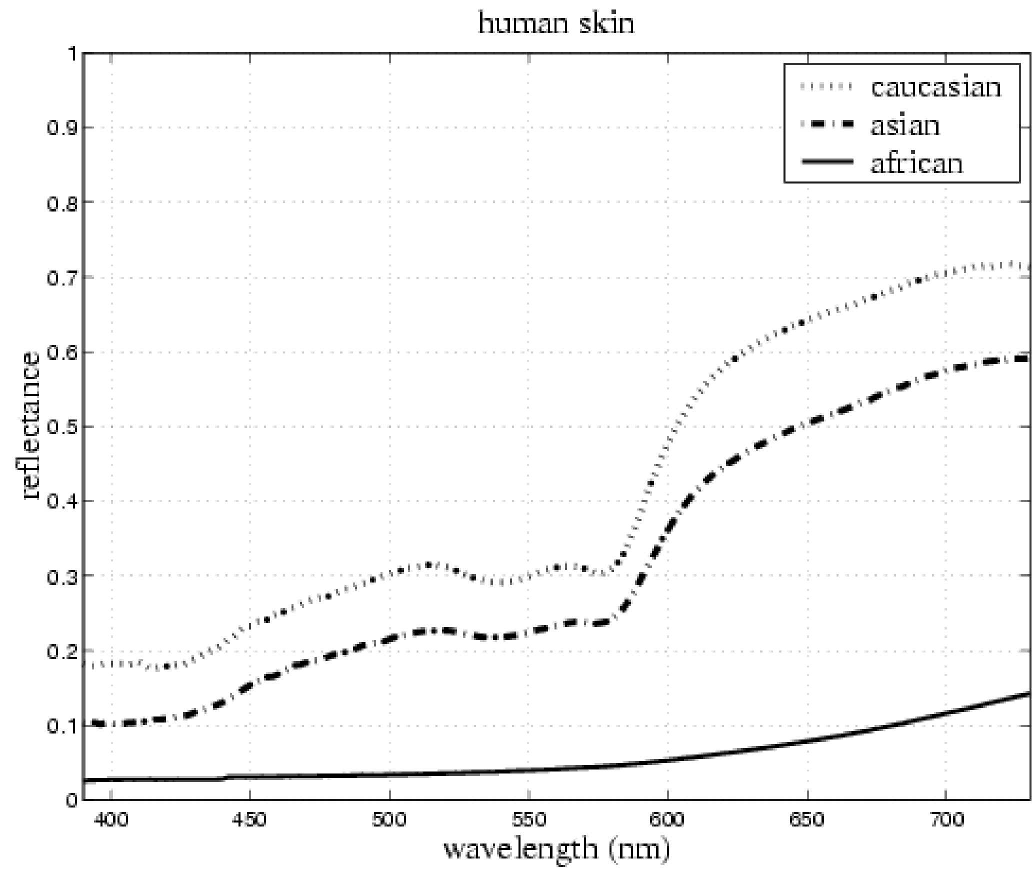
- Typical pigments found in flowers:
  - anthocyanins, carotenoids and UV-absorbing flavone
- Examples of reflectance curves for flowers:



Human Skin

- Surface reflection:
  - affected by the presence of hairs, oiliness and water content
- Subsurface reflection and transmission
  - affected by the internal distribution of tissues
- Absorption
  - affected by the presence of pigments (melanin, hemoglobin and carotene)

- Typical reflectance curves for human skin:



Art and Archival Imaging

- man-made objects decay over time
- multispectral scanning can preserve digital images of important objects in archives
  - for analysis
  - for reproduction
- scanning cameras can also be used to continuously monitor documents and works of art
- even two-dimensional objects present many challenges, as they not only have diffuse reflection, but gloss, surface texture, and so on

Art and Archival Imaging

- multispectral cameras are usually monochromatic CCD cameras with various filters
- filters may be placed between object and camera lens, or may be between light source and object
- from 3 to 12 or more filters are used
- choosing the best filters is not a trivial task, as must optimize the recorded channel information based on
  - the light source
  - the camera sensitivity
  - the object properties, eg. types of paint or ink

Art and Archival Imaging

- the recorded multispectral information then must be converted into a more useful form, usually factoring out the illuminant to give an estimate of object color or reflectance at each pixel
- even ignoring gloss and surface texture issues, errors can result from
  - uneven illumination
  - poor camera alignment
  - chromatic aberration
  - registration of multiple images (mosaicing)
  - CCD noise
  - discretization errors
- but with enough channels, and carefully-chosen algorithms, fairly accurate image reconstruction is possible

2. Perceptual Response and Color

- light energy is physical
- color is *psychophysical*:
  - partly from energy stimulating the cones in the retina
  - partly from processing by the human visual system
- often easier to measure and predict the physical component
- psychophysical aspects of color perception include
  - light adaptation and chromatic adaptation
  - color constancy
  - simultaneous contrast
- *color appearance models* attempt to characterize these qualities

Light Adaptation and Chromatic Adaptation

- *light adaptation* is the change in the visual system to compensate for different intensities of illumination
- *chromatic adaptation* is the change in the visual system to compensate for different spectral power distributions of illumination

Color Constancy

- *color constancy* is the tendency of a object to appear the same color under different levels and colors of illumination
  - a yellow banana appears yellow in sunlight, fluorescent or tungsten illumination even under strongly colored illumination
  - if the banana under different illuminants is examined in isolation, radical color shifts are seen
- the human visual system tends to factor out the overall illumination

Simultaneous Contrast

- *simultaneous contrast* is the tendency of a object to take on a hue and lightness complementary to its surround
- interaction between cones in the retina
- boundaries between colored regions are enhanced
  - Mach banding

Early Color Experiments

- Newton's experiments showed that white light could be separated by a prism into a continuous series of colors called the *spectrum*
  - color a property of the light, not the objects
  - without light, there is no color
- Maxwell quantified the wave aspects of light for electromagnetism
- Grassman's Laws showed the three-dimensional aspects of color
- Maxwell's color matching experiments demonstrated that most target colors could be matched by the additive combination of three primary light sources
- adding one of the primaries to the target allowed all colors to be matched
  - effectively subtraction of one primary from the others
- linear combinations of three primaries can match any target color

Human Visual System

- light energy passes through the lens of the eye to the *retina*
- the retina contains
  - the *blind spot*, where the optic nerve passes to the brain
  - the *fovea*, where *cone photoreceptors* are concentrated
  - *rod photoreceptors*
- rods are more sensitive to low levels of light, *scotopic vision*
- the cones are more sensitive in daylight, *photopic vision*
- cones are mostly responsible for color vision, and come in three types: those sensitive to Long, Medium, and Short wavelengths (L,M,S)

Commission Internationale de l'Eclairage (CIE)

- defined CIE Standard Observer color-matching functions in 1931 called $\bar{x}(\lambda), \bar{y}(\lambda), \bar{z}(\lambda)$
- result of color-matching experiments using three light sources at 700 nm, 546.1 nm, 435.8 nm

Trichromacy and Color-Matching Functions

- CIE color-matching functions allow conversion of SPD into CIE XYZ

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \int_V \Phi(\lambda) \begin{bmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{bmatrix} d\lambda \tag{1}$$

Color-Matching Functions in Matrix Form

- also possible to describe as matrix equation

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = [\bar{\mathbf{x}}, \bar{\mathbf{y}}, \bar{\mathbf{z}}]^t \phi \tag{2}$$

Vector Algebra

- additive color means tristimulus values can be treated with vector algebra
- sum of SPDs gives sum of tristimulus values

$$\int_V (\Phi_1(\lambda) + \Phi_2(\lambda) + \Phi_3(\lambda) + \ldots) \begin{bmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{bmatrix} d\lambda = \begin{bmatrix} X_1 \\ Y_1 \\ Z_1 \end{bmatrix} + \begin{bmatrix} X_2 \\ Y_2 \\ Z_2 \end{bmatrix} + \begin{bmatrix} X_3 \\ Y_3 \\ Z_3 \end{bmatrix} + \ldots$$

Vector Algebra

- scalar multiple of SPD gives multiple of tristimulus values

$$\int_V k\Phi(\lambda) \begin{bmatrix} \bar{x}(\lambda) \\ \bar{y}(\lambda) \\ \bar{z}(\lambda) \end{bmatrix} d\lambda = k \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

- can treat tristimulus values as vectors

Chromaticity Values

- due to light adaptation, is often convenient to separate *luminance* Y from remaining color information, *chrominance*
- CIE chromaticity values x and y are defined by

$$
\begin{aligned}
x &= \frac{X}{X+Y+Z} && (3)\\
y &= \frac{Y}{X+Y+Z} && (4)\\
z &= \frac{Z}{X+Y+Z} && (5)\\
&= 1-x-y && (6)
\end{aligned}
$$

- plots on the chromaticity or *horseshoe diagram* inherit vector space properties
- note *spectrum locus* for monochromatic colors

Perceptually Uniform Spaces

- many other device-independent 3D color spaces exist
- CIE LAB was created to be a *perceptually uniform* space
  - Euclidean distance corresponds to perceived color difference
- in fact is only pseudo-uniform
- nominal *white point* $[X_n, Y_n, Z_n]^t$ becomes $[100, 0, 0]^t$ in CIE LAB
- this white-point mapping adjusts for light adaptation
- conversion from CIE XYZ to CIE LAB factors in nonlinear response of the human visual system to luminance

