Schedule

- **12:00 – 12:15 Introduction**
  - *Prof. Nadia Magnenat-Thalmann*
- **12:15 – 13:05 Anatomical modelling from medical data**
  - *Prof. Nadia Magnenat-Thalmann and Jérôme Schmid*
- **13:05 – 13:30 Physically-based simulation of biological tissues (Part 1)**
  - *Dr. Hervé Delingette*
- **15:00 – 15:25 Physically-based simulation of biological tissues (Part 2)**
  - *Dr. Hervé Delingette*
- **15:25 – 16:15 Medical visualisation and applications**
  - *Dr. Marco Agus and J.A. Iglesias Guitián*
- **16:15 – 16:30 Conclusion and discussion**

Physically-based simulation of biological tissues

Dr. Hervé Delingette – ***INRIA, Asclepios, France***

Overview

Measuring Soft Tissue Deformation

Continuum Models of Soft Tissue

Discretization Methods

Interactive Simulation : SOFA Platform

Examples

Soft Tissue Characterization

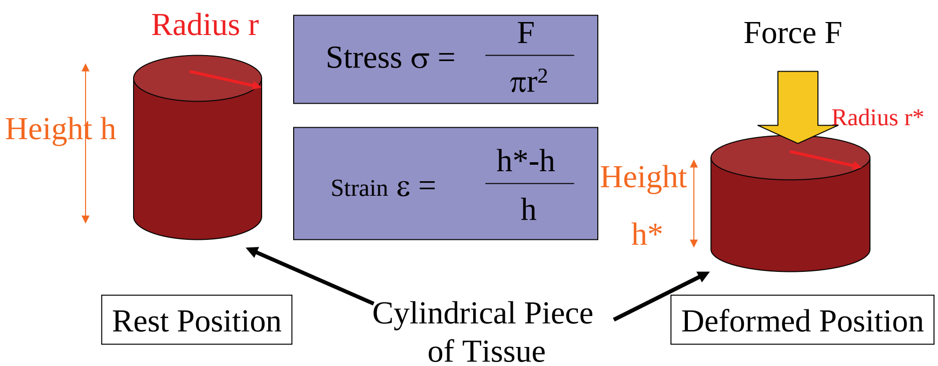
Biomechanical behavior of biological tissue is very complex

Most biological tissue is composed of several components :

- Fluids : water or blood
- Fibrous materials : muscle fiber, neuronal fibers, …
- Membranes : interstitial tissue, Glisson capsule
- Parenchyma : liver or brain

Soft Tissue Characterization

To characterize a tissue, its stress-strain relationship is studied

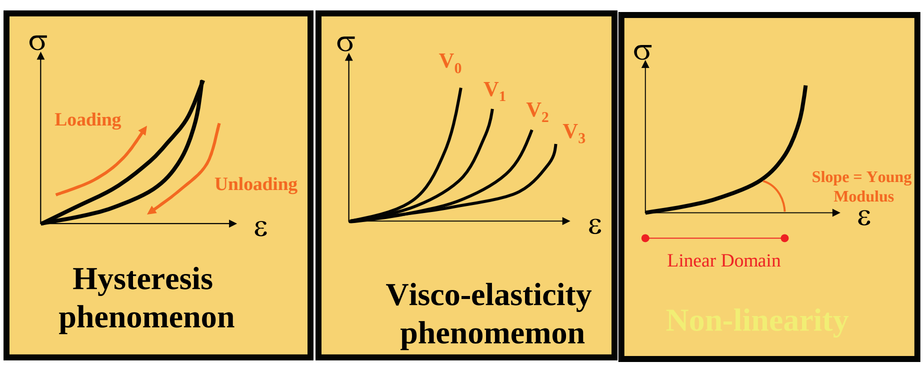


$$\text{Stress } \sigma = \frac{F}{\pi r^2}$$

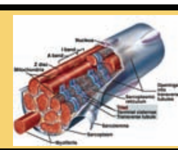
$$\text{Strain } \varepsilon = \frac{h^*-h}{h}$$

