Surface Simplification Algorithms Overview

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Simplification approaches:

- incremental methods based on local updates
  - **mesh decimation** [Schroeder et al. `92, ... + others]
  - **energy function optimization** [Hoppe et al. `93, Hoppe '96, Hoppe '97]
  - **quadric error metrics** [Garland et al. '97]
- coplanar facets merging [Hinker et al. `93, Kalvin et al. `96]
- re-tiling [Turk `92]
- clustering [Rossignac et al. `93, ... + others]
- wavelet-based [Eck et al. `95, + others]

Incremental methods based on ***local updates***

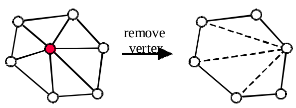
- All of the methods such that :
  - simplification proceeds as a sequence of ***local updates***
  - each update ***reduces mesh size*** and [monotonically] ***decreases*** the ***approximation precision***

- Different approaches:
  - **mesh decimation**
  - **energy function optimization**
  - **quadric error metrics**

- Local update actions:

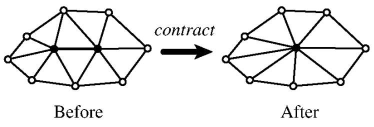
No. Faces

- **vertex removal**



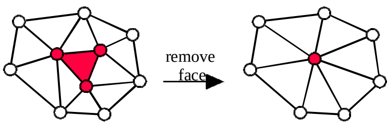
n-2

- **edge collapse**
  - **preserve location**
  - **new location**



n-2

- **triangle collapse**
  - **preserve location**
  - **new location**



n-4

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The common framework:

- **loop**
  - ***select*** the element to be deleted/collapsed;
  - ***evaluate approximation*** introduced;
  - ***update*** the mesh after deletion/collapse;

until mesh **size/precision** is satisfactory;

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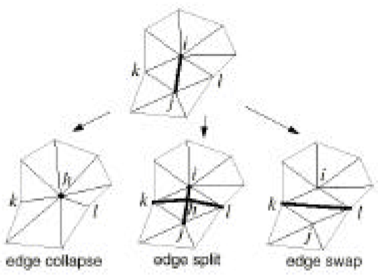
Energy function optimization

Mesh Optimization **[Hoppe et al. `93]**

- Simplification based on the iterative execution of :
  - edge collapsing
  - edge split
  - edge swap



- approximation quality evalued with an **energy function** :

  $E(M) = E_{dist}(M) + E_{rep}(M) + E_{spring}(M)$

  which evaluates geometric **fitness** and repr. **compactness**

  E_{dist} : sum of squared distances of the original points from M

  E_{rep} : factor proportional to the no. of vertex in M

  E_{spring} : sum of the edge lenghts

Algorithm structure

- outer minimization cicle (***discrete*** optimiz. probl.)
  - choose a legal action (edge collapse, swap, split) which reduces the energy function
  - perform the action and update the mesh (M_i -> M_{i+1})
- inner minimization cicle (***continuous*** optimiz. probl.)
  - optimize the vertex positions of M_{i+1} with respect to the initial mesh M_0

but (to reduce complexity)

- legal action selection is random
- inner minimization is solved in a fixed number of iterations

Mesh Optimization - *Examples*

(j) Laser range data (v = 12,745) (k) Output of phase one (l) Output of phase two

[Image by Hoppe et al.]

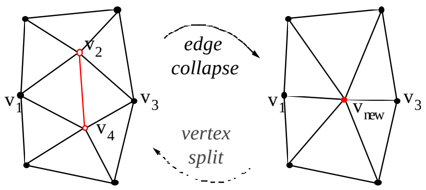
Mesh Optimization - *Evaluation*

- high quality of the results
- preserves topology, re-sample vertices
- high processing times
- not easy to implement
- not easy to use (selection of tuning parameters)
- adopts a global error evaluation, but the resulting approximation is not bounded

implementation available on the web

Progressive Meshes **[Hoppe `96]**

- execute **edge collapsing** ***only*** to reduce the *energy function*
- *edge collapsing* can be easily inverted ==> store sequence of inverse *vertex split* trasformations to support:
  - multiresolution
  - progressive transmission
  - selective refinements
  - geomorphs
- *faster* than MeshOptim.



... Energy function optimization: **Progressive Meshes** ...

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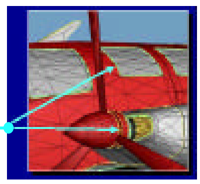
Preserving mesh ***appearance***

- shape and crease edges
- scalar fields discontinuities (e.g. color, normals)
- discontinuity curves



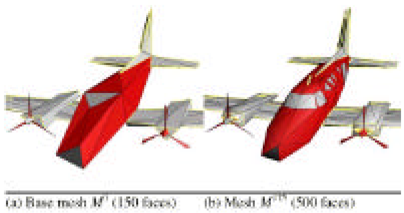
[image by H. Hoppe]

Managed by inserting two new components in the *energy function*:

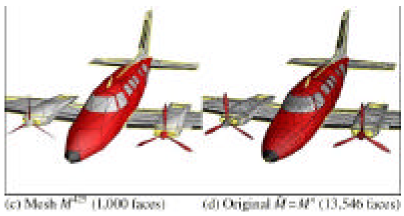
- E_{scalar}: measures the accuracy of scalar attributes
- E_{disc}: measure the geometric accuracy of discontinuity curves

... Energy function optimization: **Progressive Meshes** ...