Perceptually Uniform Spaces

- conversion from CIE XYZ to CIE LAB is given by [from Hardeberg 2001]:

$$L^* = 116 f(Y/Y_n) - 16 \quad (7)$$

$$a^* = 500\,[f(X/X_n) - f(Y/Y_n)] \quad (8)$$

$$b^* = 200\,[f(Y/Y_n) - f(Z/Z_n)] \quad (9)$$

where

$$f(\alpha) = \begin{cases} \alpha^{1/3}, & \alpha \geq 0.008856 \\ 7.787\alpha + 16/116, & otherwise \end{cases}$$

Just Noticeable Differences

- Euclidean distance in CIE LAB measures color difference
- this distance measure is called $\Delta E_{ab}(1976)$ or ΔE
- a $\Delta E$of 1.0 is often considered a Just-Noticeable Difference (JND) between two colors placed side-by-side
- there remains much debate about the correct value of ΔE to make a JND
- there is also a ΔE_{94} standard (1994)

Luminous Efficiency and Metamerism

- *photopic* observer has *spectral luminous efficiency* (or *efficacy*), $V(\lambda)$
- *scotopic* observer $V'(\lambda)$ shifted towards blue, *Purkinje shift*
- can compute response to illuminant $\Phi(\lambda)$ with

$$\int_V \Phi(\lambda) V(\lambda) d\lambda$$

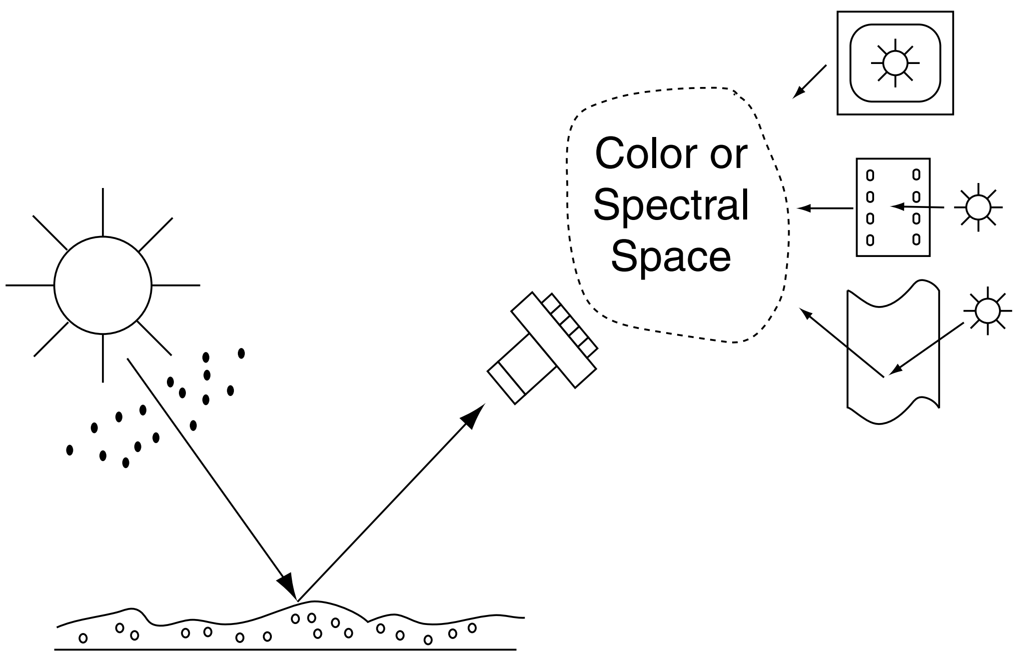
- quantities making use of $V(\lambda)$ involve the term *luminous*

Metamerism

- spectra have no color
- continuous spectra form an infinite-dimensional space
- three types of human receptors (cones)
- mapping is many-to-one
- there may be many spectra giving the same color sensation
- phenomenon is called *metamerism*
- spectra that are equivalent to black under a given illuminant are called *metameric blacks*

III. THE VIRTUAL CAMERA AND DEVICES

1. Spectra through the Virtual Camera



Spectra through the Virtual Camera

- the *virtual camera* maps light from the 3D world to the imaging plane
- simple renderers use RGB values for
  - illuminants
  - surface properties
  - intermediate calculations
  - final pixel values
- some applications require greater spectral resolution

Spectra through the Virtual Camera

- RGB can be thought of as three spectral samples
- also as coordinates relative to a three-dimensional basis

Lens Effects

- in rendering, a projective transformation maps light field data to the image plane
- this virtual camera stage is used for lens effects
- effect of a real lens is to focus light on image plane
- side effects include refraction of light at different wavelengths
- useful for simulation
  - lens models
  - chromatic aberration
  - flare in special effects
- must also map spectra to device coordinates

Lens Effects

- maximize effectiveness of spectral rendering by considering output devices
- often monitor is not the final destination
- film stocks
- print (under one or several potential illuminants)

Filters

- in theatre, photography and film, filters are important to creating specific lighting effects
- RGB values poorly simulate light filtration
- with spectral rendering, more accurate rendering is possible

Filters

- easy to manipulate the entire image by multiplying spectra by filter transmittance
  - Bouguer's Law (or Lambert's law of absorption)
- this allows balancing for appropriate film stock
- can also manipulate individual lights

Virtual Camera vs Digital Camera

- digital camera similar to virtual camera
- light enters lens, strikes image plane
- is rasterized into array of pixels
  - digital cameras often use RGGB array with interpolation, rather than true RGB for each pixel
  - more expensive cameras use the full array

Virtual Camera vs Digital Camera

- digital cameras suffer from some defects
  - linearity of array makes high dynamic range imaging difficult
  - dark current noise
  - blooming in CCD array
- can avoid these artifacts in virtual camera

Managing Spectral Data

- spectral data usually treated as a vector
- more specifically, array of coordinates relative to some basis
- how many dimensions are needed?
- how to choose basis?
- how much precision is needed in coordinates?

Managing Spectral Data

- multispectral cameras exist
  - usually monochrome CCD cameras with many filters
  - 16 or more channels
- spectrophotometers can give 1 nm samples: 300 dimensions
- very costly to maintain high-resolution images with this much information per pixel
- used for specialized image archiving, research

Managing Spectral Data

- prefer instead to have sensible spectral basis
  - basis for typical application reflectances
  - basis for typical application illuminants
- typical reflectances, transmittances often have smooth shape, few high frequencies

Principal Component Analysis

- statistical technique
- used for many applications where principal axes of variance are required
- can be used for compression
- usually implemented with Singular Value Decomposition (SVD)

Principal Component Analysis

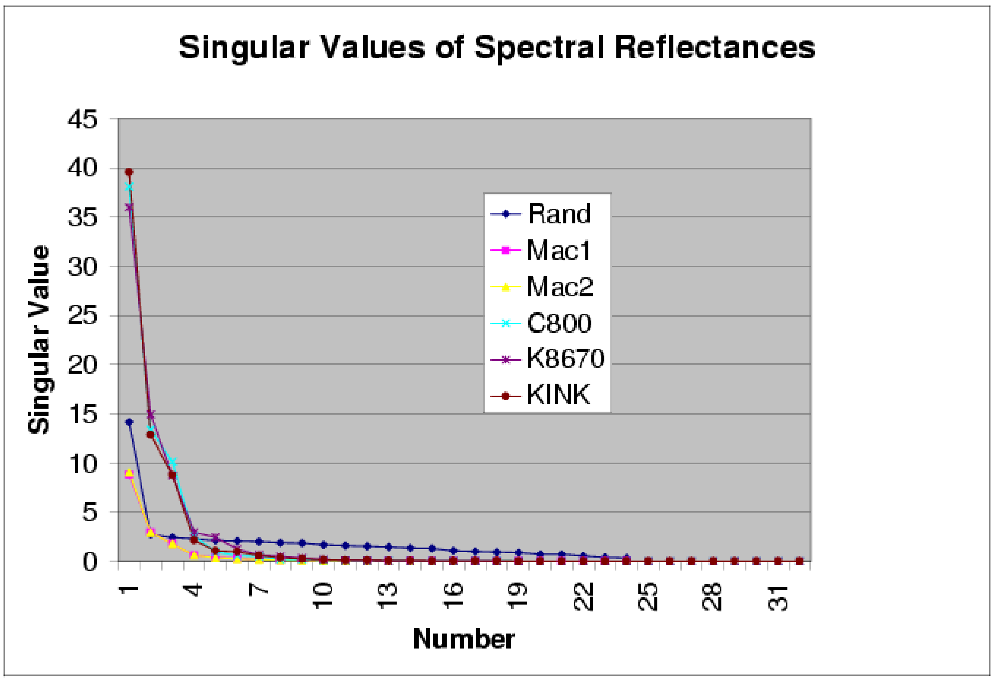
- singular value decomposition of matrix $M_{m \times n}$ gives
  - $U_{m \times m} S_{m \times n} V^t_{n \times n} = M_{m \times n}$
  - columns of orthogonal $U_{m \times m}$ are eigenvectors of symmetric MM^t
  - columns of orthogonal $V_{n \times n}$ are eigenvectors of symmetric $M^t M$
  - $S_{m \times n}$ is matrix with general diagonal entries w_i, the *singular values* of M