Soft Tissue Characterization

In stress-strain relationships there are :



Anisotropy



Parameter estimation

Complex for biological tissue :

- Heterogeneous and anisotropic materials
- Tissue behavior changes between in-vivo and in-vitro
- Ethics clearance for performing experimental studies
- Effect of preconditioning
- Potential large variability across population

Soft Tissue Characterization

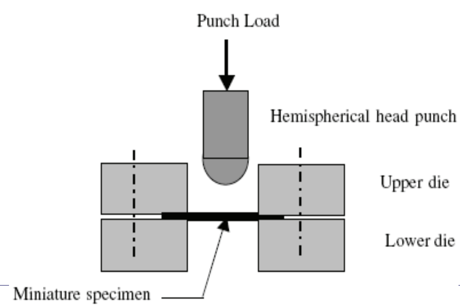
Different possible methods

- In vitro rheology
- In vivo rheology
- In silico rheology
- Elastometry

Soft Tissue Characterization

In vitro rheology

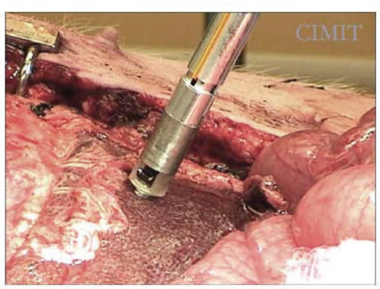
- can be performed in a laboratory. Technique is mature
- Not realistic for soft tissue (perfusion, …)



Soft Tissue Characterization

In vivo rheology

- can provide stress/strain relationships at several locations
- Influence of boundary conditions not well understood

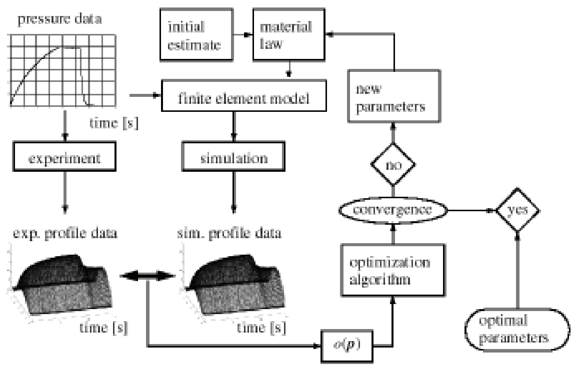


Source : Cimit, Boston USA

Soft Tissue Characterization

In silico rheology (Inverse Problems)

- well-suited for surgery simulation (computational approach)
- require the geometry before and after deformation



Soft Tissue Characterization

Elastometry (MR, Ultrasound)

- mesure property inside any organ non invasively
- validation ? Only for linear elastic materials

Source Echosens, Paris

Soft Tissue Characterization

May be difficult to find “reliable” soft tissue material parameters

Example : Liver soft tissue characterization

First Author	Experimental Technique	Liver Origin	Young Modulus (kPa)
Yamashita [111]	Image-Based	Human	Not Available
Brown [15]	*in-vivo*	Porcine Liver	≈ 80
Carter [17]	*in-vivo*	Human Liver	≈ 170
Dan [27]	*ex-vivo*	Porcine Liver	≈ 10
Liu [62, 61]	*ex-vivo*	Bovine Liver	Not Available
Nava [76]	*in-vivo*	Porcine Liver	≈ 90
Miller [74]	*in-vivo*	Porcine Liver	Not Available
Sakuma [92]	*ex-vivo*	Bovine Liver	Not Available

Table 2: [Must use a “proper” model to estimate its parameters] out the biomechanical properties of the liver.