Progressive Meshes
Examples



(a) Base mesh M^0 (150 faces) (b) Mesh M^{175} (500 faces)



(c) Mesh M^{425} (1,000 faces) (d) Original $\hat{M} = M^n$ (13,546 faces)

... Energy function optimization: Progressive Meshes...

Progressive Meshes - *Evaluation*

- high quality of the results
- preserves topology, re-sample vertices
- not easy to implement
- not easy to use (selection of tuning parameters)
- adopts a global error evaluation, not-bounded approximation
- preserves vect/scalar attributes (e.g. color) **discontinuities**
- supports **multiresolution** output, geometric morphing, **progressive transmission, selective** refinements
- much **faster** than MeshOpt.

will be available in MS DirectX 5.0 graphics interface

Decimation

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Mesh Decimation **[Schroeder et al'92]**

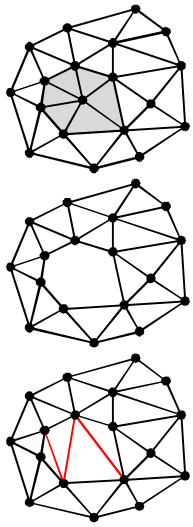
- Based on controlled removal of ***vertices***
- Classify vertices as ***removable*** or ***not*** (based on local topology / geometry and required precision)

Loop

- choose a *removable* vertex $\mathbf{v}_i$
- delete $\mathbf{v}_i$ and the incident faces
- re-triangulate the hole

until

no more removable vertex **or** reduction rate fulfilled



... Decimation ...

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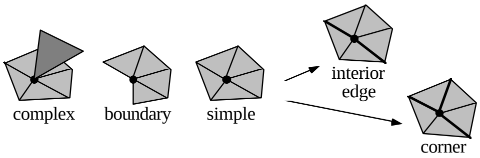
- General method (manifold/non-manifold *input*)
- Algorithm phases:
  - topologic classification of vertices
  - evaluation of the decimation criterion (error evaluation)
  - re-triangulation of the removed triangles patch

Topologic classification of vertices

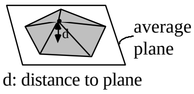
- for each vertex: find and characterize the loop of incident faces

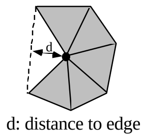


- *interior edge*: if dihedral angle between faces $< k_{angle}$
  (k_{angle} : user driven parameter)
- *not-removable vertices*: complex, [corner]

Decimation criterion -- a vertex is *removable* if:

- **simple** vertex:
  if distance ***vertex - face loop average plane***
  is lower than ε_{max}
- **boundary / interior / corner** vertices:
  if distance ***vertex - new boundary/interior edge***
  is lower than ε_{max}

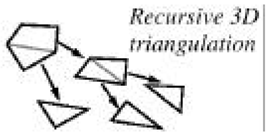




- adopts *local evaluation* of the approximation!!
- ε_{max} : value selected by the user

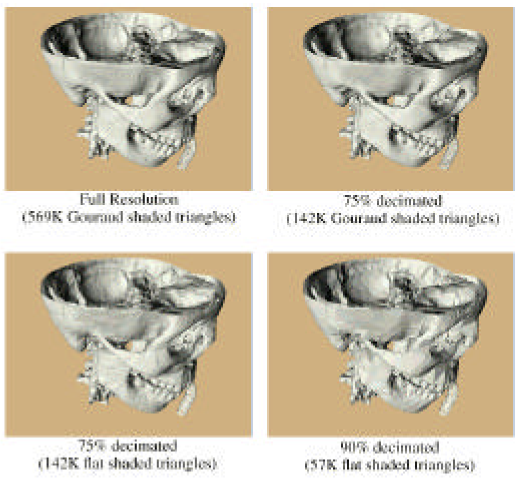
Re-triangulation

- face loops in general non planar !   (but star-shaped)
- adopts ***recursive loop splitting*** re-triangulation



- control *aspect ratio* to ensure simplified mesh quality
- for each vertex removed:
  - *if* simple or boundary vertex ==> 1 loop
  - *if* interior edge vertex ==> 2 loops
  - *if* boundary vertex ==> - 1 face
  - *otherwise* ==> - 2 faces

Decimation - *Examples*



(images by W. Lorensen)

Original Mesh Decimation - *Evaluation*

- good efficiency (speed & reduction rate)
- simple implementation and use
- good approximation
- works on huge meshes
- preserves topology; vertices are a subset of the original ones
- error is ***not*** bounded (local evaluation ==> accumulation of error!!)