Principal Component Analysis

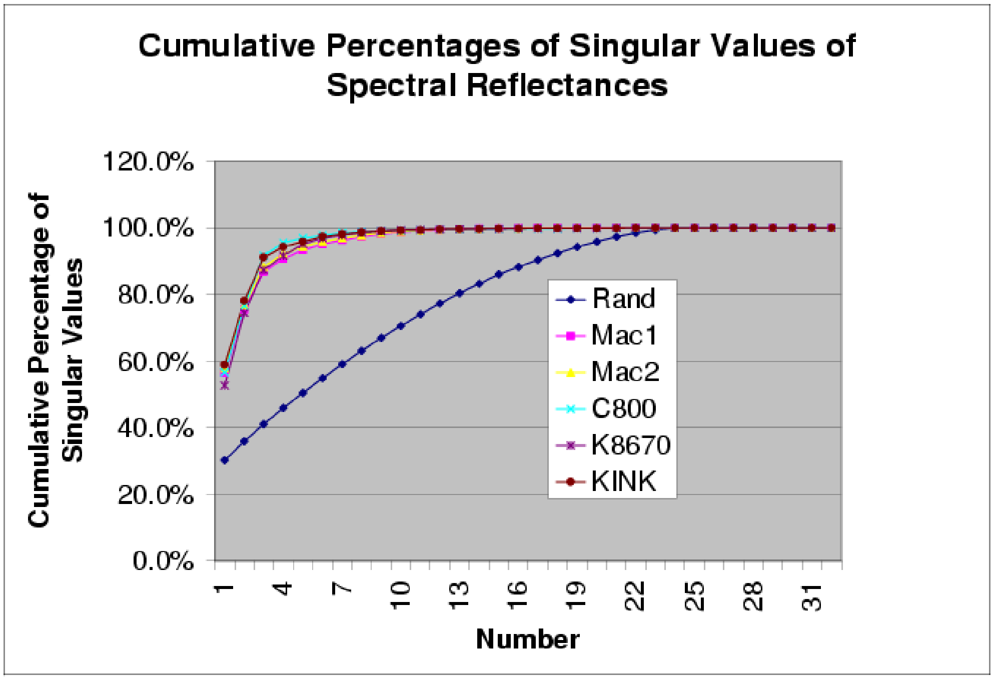
- axes are determined in order of significance
- diagonal entries of S decrease, and reveal how much variation is accounted for in each axis
- can use columns of U as basis
- for natural reflectances, there is debate over how many bases are sufficient: it depends on the application
- 6 often account for at least 95% of the variation
  - can add singular values to determine total contribution
- up to 20 or more are used

Decreasing Singular Values

1. Random reflectances
2. Macbeth color checker (1)
3. Macbeth color checker (2)
4. Canon CLC800 laser printer
5. Kodak 8670 dye diffusion thermal printer
6. Kodak inkjet printer



Increasing Accumulation of Singular Values



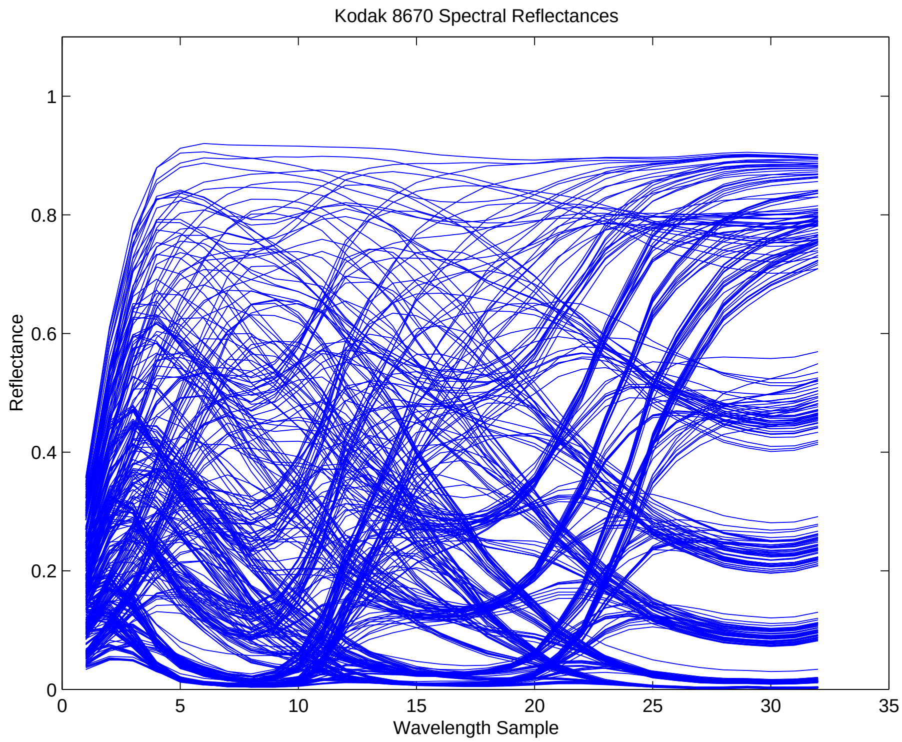
Linear Reflectance Spaces

- can form an actual "color space" from the coefficients
- early work by Paeth [Paeth (1993)]
- raster images become basis plus 6–20 coefficients per pixel
- can examine how this data captures image information

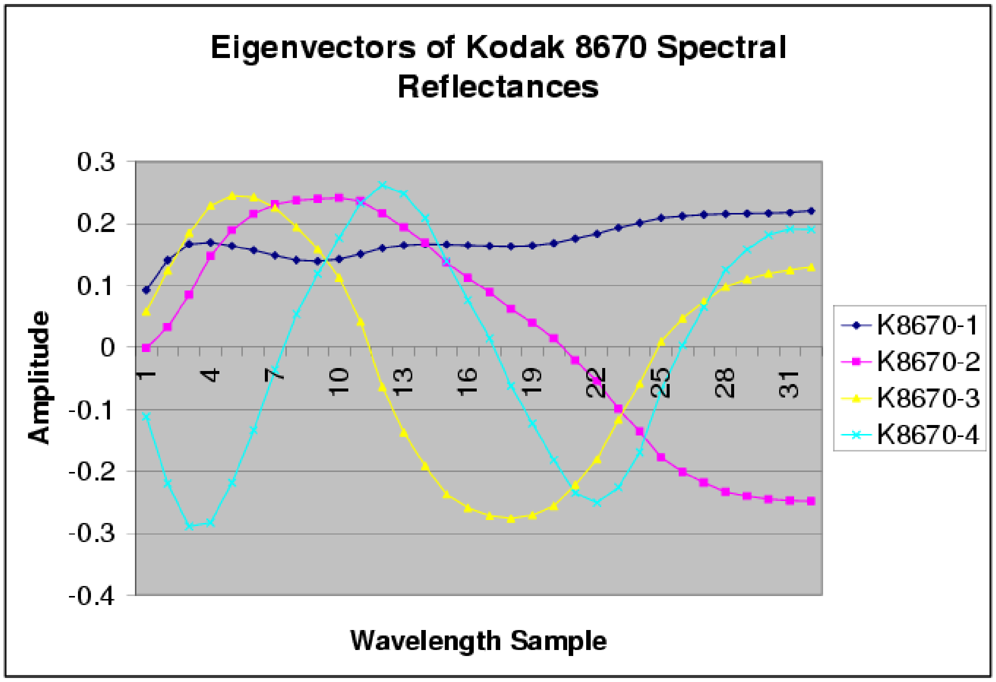
Linear Reflectance Spaces

- often first few reflectance bases have characteristic shapes, as U matrix from SVD is orthogonal
  - first is relatively uniform
  - second is sine-like
  - third is cosine-like
- can instead force the first few bases to be ideal flat, sine, cosine
- gives a standarized linear reflectance space

Kodak 8670 Dye Diffusion Thermal Printer Reflectances



Kodak 8670 Dye Diffusion Thermal Printer Reflectance Basis



Linear Reflectance Spaces

- linear reflectance space has neutral axis (like CIE LAB)
- black and white points at ends of neutral axis
- double cone shape has belt of saturated colors around middle
- first few reflectance coefficients correlated with color space
  - color space methods can be used with linear reflectance spaces

2. Device Characterization and Gamuts

- most devices have analog components with continuous behavior
  - electron guns in monitor
  - dye diffusion system, ink jets in printer
  - optoelectronic components in scanner, digital camera
- device inputs/outputs usually have discrete behaviour
  - monitor, scanner, camera RGB
  - printer CMY, CMYK or RGB
- model discrete or continuous behavior?

Ideal Characterization Functions

- can measure analog properties of devices (necessary to build them), but inconvenient generally
- instead take discrete measurements
- assume these measurements sample a continuous function that models device behavior
  - is reasonable approach, due to analog device components
  - expect continuous and smooth device behaviour between samples
  - if not, device will be unpopular
- never actually access the continuous behavior, due to discrete channels of device
- watch for quantization issues

Ideal Characterization Functions

- assuming underlying smooth function is convenient mathematically
  - can analyze behaviour of vector field, $f : \mathcal{R}^3 \rightarrow \mathcal{R}^3$
  - can differentiate to get Jacobian
  - can choose sensible interpolants based on local smoothness
  - can invert with some hope of efficiency, robustness
- what is the ideal characterization function?
- what operations are needed with the functions?