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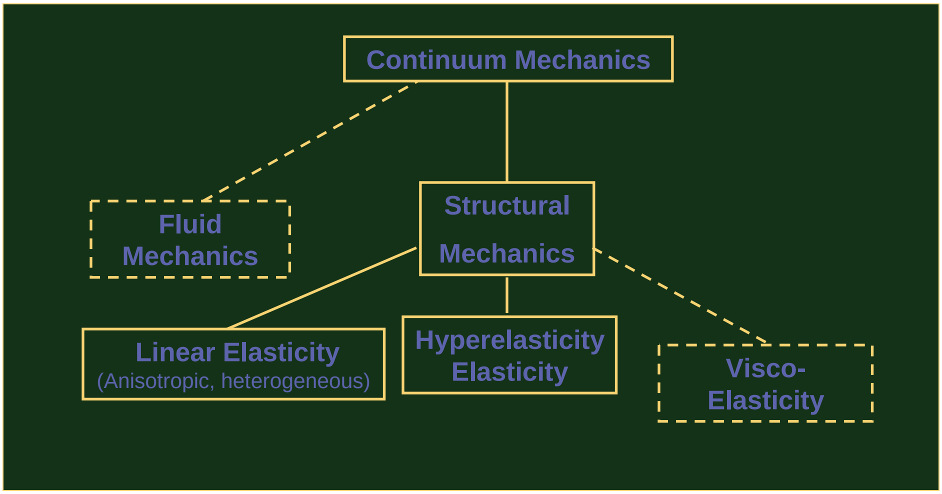
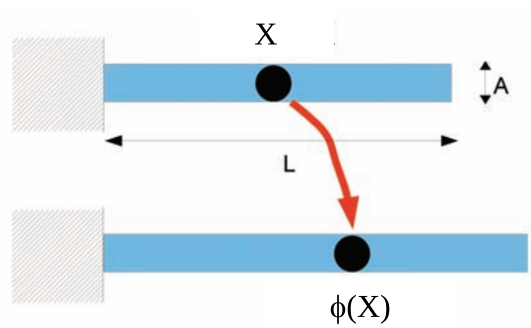
Continuum Mechanics



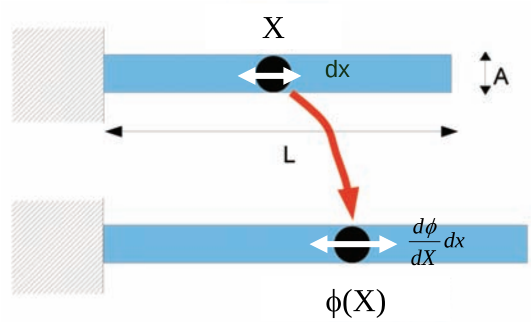
1D Elasticity



Point X is deformed into point $\phi(X)$

How much deformation around point X ?

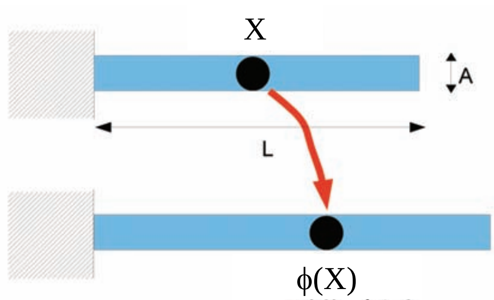
1D Elasticity : stretch ratio



Rest length : 2 dx New length : ϕ(x+dx) – ϕ(x-dx)

Stretch ratio at X is $s(X) = \frac{d\phi}{dX}$

1D Elasticity : strain energy



What is the energy

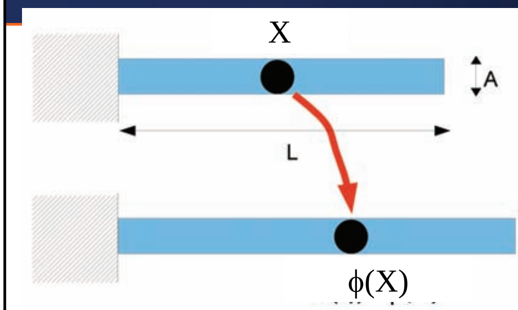
necessary to deform the bar ?