*implemented in the **Visualization Toolkit** (**VTK**), public domain*

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Enhancing Mesh Decimation

- Improve approximation precision, ensure bounded error
  - **bounded** error [Cohen'96, Gueziec'96]
  - **global error** evaluation [Soucy'96, Bajaj'96,Klein'96,Ciampalini'97, +...]
  - smarter **re-triangulation** (edge flipping) [Bajaj'96, Ciampalini'97]
- Multiresolution, dynamic LOD [Ciampalini'97]
- Decimate other entities
  - **edges** (collapse into vertices) [Gueziec'95-'96,Ronfard'96, Algorri96]
  - **faces** (collapse into vertices) [Hamann '94]
- Preserve color and attributes info [Soucy'96, Cohen et al 98, Cignoni etal 98, +....]
- Topology simplification [Lorensen 97]
- Extension to 3D meshes (tetrahedral meshes) [Renze'96, Trotts etal 98, Staadt et al 98]

Approximation Error Evaluation

User' viewpoint:
- simple to grasp
- simple to drive

Classification of simplification methods based on ***approximation error*** evaluation euristics:

- **locally-bounded** error, based on mesh distances [ex. standard Mesh Decimation]
- **globally bounded** error, based on mesh distances [ex. Envelopes + enhanced Decimation + others]

→ *very handy*

- control based on **mesh characteristics** [ex. vertex proximity, mesh curvature] → *may be misleading*
- **energy function** evaluation [ex. Mesh Optim. , Progr. Meshes] → *not easy, many parameters to be selected*

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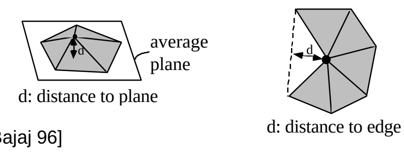
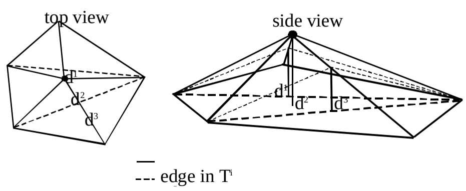
Heuristics proposed for ***local error evaluation***:

- *approximate* evaluation [Schroeder 92]



- *correct* evaluation [Bajaj 96]
  given two linear patches ==>
  the max value of meshes' distance is either on edges' intersections or on internal vertices



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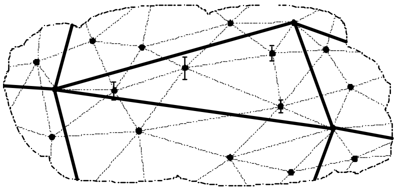
Heuristics proposed for ***global error evaluation***:

- ***accumulation*** *of local errors* [Ciampalini97]
  fast, **but** approximate

- ***vertex--to--simplified mesh*** **distance** [Soucy96]
  requires storing which of the original vertices maps to each simplified face;
  very near to exact value (but large under-estimation in the first steps)



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... Heuristics proposed for ***global error evaluation***:

- ***input mesh -- to -- simplified mesh <u>edges</u>*** **distance** [Ciampalini97]
  - for each internal edge:
    - select sampling points $\mathbf{p}_i$ (regularly/random)
    - evaluate distance d(M_0, $\mathbf{p}_i$)

  sufficiently precise and efficient in time

- ***input mesh -- to -- simplified mesh*** **distance** [Klein96]
  precise, **but** more complex in time

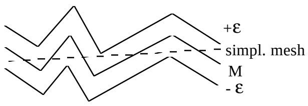
- **use *envelopes*** [Cohen et al.'96]
  precise, no self-intersections **but** complex in time and to be implemented

Simplification Envelopes

[Cohen et al.'96]

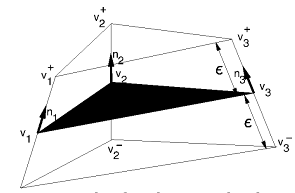
- given the input mesh ***M***
  - build two envelope meshes M_- and M_+ at distance **-e** and **+ e** from ***M*** ;
  - simplify M (following a decimation approach) by enforcing the decimation criterion:

    a candidate vertex may be removed **only if** the new triangle patch does not intersect neither M_- or M_+



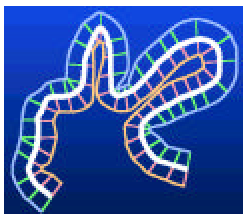
- by construction, envelopes do not self-intersect

  ==> simplified mesh is **not self-intersecting** !!



the fundamental prism

- distance between envelopes becomes smaller near the bending sections, and simplification harder
- **border tubes** are used to manage open boundaries



(drawing by A. Varshney)