Using Characterization Functions: Evaluation

- for an output device, have mapping $f : \mathcal{D}_O \rightarrow \mathcal{C}$
- $\mathcal{D}_O$ is domain of output device
- $\mathcal{C}$ is colour space
- function evaluation predicts or *proofs* device behavior
- building functions of this type involves characterization by model, or by measurement and interpolation
  - fairly straightforward
  - don't usually need to evaluate function at out-of-gamut points (eg negative RGB)
  - challenge is efficiency

Using Characterization Functions: Inversion

- output device characterization is usually *inverted*, however
- want $f^{-1} : \mathcal{C} \rightarrow \mathcal{D}_O$
- inverse function answers question: how to output color c?
- characterization function should do more than fit device data, should also be easy to invert
- must first check that device measurements are well-behaved
  - is the device behavior inherently invertible?
  - quantization, device noise may suggest not
- may prefer invertibility of characterization over accurate data fitting

Using Characterization Functions: Composition

- for *color management*, must ship color data between devices
- characterize monitor with $f_M : \mathcal{D}_M \to \mathcal{C}$
- characterize printer with $f_P : \mathcal{D}_P \to \mathcal{C}$
- to proof printer colors on monitor, want system characterization $f = f_M^{-1} \circ f_P : \mathcal{D}_P \to \mathcal{D}_M$
- to print monitor colors accurately, want system characterization $f = f_P^{-1} \circ f_M : \mathcal{D}_M \to \mathcal{D}_P$
- need *composition* of characterization functions

Using Characterization Functions: Gamut Mapping

- may not want exact colorimetric reproduction between devices
  - devices have different *gamuts*
  - *gamut mapping* may be necessary to adjust colors to suit new device, new viewing environment
  - preserve user's *rendering intent*
- the gamut mapping is also applied by composition
  - to print monitor colors with gamut mapping g, want system characterization $f = f_P^{-1} \circ g \circ f_M : \mathcal{D}_M \to \mathcal{D}_P$

Using Characterization Functions: Projection

- one remaining difficulty: may want to output colors device can't handle
- such colors are outside the device gamut: *out-of-gamut colors*
- for an output device, characterization function $f : \mathcal{D}_{\mathcal{O}} \rightarrow \mathcal{C}$
- have inverse characterization function $f^{-1} : \mathcal{C} \rightarrow \mathcal{D}_O$
- usually try to have $f \circ f^{-1} \equiv I$ and $f^{-1} \circ f \equiv I$
  - tricky in practice due to quantization errors and numerical problems
  - must be especially careful at gamut edges
- inverse function can't always handle colors outside range of f
- need separate *projection* function to project out-of-gamut colors onto or into the device gamut

Using Characterization Functions: Projection

- projection often slower than optimized evaluation or inversion
- prefer to work on unique image colors (not do expensive calculation for each pixel)
- many projection approaches
  - clipping out-of-gamut colors to gamut edge versus compressing all colors
- makes representation of exterior of gamut very important

Using Characterization Functions: Summary

- operations on characterization functions include
  - *evaluation*
  - *inversion*
  - *composition*
- additional function of *projection*
- essentially want an algebraic *group* of functions
  - group has functions $\{f\}$ as elements, composition ($\circ$) as binary operation
  - domain and range of functions must be identical ($\mathcal{R}^3$ will do)
  - group has closure (functions are one-to-one and onto)
  - projection is method of forcing closure
- group concept is theoretically attractive, hard to ensure in practice

Linear Characterization Functions

- best possible model is linear: $f(\vec{x}) = M\vec{x}$
  - infinitely differentiable
  - efficient to evaluate
  - easy to invert (if invertible matrix)
  - composition $\rightarrow$ product of invertible matrices
    - extremely efficient: multiple operations compress to one matrix
  - if functions map all of $\mathcal{R}^3$, no need for projection
  - actually do have a group

Monitor Characterization Function

- never quite have ideal linear model in practice
- monitor characterization is close
- but limited range of RGB awkward
- mapping normalized RGB to normalized XYZ is one way of ensuring closure:
  - domain, range of f now unit cube
  - nonlinearity swept under carpet by LUTs
  - use normalization factors to return to reality
- other devices are unfortunately not so well-behaved
- piecewise linear approaches are possible

Characterization by Model: CRT Monitors

- cathode ray tubes (CRTs)
- same technology as color television
  - red, green, blue phosphors on screen
  - phosphors excited by R, G, B electron guns
  - phosphor luminance proportional to gun voltage to some power

CRT Monitors

- frame buffer → display [from Berns (2000)]

$$R_{lin} = \left[k_{g,r} \left(\frac{RLUT[d_r]}{d_{max}} \right) + k_{o,r} \right]^{\gamma_r}$$

- frame buffer holds d_r for red channel
- for 8-bit channels $d_{max} = 255$
- red gain and offset, $k_{g,r}$ and $k_{o,r}$
- gamma γ_r
- result is linearized R_{lin} value, if RLUT is built correctly
- similar equations for G_{lin} and B_{lin}
- can use matrix to convert $R_{lin}, G_{lin}, B_{lin}$ to CIE XYZ

CRT Monitors

- attempt to build RLUT, GLUT, BLUT so as to compensate for gamma
- curve is to the power $1/\gamma$

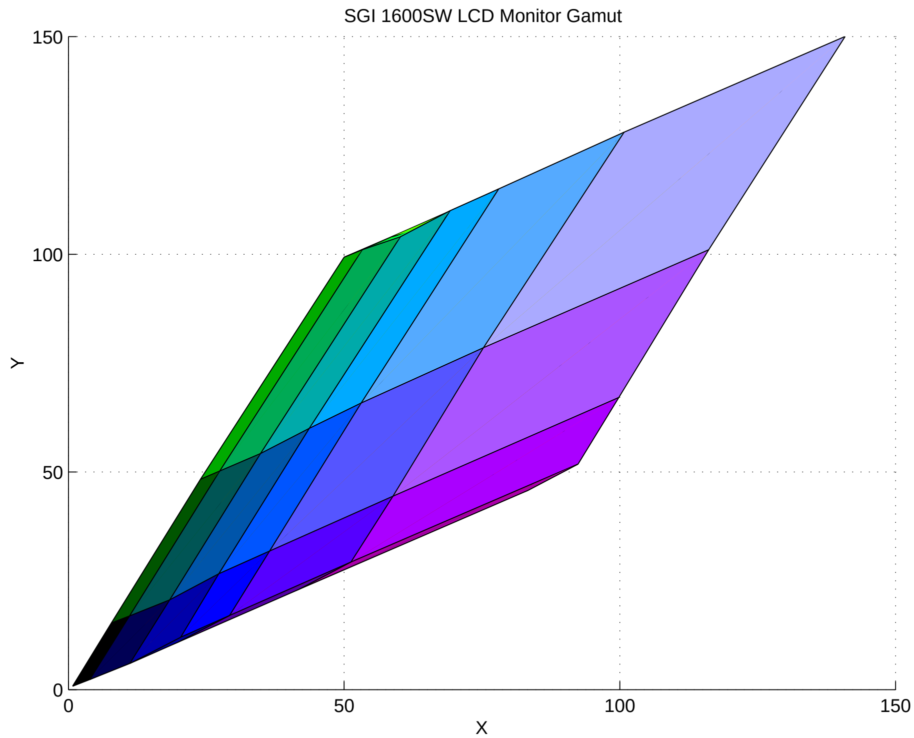
Monitor Metamerism

- note that matching monitor XYZ to object XYZ is risky
  - viewing conditions are often different
  - there is considerable metamerism with monitor
- could use larger matrix to convert to spectral representation

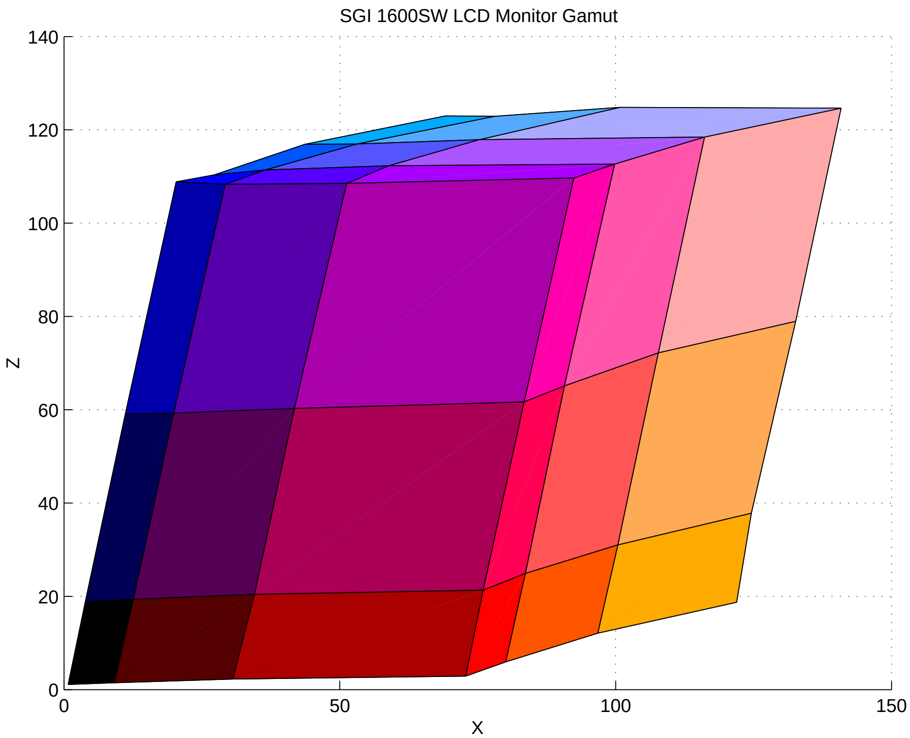
Characterization by Model: Printers

- challenging area due to variety of technologies, both subtractive and additive mixing, and errors due to device noise and quantization
- characterization by model
  - Neugebauer model is often the first step (intended for halftoning)
  - Emmel/Hersch model for inkjet printers
  - Berns model for dye diffusion thermal transfer printers
- characterization by measurement
  - more generic approach