Deformation energy W depends "how stretched" the bar is

⟺

W depends on strain ε = distance between s and 1

1D Elasticity : strain



Different choices of strain

$$\varepsilon(s)=\frac{1}{\alpha}\left(s^{\alpha}-1\right) \quad \text{For } \alpha>0$$

$$\varepsilon(s)=s-1 \quad \text{For } \alpha=1 \quad \text{Engineering strain}$$

$$\varepsilon(s)=\frac{1}{2}\left(s^{2}-1\right) \quad \text{For } \alpha=2 \quad \text{Green-Lagrange strain}$$

$$\varepsilon=\log s \quad \text{For } \alpha=0 \quad \text{Henky strain}$$

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1D Elasticity : stress

Stress is the energy conjugate of strain

$$\sigma=\frac{\partial W}{\partial \varepsilon} \qquad \varepsilon=\frac{\partial W}{\partial \sigma}$$

Extensive Variable	Intensive Variable
Position	Force
Angle	Torque
Volume	Pressure
Strain	*Stress*

For $\alpha=1$ (First Piola-Kirchhoff) nominal stress

For $\alpha=2$ Second Piola-Kirchhoff stress

For $\alpha=0$ Cauchy stress

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St Venant Kirchhoff Material

Basic Material :

- W is a quadratic function of strain
- Stress is proportional to strain $\sigma = \frac{\partial W}{\partial \varepsilon}$

1D case : λ is the material stiffness

$$W = \int_\Omega \frac{1}{2}\sigma\varepsilon = \int_\Omega \frac{\lambda A}{2}\varepsilon^2 dX = \int_\Omega \frac{\lambda A}{2\alpha^2}\left(\left(\frac{d\phi}{dX}\right)^\alpha - 1\right)^2 dX$$

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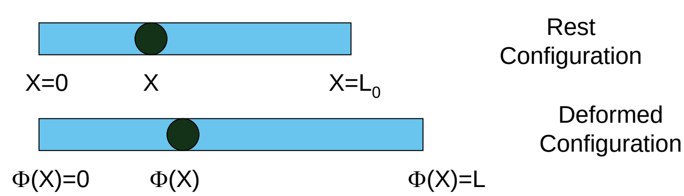
1D Elasticity : discretization

Represent the bar with a single segment



Stretch Ratio $s = \frac{L}{L_0}$

Strain $\varepsilon = \frac{1}{\alpha}\left(\frac{L^\alpha}{L_0^\alpha} - 1\right)$

Strain Energy $W = \frac{\lambda A L_0^{1-2\alpha}}{2\alpha^2}\left(L^\alpha - L_0^\alpha\right)^2$

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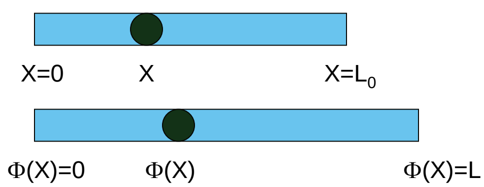
1D Elasticity : discretization

Represent the bar with a single segment



For $\alpha = 1$ $\quad W = \frac{\lambda A}{2L_0}(L - L_0)^2$ ➡ (Quadratic) Spring Energy

For $\alpha = 2$ $\quad W = \frac{\lambda A}{8L_0^3}(L^2 - L_0^2)^2$ ➡ Biquadratic Spring Energy