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Simplification Envelopes - *Evaluation*

- works on manifold surface **only**
- bounded approximation
- construction of envelopes and intersection tests are not cheap
- **>** three times more RAM (input mesh + envelopes + border tubes)
- preserve topology, vertices are a subset of the original, prevents self-intersection

available in public domain

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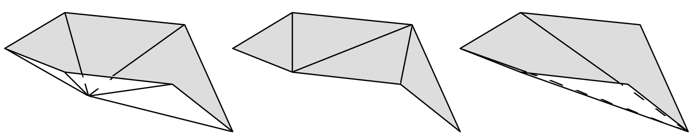
For all methods based on re-triangulation, approximation depends on **new patch quality**

- control new triangles' **aspect ratio**, to avoid slivery faces [equiangularity]
- adopt **edge flipping** to improve mesh quality [Bajaj'96, Ciampalini97]
  - build a first triangulation and, through a **greedy** optimization process based on edge flipping, adapt it to the original mesh
  - **global error** estimate is needed to support flipping

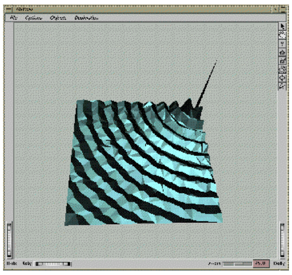


Original — A triangulation — A better triangulation

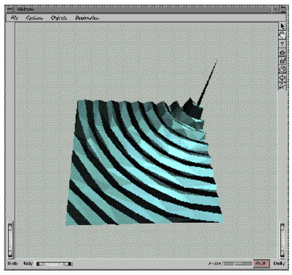
Mesh approximation improvement due to edge flipping **(Jade2.0 code)**

- original mesh: 28,322 triangles
- simplified meshes: same approximation error

no flipping: 1004 faces



with flipping: 528 faces



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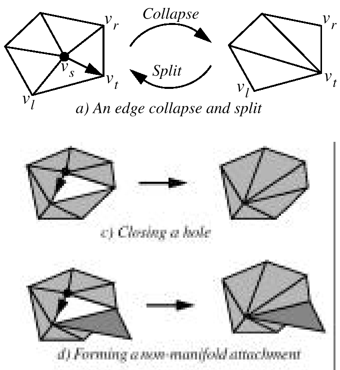
Topology Modifying Progressive Decimation

[Schroeder Vis97]

- *topology preservation*: a limiting factor in overall reduction capability
- adopts a **progressive-mesh** approach on top of an **edge-collapse** based mesh decimator
- atomic action: **edge collapse** encoded for progressive storage, transmission, and reconstruction **==>** holes may close, non-manifold attachments may form
- uses a priority queue to store candidate vertices
- *available in the **vtk** system*



Jade 2.0 (Multiresolution Glob. Err. Decim.)

[Ciampalini et al.'97]

- **Goals:**
  - ***Speed***
  - ***Precision*** global error management
  - ***Simpl. Efficiency*** good compression ratio
  - ***Generality*** not orientable, not manifold surfaces
  - ***Multiresolution output***
  - ***Ease of use*** given a target **# vertices** ==> "best" quality mesh
    given a target **approx.error** ==> "smallest" mesh
- **code in the public domain** (SGI executables only)

Candidate vertices selection

- *vertex **classification***: same as standard Mesh Decimation
- uses an **heap** to store candidate vertices in order of error
  - *heap initialization*: for all vertices, simulate removal and evaluate approximation introduced
- evaluation of the ***error*** introduced while removing a vertex:
  - approximated *input_mesh---to---simpl_mesh* distance
  - integrated with *edge flipping* test
- *vertex **selection*** for removal:
  - in order of **increasing error** (from ***heap***)
  - decimating sorted vertices improves mesh quality and is crucial to support multiresolution

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```
Algorithm Jade 2.0 (M0,target_err,S)
   Var VH: Heap;         \{vertex heap, sorted by increasing error\}
     S:= M0;
     \{initialize the heap VH: \}
     FOR EACH vertex vi in M0 DO
        compute error ei associated to the removal of vi (includes
        re-triangulation but not mesh update);
        insert (vi,ei) in VH;

     \{main cycle: \}
     REPEAT
        pop first candidate v from VH;
        delete from simplified mesh S;
        retriangulate the hole in S;
        err := current_approximation(S);
        check error in VH for the vertices on the border of the
        re-triangulated hole (and, in case, update heap VH );