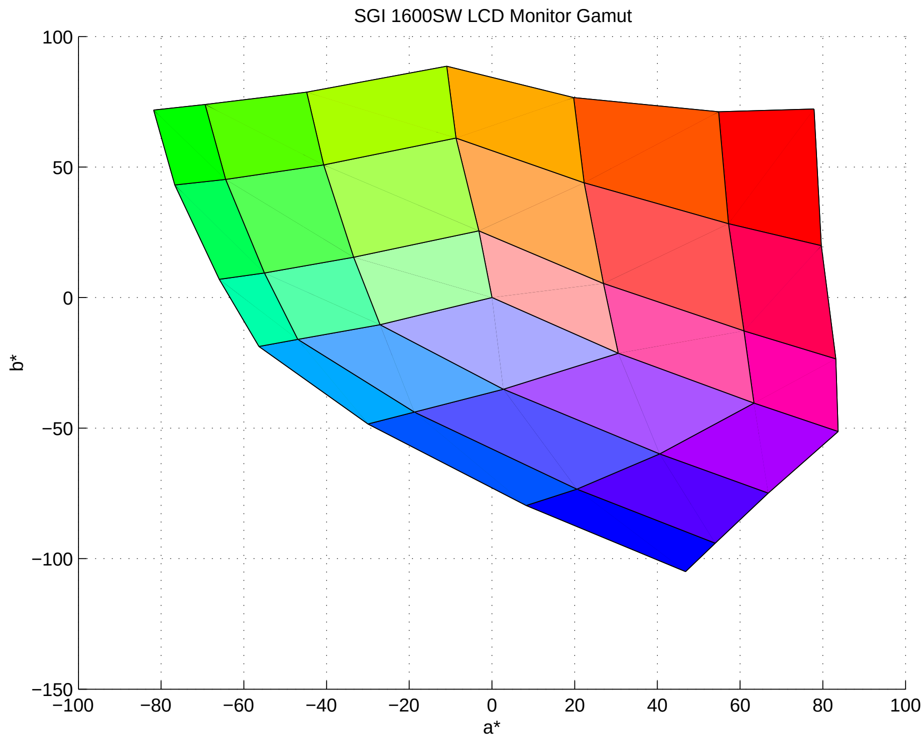
SGI 1600SW LCD Monitor Gamut (CIE XYZ)



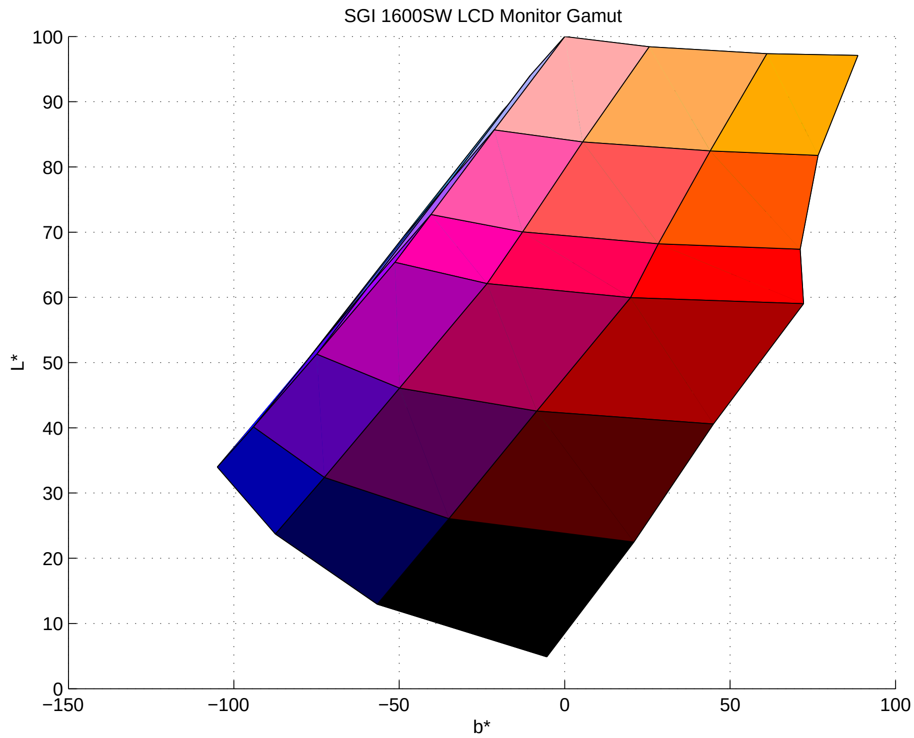
SGI 1600SW LCD Monitor Gamut (CIE XYZ)



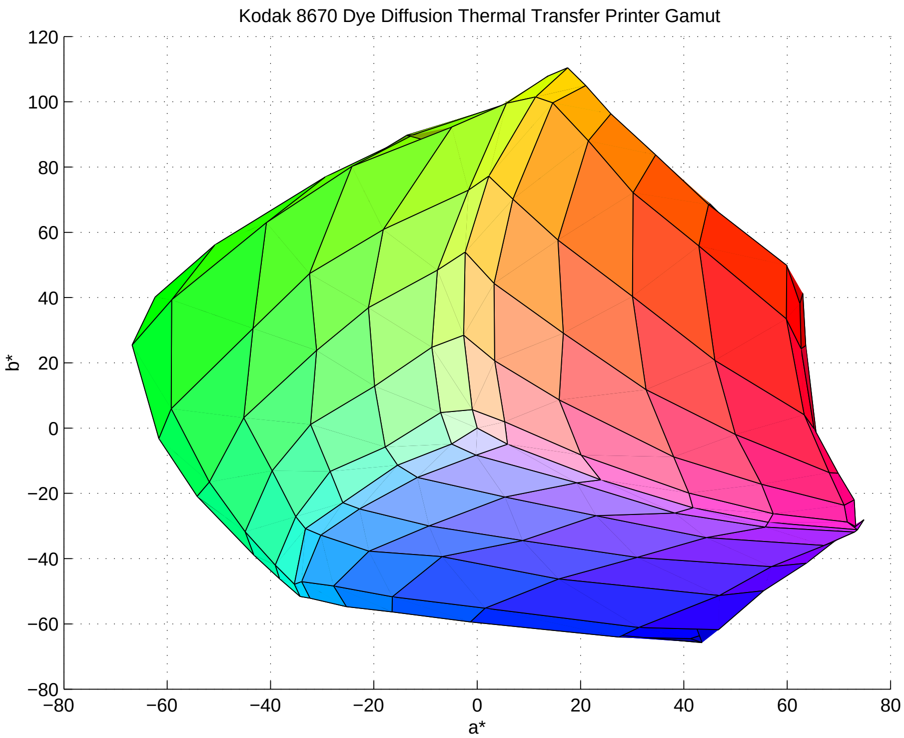
SGI 1600SW LCD Monitor Gamut (CIE LAB)



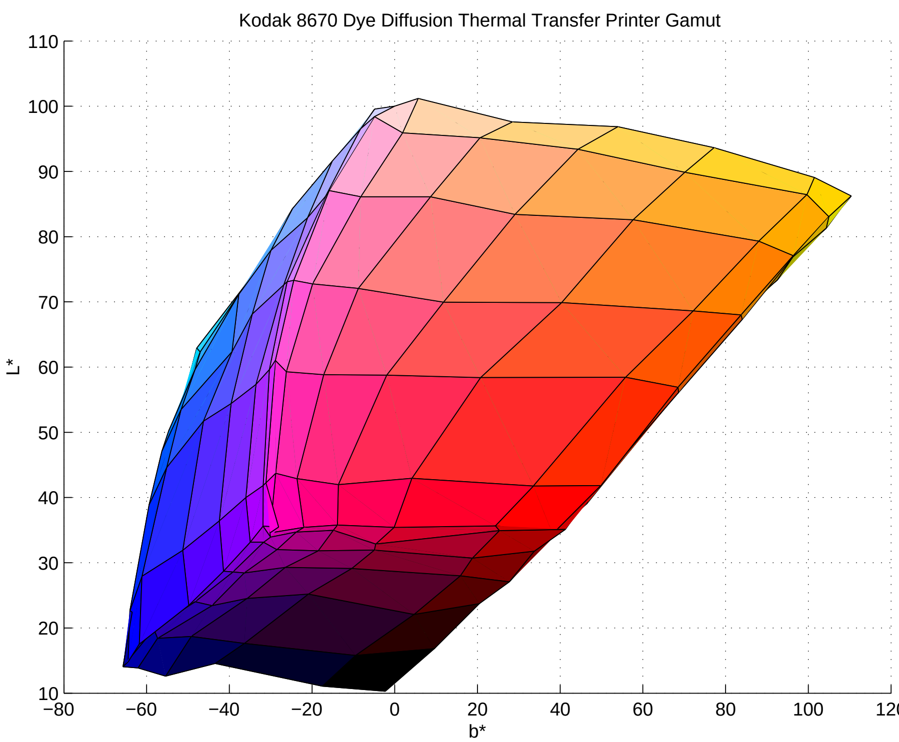
SGI 1600SW LCD Monitor Gamut (CIE LAB)