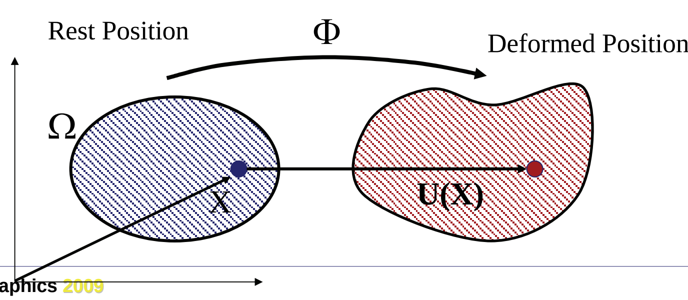
3D Elasticity

Deformation Function

$$X \in \Omega \mapsto \phi(X) \in \Re^3$$

Displacement Function

$$U(X) = \phi(X) - X$$

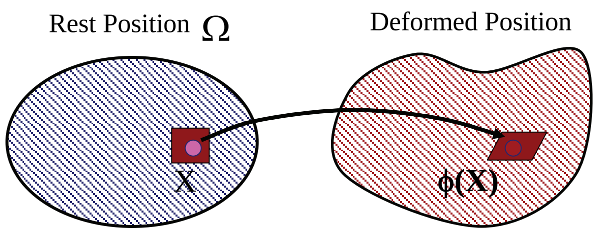


Deformation Gradient

The local deformation is captured by the deformation gradient :

$$F = \frac{\partial \phi}{\partial X}$$

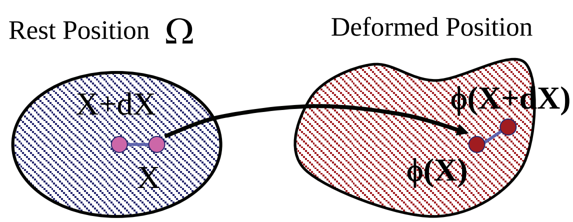
$$F_{ij} = \frac{\partial \phi_i}{\partial X_j} = \begin{bmatrix} \frac{\partial \phi_1}{\partial X_1} & \frac{\partial \phi_1}{\partial X_2} & \frac{\partial \phi_1}{\partial X_3} \\ \frac{\partial \phi_2}{\partial X_1} & \frac{\partial \phi_2}{\partial X_2} & \frac{\partial \phi_2}{\partial X_3} \\ \frac{\partial \phi_3}{\partial X_1} & \frac{\partial \phi_3}{\partial X_2} & \frac{\partial \phi_3}{\partial X_3} \end{bmatrix}$$



F(X) is the local affine transformation that maps the neighborhood of X into the neighborhood of ϕ(X)

Stretch Tensor

Distance between point may not be preserved



Distance between deformed points

$$(ds)^2 = \|\phi(X + dX) - \phi(X)\|^2 \approx dX^T (\nabla \phi^T \nabla \phi) dX$$

Right Cauchy-Green Deformation tensor

$$C = \nabla \phi^T \nabla \phi$$

Measures the change of metric in the deformed body

Strain Tensor

Example : Rigid Body motion entails no deformation

$$\phi(X) = RX + T$$

$$F(X) = \nabla\phi(X) = R \qquad C = R^T R = Id$$

Strain tensor captures the amount of deformation

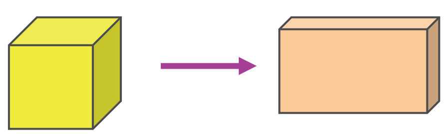
- It is defined as the "distance between C and the Identity matrix"

$$E = \frac{1}{2}\left(\nabla\phi^T \nabla\phi - Id\right) = \frac{1}{2}\left(C - Id\right)$$

Strain Tensor

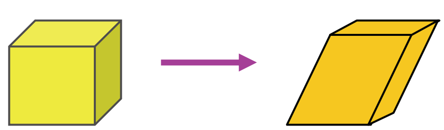
Diagonal Terms : ε_i

- Capture the length variation along the 3 axis



Off-Diagonal Terms : γ_i

- Capture the shear effect along the 3 axis



$$E = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{bmatrix}$$

Analogy 1D-3D Elasticity

1D Elasticity		3D Elasticity	
Deformation Gradient	$\frac{d\phi}{dX}$	Deformation Gradient	$\nabla\phi(X)$
Square Stretch Ratio	$s^2 = \left(\frac{d\phi}{dX}\right)^2$	RCG-Deformation Tensor	$C = \nabla\phi^T \nabla\phi$
Green Strain	$\varepsilon(s) = \frac{1}{2}(s^2 - 1)$	Green Strain Tensor	$E = \frac{1}{2}(\nabla\phi^T \nabla\phi - Id)$
SVK Strain Energy	$w(X) = \frac{\lambda A(\varepsilon(s))^2}{4}$	SVK Strain Energy	$w(X) = \frac{\lambda}{2}(tr\,E)^2 + \mu\,tr\,E^2$