     UNTIL err <= target_err;
   END;
```


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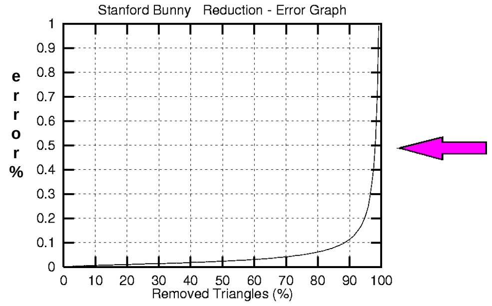
Results

- Simplification times ~= linear with mesh size



no staircase abrupt error increase
(fundamental for the quality of the multiresolution output)

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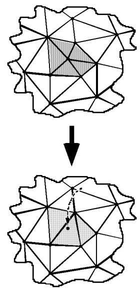
Construction of a multiresolution model

Keep the ***history*** of the simplification process :

- when we remove a vertex we have **dead** and **newborn** triangles
- assign to each triangle *t* a ***birth error*** t_b and a ***death error*** t_d equal to the error of the simplified mesh just before the removal of the vertex that caused the birth/death of *t*



By storing the ***simplification history*** (faces+errors) we can simply extract ***any approximation level*** in real time

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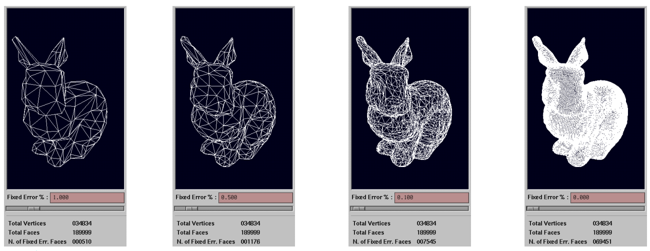
Real-time resolution management

- by extracting from the ***history*** all the triangles t_i with

  $$t_b \le \varepsilon < t_d$$

  we obtain a model $\mathbf{M}_\varepsilon$ which satisfies the approximation error ε
- mantaining the whole *history* data structure costs approximately 2.5x - 3x the full resolution model



real-time LOD

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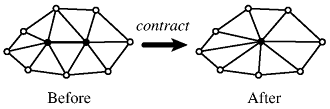
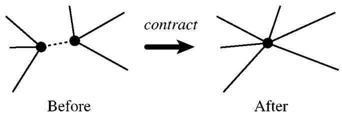
Simplification using Quadric Error Metrics

[Garland et al. Sig'97]

- Based on incremental **edge-collapsing**



- **but** can also collapse vertex couples which are **not connected** (topology is not preserved)



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Geometric error approximation is managed by simplifying an approach based on **plane set distance** **[Ronfard,Rossignac96]**

- INIT_time: store for each vertex the set of incident planes
- Vertex_Collapsing $(v_1, v_2) \Rightarrow v_{new}$
  - plane_set (v_{new}) = union of the two **plane sets** of v_1, v_2
  - collapse only if v_{new} is not "farther" from its plane set than the selected target error **ε**

criticism:

- storing plane sets and computing distances is not cheap !

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Quadric Error Metrics solution:

- quadratic distances to planes represented with **matrices**
  - plane sets merge *via* matrix sums
  - very efficient evaluation of error *via* **matrix operations**

  but
  - triangle size is taken into account only in an approximate manner (orientation only in Quadrics + weights)

Algorithm structure:

- select valid vertex pairs (upon their distance),

  insert them in an heap sorted upon minimum cost;
- **repeat**
  - extract a valid pair v_1, v_2 from heap and contract into v_{new};
  - re-compute the cost for all pairs which contain v_1 or v_2 and update the heap;

  until sufficient reduction/approximation **or** heap empty

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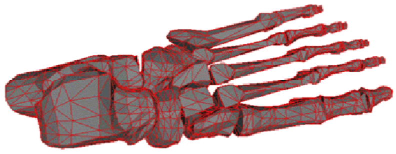
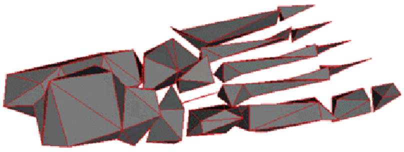
An example

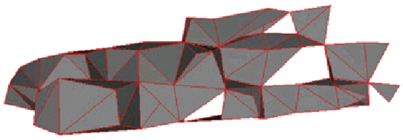