Kodak 8670 Dye Diffusion Thermal Printer Gamut (CIE LAB)



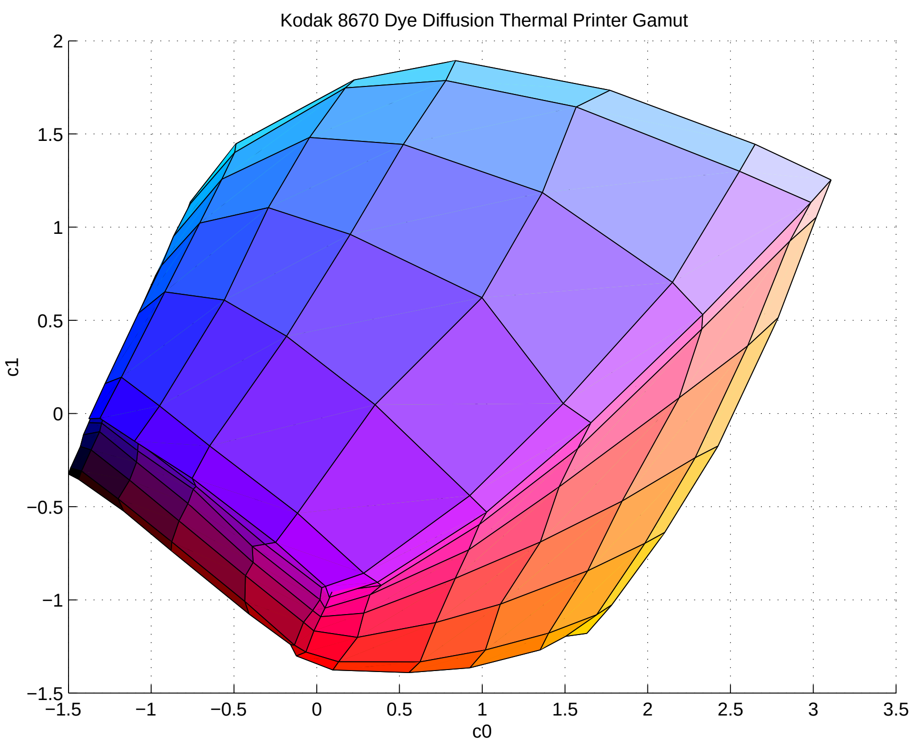
Kodak 8670 Dye Diffusion Thermal Printer Gamut (CIE LAB)



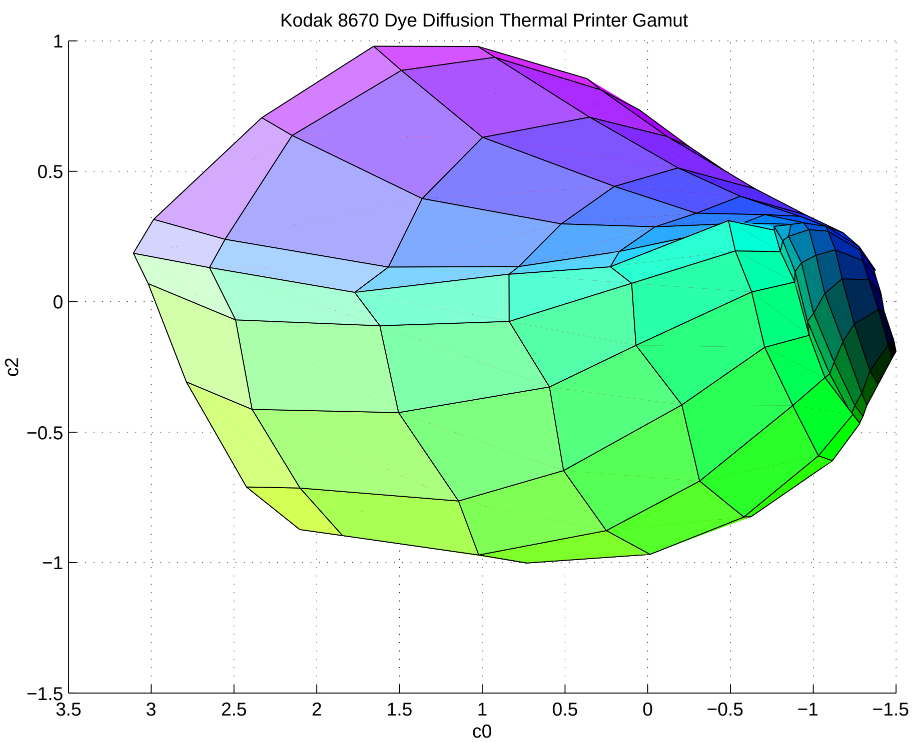
Spectral Gamuts

- linear reflectance coefficients
  - high-dimensional space
  - correlated with color spaces
  - greater accuracy with more coefficients

Kodak 8670 Dye Diffusion Thermal Printer Gamut (3 Reflectance Coefficients)



Kodak 8670 Dye Diffusion Thermal Printer Gamut (3 Reflectance Coefficients)



IV. GAMUT MAPPING AND COLOR MANAGEMENT
1. Gamut Mapping

- for characterization by measurement of output devices, often map rectilinear device input samples to unstructured points in a color space
  - easy and fast forward mapping f
  - inverse mapping f^{-1} harder
    - helps if f is smooth
    - still require root finder
- unstructured data to unstructured data is a more symmetric problem
  - regression commonly used
  - can correct errors with table interpolation methods
- still want this kind of composition, gamut mapping

$$f = f_P^{-1} \circ g \circ f_M : \mathcal{D}_M \to \mathcal{D}_P$$

Characterization by Measurement

- for output device, measure colors produced
- characterize device with $f(\mathbf{x}) = \mathbf{y}$, where $\mathbf{x}, \mathbf{y} \in \mathcal{R}^3$
- higher dimensions are possible
  - printing presses, or six-ink printers
  - modified data, as with linear reflectance models
- must develop interpolation methods for functions of this type

LUTs for Output Devices

- model is $f(\mathbf{x}) = \mathbf{y}$
- $\mathbf{x}$ is device inputs like RGB or CMY
- $\mathbf{y}$ is device output colors, measured in a space like CIE XYZ
- easier to model than input devices
  - have complete control over device inputs to be measured
  - can choose samples $\mathbf{x}_i$ in convenient grid
  - input channels have well-defined range (like 0 to 255 for 8-bit RGB)

Look-Up Tables (LUTs)

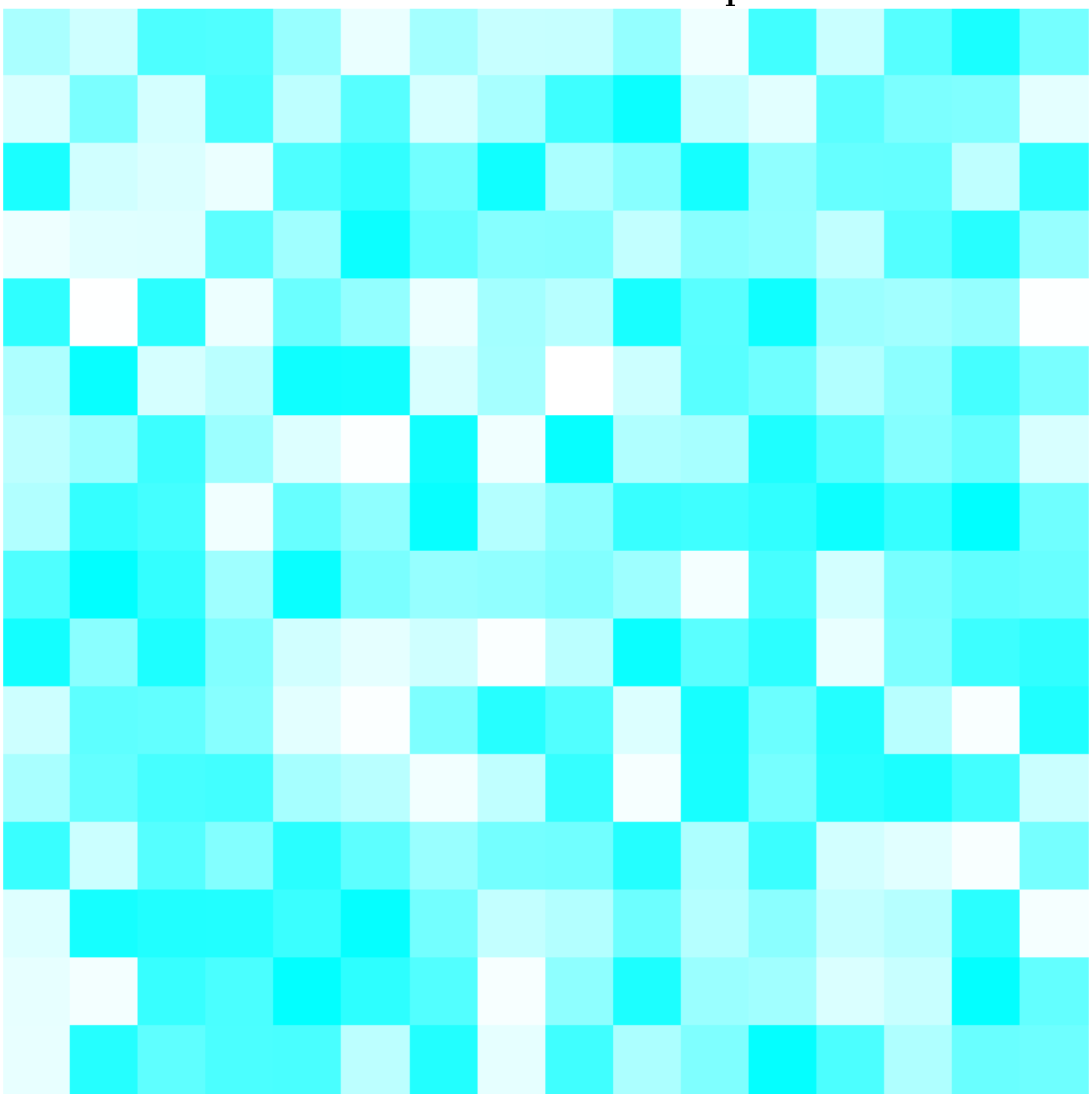
- devices deal with quantized color components
- 8 bits of R, G, B gives 24-bit color ("true color")
- could store table of $2^{24} \approx 16$ million $\mathbf{y}$ values
  - $\mathbf{x}$ values implied by table indexes
  - if quantize $\mathbf{y}$ with 3 bytes also, gives 48 MB table, is possible