Linearized Strain Tensor

Use displacement rather than deformation

$$\nabla\phi(X) = Id + \nabla U(X)$$

$$E = \frac{1}{2}\left(\nabla U + \nabla U^T + \nabla U^T \nabla U\right)$$

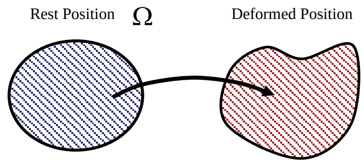
Assume small displacements

$$E_{Lin} = \frac{1}{2}\left(\nabla U + \nabla U^T\right)$$

Hyperelastic Energy

The energy required to deform a body is a function of the invariants of strain tensor E :

- Trace E = I_1
- Trace E*E= I_2
- Determinant of E = I_3



$$W(\phi) = \int_{\Omega} w(I_1, I_2, I_3)\, dX$$

Total Elastic Energy

Linear Elasticity

Isotropic Energy

$$w(X) = \frac{\lambda}{2}(tr\, E_{Lin})^2 + \mu\, tr\, E_{Lin}^2$$

Hooke's Law

(λ, μ) **: Lamé coefficients**

$w(X)$ **: density of elastic energy**

Advantage :

- Quadratic function of displacement

$$w = \frac{\lambda}{2}(div\, U)^2 + \mu \|\nabla U\|^2 - \frac{\mu}{2}\|rot\, U\|^2$$

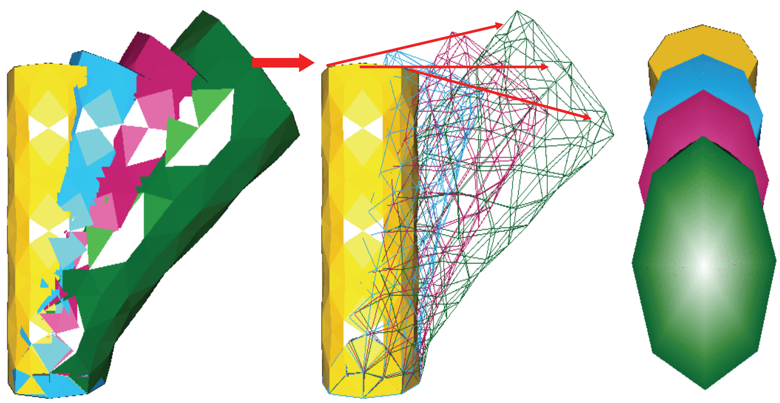
Drawback :

- Not invariant with respect to global rotation

Extension for anisotropic materials

Shortcomings of linear elasticity

Non valid for «large rotations and displacements»



St-Venant Kirchoff Elasticity

Isotropic Energy

$$w(X) = \frac{\lambda}{2}(tr\,E)^2 + \mu\, tr\,E^2$$

(λ, μ) **: Lamé coefficients**

Advantage :