- **Original.** Bones of a human's left foot (4,204 faces).
- Note the many separate bone segments.

- **Edge Contractions.** 250 face approximation.
- Bone seg-ments at the ends of the toes have disappeared; the toes appear to be receding back into the foot.





- **Clustering.** 262 face approximation.



[Images by Garland and Heckbert]

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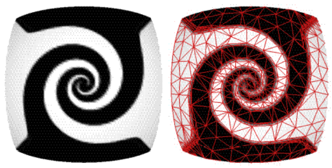

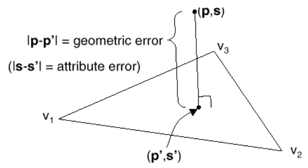

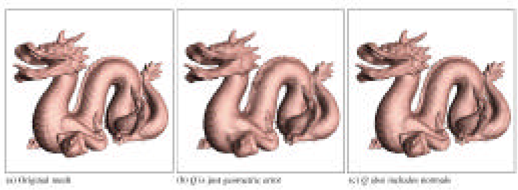

Quadric can be exteneded to take into account:

- color and texture attributes error are computed by projecting them in R^{3+m} [Garland 98]
- by computing attribute error as the squared deviation between original value and the value interpolated [Hoppe 99]

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Quadric Error Metrics -- *Evaluation*

- iterative, incremental method
- error is bounded
- allows topology simplification (aggregation of disconnected components)
- results are very high quality and ***times incredibly short***
- Various commercial packages use this technique (or variations)

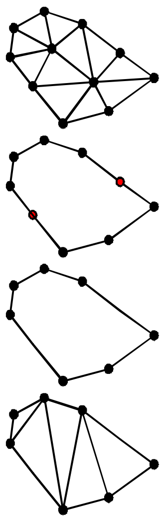
Not-incremental methods:

- coplanar facets merging [Hinker et al. `93, Kalvin et al. `96]
- re-tiling [Turk `92]
- clustering [Rossignac et al. `93, ... + others]
- wavelet-based [Eck et al. `95]

Coplanar Facets Merging

Geometric Optimization **[Hinker '93]**

- Construct nearly co-planar sets (comparing normals)
- Create edge list and remove duplicate edges
- Remove colinear vertices
- Triangulate resultant polygons



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Geometric Optimization - Evaluation

- simple and efficient heuristic
- evaluation of approximation error is highly inaccurate and not bounded
  (error depends on relative size of merged faces)
- vertices are a subset of the original
- preserves geometric discontinuities (e.g. sharp edges) and topology

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Superfaces

[Kalvin, Taylor '96]

- group mesh faces in a set of *superfaces*:
  - iteratively choose a seed face f_i as the current *superface* Sf_j
  - find by propagation all faces adjacent to f_i whose vertices are at distance $\varepsilon/2$ from the mean plane to Sf_j and insert them in Sf_j
  - moreover, to be merged each face must have orientation similar to those of others in Sf_j
- straighten the *superfaces* border
- re-triangulate each *superface*

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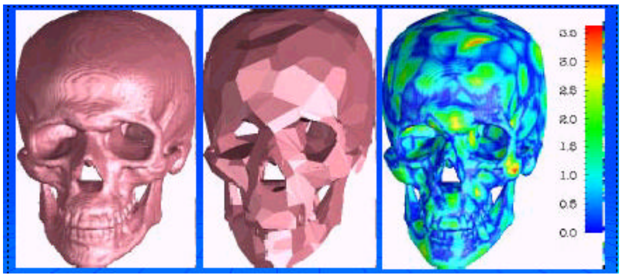
Superfaces - an example

- Simplification of a human skull (fitted isosurface),
  images courtesy of IBM



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Superfaces **- Evaluation**

- slightly more complex heuristics
- evaluation of approximation error is more accurate and bounded
- vertices are a subset of the original ones
- preserves geometric discontinuities (e.g. sharp edges) and topology

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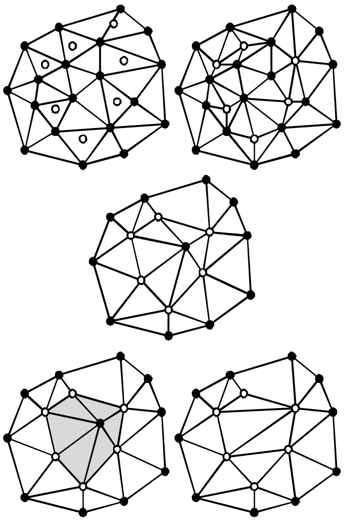
Re-tiling

Re-Tiling **[Turk `92]**