Look-Up Table for All Possible Inputs

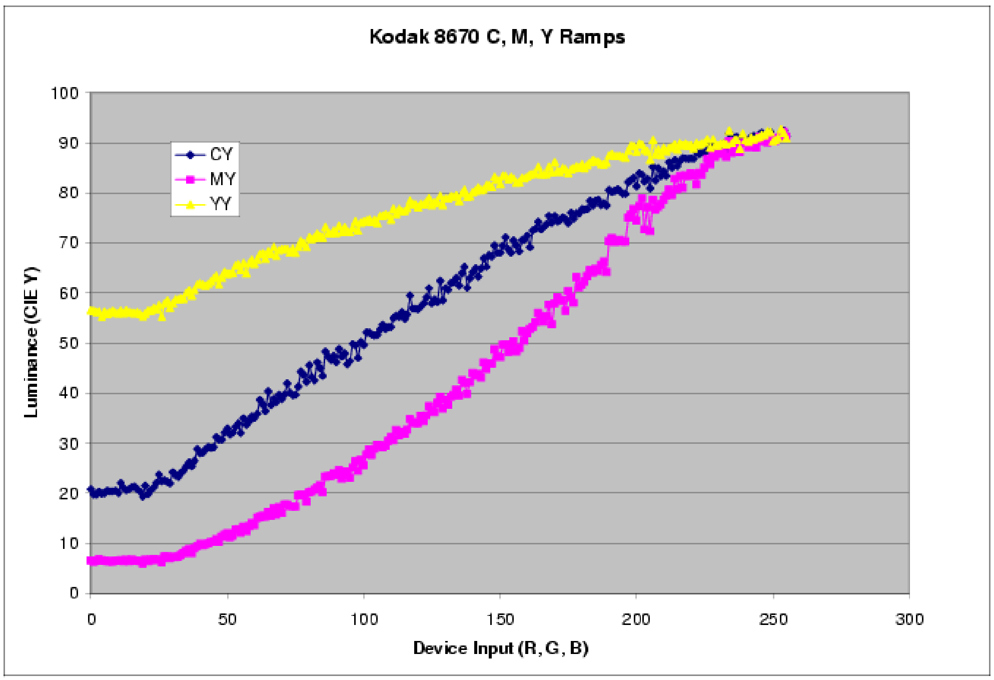
- Advantages:
  - no interpolation needed
  - very fast
- Disadvantages:
  - impractical to measure 16 million entries (although could measure fewer, interpolate with spline)
  - doesn't scale well to more bits, more dimensions
  - prefer to use less memory
  - device may be too noisy to require this kind of precision (eg most printers)

CMY Ramps

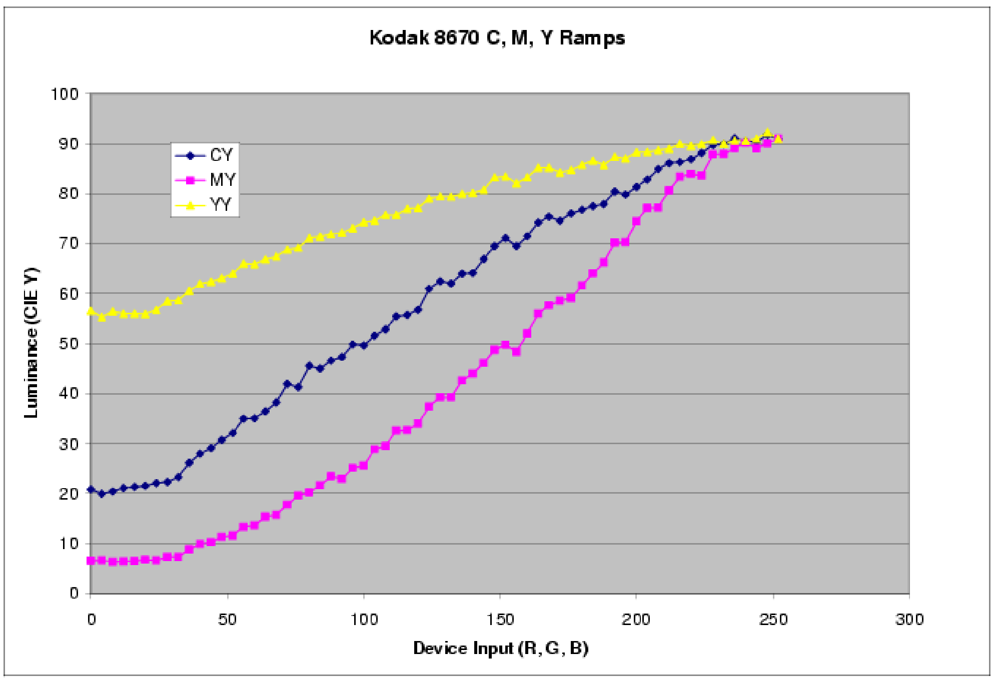
Randomized Color Samples



Kodak 8670 Dye-SubPrinter (8 Bits)



Kodak 8670 Dye-SubPrinter (6 Bits)



Look-Up Tables with Interpolation

- look-up tables for all possible inputs may not be needed
- device and measurement noise casts doubt on data
  - must take multiple measurements of single image to find instrument variance
  - must take measurements of multiple images to find device variance
- smooth interpolation of measured data is reasonable approach

Measurement

- when measuring anything, should keep track of
  - number of measurements
  - for each component
    - min and max
    - mean
    - variance or standard deviation
- can determine from this data whether normal distribution is appropriate

Interpolation/Approximation with LUTs

- find distribution of errors in device colours
- put error bars on graphs
- find interpolation/approximation method that
  - has appropriate smoothness for device
  - passes through error bars
    - approximating noisy data, or
    - interpolating averaged, accurate data
  - is sufficiently fast

Linear Interpolation in 1D

- have samples x_j for various j
- know $y_j = f(x_j)$
- want $\hat{f}(\cdot) \approx f(\cdot)$
- algorithm for computing $\hat{y} = \hat{f}(x)$:
  1. Find interval $[x_i, x_{i+1}]$ enclosing x, so that $x_i \leq x < x_{i+1}$
  2. Compute affine combination: $x = \bar{\alpha} x_i + \alpha x_{i+1}$, where

$$\bar{\alpha} = 1 - \alpha \tag{10}$$

$$\alpha = \frac{x - x_i}{x_{i+1} - x_i} \tag{11}$$

  3. Apply same affine combination to corresponding y_i, y_{i+1}

$$\hat{y} = \bar{\alpha} y_i + \alpha y_{i+1} \tag{12}$$

Linear Interpolation in 1D

- linear interpolation
  - is fast
  - interpolates the data
  - has undefined derivatives at data points (is not smooth)
  - is a linear (degree one) spline curve

Bilinear Interpolation in 2D

- looking for function approximating $f(x, y) = z$
- have samples x_i in x and y_j in y
- know function values at Cartesian product grid points, $f(x_i, y_j) = z_{i,j}$
- algorithm for computing $\hat{z} = \hat{f}(x, y)$:

  1. Find interval $[x_i, x_{i+1}]$ enclosing x, so that
  2. Find interval $[y_j, y_{j+1}]$ enclosing y, so that $y_j \leq y < y_{j+1}$
  3. Compute affine combination: $x = \bar{\alpha} x_i + \alpha x_{i+1}$
  4. Compute affine combination: $y = \bar{\beta} y_j + \beta y_{j+1}$
  5. Apply same affine combination to corresponding $z_{i,j}$

$$\begin{aligned} \hat{z} &= \bar{\alpha}\left(\bar{\beta} z_{i,j} + \beta z_{i,j+1}\right) + \alpha\left(\bar{\beta} z_{i,j+1} + \beta z_{i+1,j+1}\right) && (13) \\ &= \sum_{I=0}^{1}\sum_{J=0}^{1}[\alpha, \beta]^{<I,J>} z_{i+I,j+J} && (14) \end{aligned}$$

Bilinear Interpolation in 2D

- can implement recursively
  - 1D interpolation of two 1D interpolation problems
  - efficient and easy to code
- or can use explicit sum of $2^2 = 4$ vertices of enclosing cell

Trilinear Interpolation in 3D

- can implement recursively
  - 1D interpolation of two 2D interpolation problems
- or can use explicit sum of $2^3 = 8$ vertices of enclosing cell

$$\hat{w} = \sum_{I=0}^{1}\sum_{J=0}^{1}\sum_{K=0}^{1}[\alpha, \beta, \gamma]^{<I,J,K>} w_{i+I,j+J,k+K} \qquad (15)$$

Other Linear Interpolation Methods

- trilinear interpolation is degree one tensor product spline
- may think next step in complexity is degree two (quadratic) tensor product spline
- there is an intermediate method: *sequential linear interpolation (SLI)* [Allebach et al]
- idea is to allow new samples with each recursive call, for adaptive refinement

Sequential Linear Interpolation

- for bilinear case, wanting $\hat{z} = f(x, y)$
- first step is as with linear interpolation, find enclosing $[x_i, x_{i+1}]$
- let $\hat{z} = \bar{\alpha}\bar{g}_i(y) + \alpha g_i(y)$
- function $\bar{g}(y)$ performs linear interpolation on samples $\{\bar{y}_j\}$
- function $g(y)$ performs linear interpolation on samples $\{y_k\}$, a different set
- can generalize to trilinear case
- fairly easy to implement in hardware

Smooth Interpolation

- tensor product spline methods
  - generalizes well to higher dimensions
  - smooth functions
  - additional complexity
- simplex spline methods
  - Delaunay triangulation or tetrahedrization
  - barycentric interpolation
  - generalizing to higher dimensions, higher degrees is harder

Using a Data Fitter to Avoid Inversion

- characterization by measurement can use a data fitting algorithm
  - regression plus table interpolation is essentially a fitter
- in cases like scanner characterization, there is no structure to the data points in either space
- so there is no advantage to the forward mapping (as with trilinear interpolation over a grid)
- if mapping between spaces of equal dimension, like $\mathcal{R}^3 \rightarrow \mathcal{R}^3$, can
  - treat as a symmetric problem
  - fit in either direction
  - fit to avoid needing an explicit inversion
- composition of many mappings can be re-fit, expressed as one mapping

Linear Regression

- linear regression without constant term

$$XYZ_{pred} = M_{3\times3} * RGB$$

- linear regression with constant term

$$XYZ_{pred} = M_{3\times4} * RGB$$

- can be extended by adding higher degree terms

Quadratic Regression

- linear fit not usually good enough for scanners
- can use more than just $R, G, B, 1$ as terms
- add in $R^2, G^2, B^2, RG, GB, RB$ for *quadratic regression*

$$XYZ_{pred} = M_{3\times10} * RGB2$$

Cubic Regression

- add in $R^3, R^2G, R^2B, RG^2, RGB, RB^2, G^3, G^2B, GB^2, B^3$ for *cubic regression*

$$XYZ_{pred} = M_{3\times 20} * RGB3$$

Optimization

- mapping image pixels through polynomial can be expensive
- helps to identify unique RGB values with a color quantization algorithm
  - simple binning or hashing methods also possible
- can do matrix operations in hardware
- or can use look-up table if $x \in [0, 255]$
  - let $KLUT[x] = k * x$
  - let $QLUT[x] = x * x, \ldots$
  - can implement conversion RGB $\rightarrow$ RGB2 with look-up tables
  - matrix multiplication becomes adds and look-ups

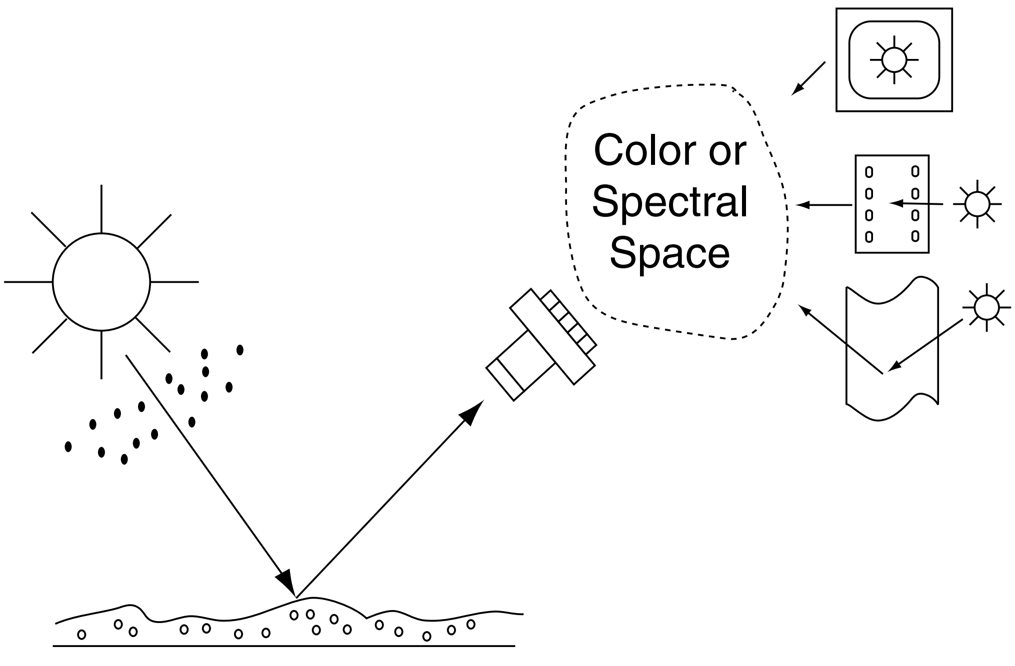
Regression and Sequential Linear Interpolation

- linear regression gives first approximation to data
- sequential linear interpolation accounts for residual
- both methods scale to higher dimensions
- both methods can yield invertible functions
- regression can also be paired with spline methods

Spline Interpolation

- piecewise linear approach is a degree one spline
- can generalize to
  - higher degree
  - Bezier curves
  - B-splines

Gamut Mapping



Gamut Mapping

- guidelines from Stone, Cowan, Beatty (1988)
  1. The gray axis of the image should be preserved.
  2. Maximum luminance contrast is desirable.
  3. Few colors should lie outside the destination gamut.
  4. Hue and saturation shifts should be minimized.
  5. It is better to increase than to decrease the color saturation.

Black- and White-Point Mapping

- mapping black point and white point is first step is aligning the neutral axes
- getting equal steps of gray to match is second step (Lamming and Rhodes)

Out-of-gamut Projection

- composition of functions may give some points outside the destination device gamut
- must project them into the gamut, using either
  - a continuous method (*gamut compression*)
  - a clipping method
- Stone, Cowan, Beatty recommend a continuous method that leaves a few points outside the gamut
  - desaturation
  - contrast scale factor
  - the umbrella transform

Geometric Methods

- computational geometry approach to avoid out-of-gamut colours, perform tetrahedral interpolation and extrapolation [Hardeberg (2001)]
- gamut-mapping can be carried out in linear reflectance space
- virtual camera as a gamut-mapping agent

2. Challenges in Multispectral Management

- color management systems are evolving
- they have not been the magic bullet to solve all color problems
- but they are a helpful first step
  - for simple "snapshot-quality" color reproduction
  - to consolidate and identify the problem areas
  - to provide a framework for future development

Challenges in Multispectral Management

- challenges include
  - greater accuracy of color reproduction
  - more adaptability to new input and output technologies
    - more channels
    - greater bit depth
    - greater resolution
    - variable/adaptive channels, resolutions
  - better gamut mapping and gamut compression
  - improved color appearance models
  - better quantification of rendering intent

Working with Spectral and High-Dimensional Data

- have seen many applications of spectral data
  - simulation of natural phenomena (sky, aurorae, ocean)
  - spectral acquisition (spectrophotometers, multispectral camera)
  - modeling of color devices (printers, film)
- is also possible to improve colorimetric accuracy with spectral calculations
  - extra scanner channels
  - high-dimensional models of printers
  - using extra degrees of freedom for matching in various viewing environments

Conclusions

For *multispectral management*:

- characterize devices spectrally
- choose optimal spectra/reflectances for SVD
- work in common spectral space
  - device gamuts containing image gamuts
  - simulation spectra
- adjust virtual camera for "exposure" that suits intersection of gamuts,
- adjust simulation for best gamut usage
- find best reproduction for user's *spectral rendering intent*
- implement mappings efficiently with characterization by model or by measurement

Open Problems

- many open problems remain
  - best choice of basis
  - dimensionality of basis
  - precision of coefficients
  - working with linear reflectance gamuts: gamut mapping, compression
  - managing spectral data in color management environment
  - exploiting metamerism, different viewing conditions

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