- Distribute a new set of vertices into the original triangular mesh (points positioned using repulsion/relaxation to allow optimal surface curvature representation)
- Remove (part of) the original vertices
- Use local re-triangulation

no info in the paper on time complexity!



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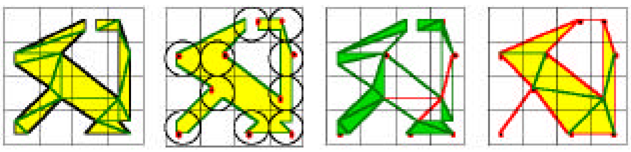
Clustering

Vertex Clustering **[Rossignac, Borrel `93]**

- detect and unify *clusters* of nearby vertices
  (discrete gridding and coordinates truncation)
- all faces with two or three vertices in a cluster are removed
- does not preserve topology (faces may degenerate to edges, genus may change)
- approximation depends on grid resolution



(figure by Rossignac)

Clustering -- Examples (1)

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- Simplification of a table lamp,
  IBM 3D Interaction Accelerator, courtesy IBM

10,108 facets 1,383 facets 474 facets 46 facets

Clustering -- Examples (2)

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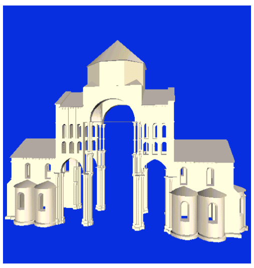
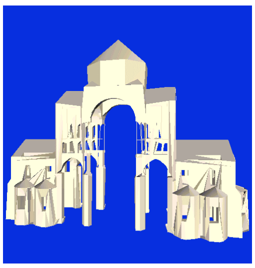
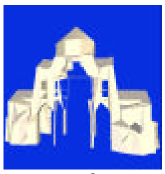
- Simplification of a portion of Cluny Abbey,
  IBM 3D Interaction Accelerator, courtesy IBM France.



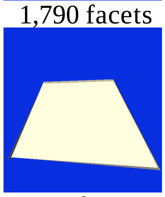
46,918 facets



6,181 facets



1,790 facets



16 facets

Clustering - *Evaluation*

- high efficiency (but timings are not reported in the paper)
- very simple implementation and use
- low quality approximations
- does not preserve topology
- error is bounded by the grid cell size

part of IBM 3D Interaction Accelerator

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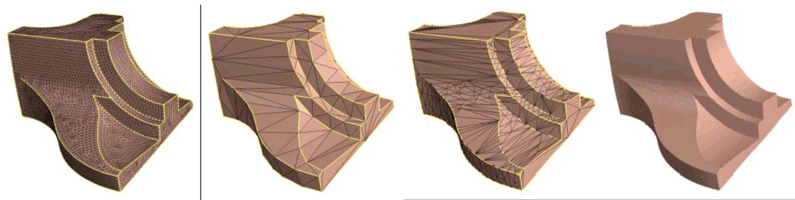
Multiresolution Analysis

[Eck et al. '95, Lounsbery'97]

- Based on the ***wavelet*** approach
  - simple base mesh
  - + local correction terms (wavelet coefficients)
- Given input mesh M:
  - ***partition*** : build a low resolution base mesh K_0 with tolerance ε_1
  - ***parametrization*** : for each face of K_0 build a parametrization on the corresponding faces of M
  - ***resampling*** : apply $\boldsymbol{j}$ recursive quaternary subdivision on K_0 to build by parametrization different approximations $\boldsymbol{K_j}$
- Supports:
  bounded error, compact multiresolution repr., mesh editing at multiple scales

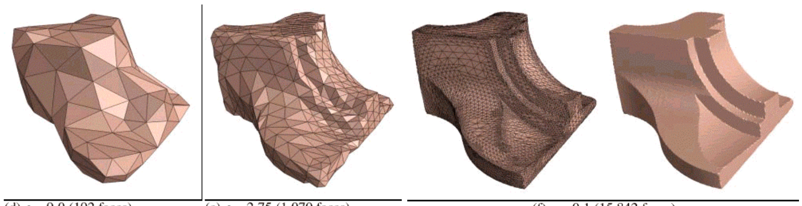
Hoppe's experiment: comparative eval. of quality of multiresolution representation

- Progressive Meshes



(a) $\hat{M}$ (12,946 faces) (b) M^{75} (200 faces) (c) M^{475} (1,000 faces)

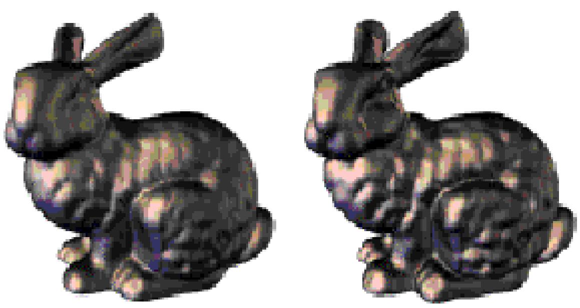
- Multiresolution Analysis



(d) $\epsilon = 9.0$ (192 faces) (e) $\epsilon = 2.75$ (1,070 faces) (f) $\epsilon = 0.1$ (15,842 faces)

Multires Signal Processing for Meshes [Guskov, Swelden,Schroeder 99]

- Still the ***Partition, Parmetrization and Resampling*** approach but the original mesh connectivity is retained:
  - partition is done on the simplified mesh
  - use of a ***non-uniform relaxation procedure*** (instead of standard triangle quadrisection) that mimics the inverse simplification process
  - Possibility of using signal processing techniques on mesh (eg. Smoothing, detail enhancement …)



[image by courtesy of Guskov et al. 99]

Preserving detail on simplified meshes

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- Problem Statement :

  how can we preserve in a *simplified* surface
  the **detail** (or **attribute value**)
  defined on the *original* surface ??

- What one would preserve:
  - **color** (per-vertex or texture-based)
  - **small variations of shape curvature** (bumps)
  - **scalar fields**
  - **procedural textures** mapped on the mesh

... Preserving detail on simplified meshes ...

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Approaches proposed in literature are:

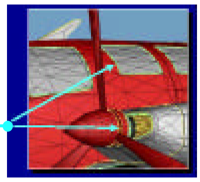
- **integrated** in the simplification process
  (ad hoc solutions **embedded** in the simplification codes)

- **independent** from the simplification process
  (post-processing phase to restore attributes detail)

Integrated approaches:

- attribute-aware simplification
  - do not simplify an element ***e*** **IF** ***e*** is on the boundary of two regions with different attribute values

  or
  - use an enhanced multi-variate approximation evaluation metrics (shape+color+…) [Hoppe96,GarHeck98,Frank etal98, Cohen etal98]



(image by H. Hoppe)

- store removed detail in textures
  - *vertex-based* [Maruka95 , Soucyetal96]
  - *texture-based* [Krisn.etal96]
- preserve **topology** of the attribute field [Bajaj et al.98]

Simplification-Independent approach:

our Vis'98 paper [Cignoni etal 98]

- ***higher generality:*** attribute/detail preservation is not part of the simplification process
- performed as a ***post-processing*** phase (after simplification)
- any attribute can be preserved, by constructing an ad-hoc ***texture map***

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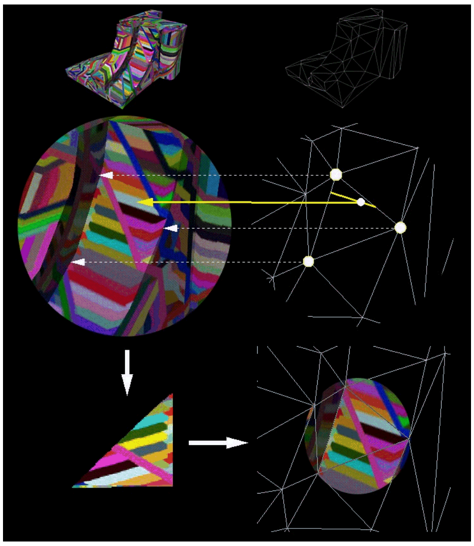
A simple idea:

- for each simplified face:
  - detect the original detail
  - code it into a triangular texture map
- pack all textures patches in a std. rectangular texture



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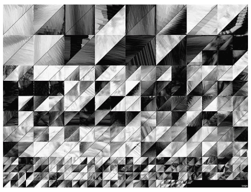
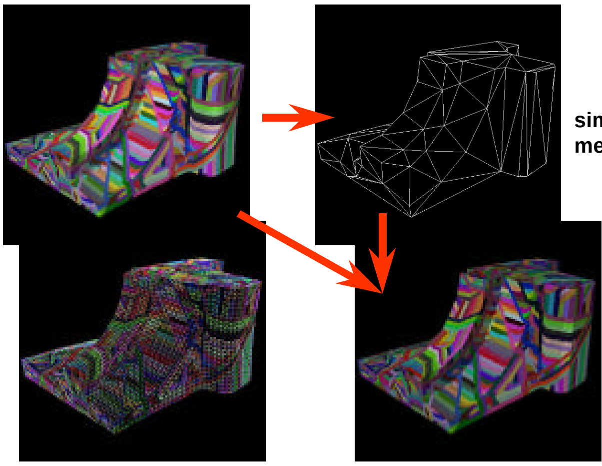
More in detail:

- For each triangular face produce a texture patch, which encodes the "detail" of **S** lost in $\mathbf{S_1}$
  - scan-convert each face of simplified mesh $\mathbf{S_1}$
    - for each sample point **p**:
      - find the corresponding point **p'** on original **S**
      - **compute** the attribute value in **S** on **p'**
      - **store** this value in a **triangular texture patch**
- Texture patches are stored in an efficient manner into a single, rectangular texture
- Use std. texture mapping (sw/hw) to render in real time



Times: tens of seconds

- an example of ***color*** preservation

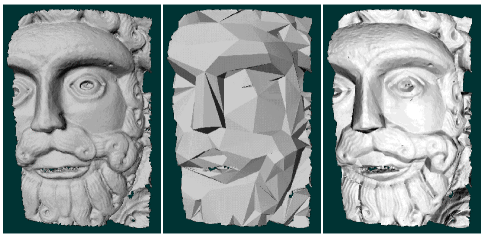


original mesh (per-vertex color)

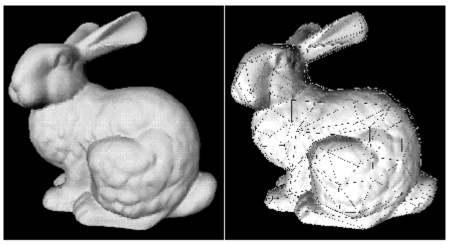
simplified mesh

simplified mesh with textured color

- example of ***geometric detail*** preservation by ***displacement mapping***



Original 20k face
simplified 500 face



Original 60k faces
simplified 250 faces