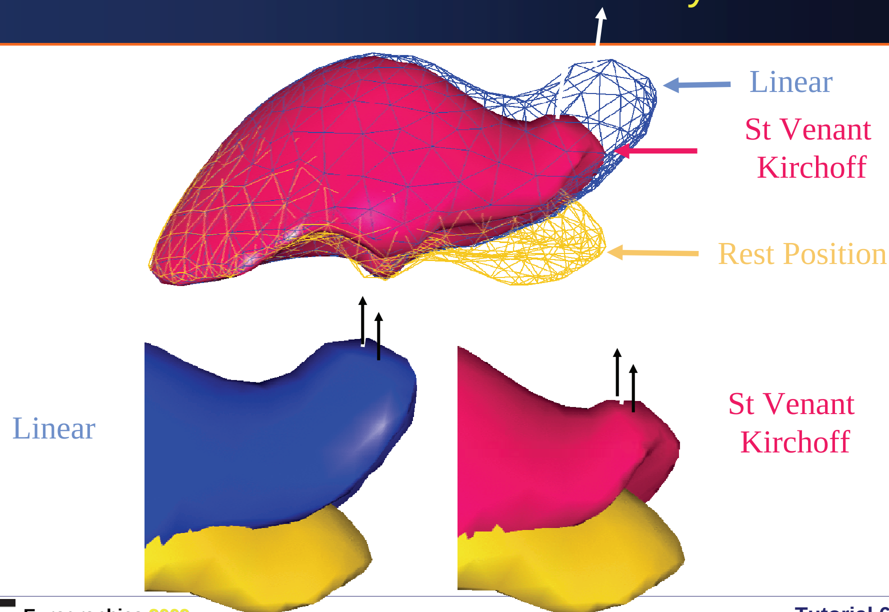
- Generalize linear elasticity
- Invariant to global rotations

Drawback :

- Poor behavior in compression
- Quartic function of displacement

Extension for anisotropic materials

St Venant Kirchoff vs Linear Elasticity



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Other Hyperelastic Material

Neo-Hookean Model

$$w(X) = \frac{\mu}{2} trE + f(I_3)$$

Fung Isotropic Model

$$w(X) = \frac{\mu}{2} e^{trE} + f(I_3)$$

Fung Anisotropic Model

$$w(X) = \frac{\mu}{2} e^{trE} + \frac{k_1}{k_2}\left(e^{k_2(I_4-1)} - 1\right) + f(I_3)$$

Veronda-Westman

$$w(X) = c_1\left(e^{\gamma trE}\right) + c_2\, trE^2 + f(I_3)$$

Mooney-Rivlin :

$$w(X) = c_{10} trE + c_{01} trE^2 + f(I_3)$$

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Examples

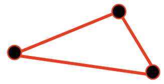
Discretisation techniques

Four main approaches :

- Volumetric Mesh Based
- Surface Mesh Based
- Meshless
- Particles

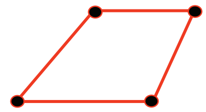
Different types of meshes

Surface Elements :



Triangle

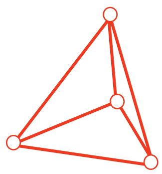
3, 12 nodes and more



Quad

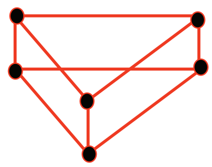
4, 8 nodes and more

Volume Elements



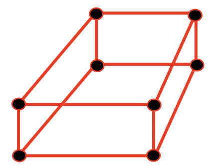
Tetrahedron

4, 10 nodes



Prismatic

6, 15 nodes and more



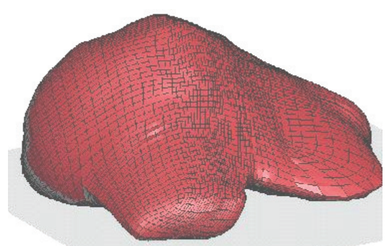
Hexahedron

8, 20 nodes and more

Structured vs Unstructured meshes

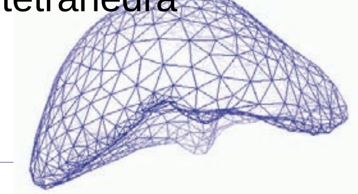
Example 1 : Liver meshed with hexahedra

3 months work
(courtesy of ESI)



Example 2: Liver meshed with tetrahedra

Automatically generated (1s)



Volumetric Mesh Discretization

Classical Approaches :

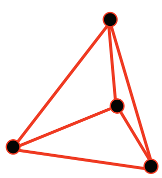
- Finite Element Method (weak form)
- Rayleigh Ritz Method (variational form)
- Finite Volume Method (conservation eq.)
- Finite Differences Method (strong form)

FEM, RRM, FVM are equivalent when using linear elements

Rayleigh-Ritz Method

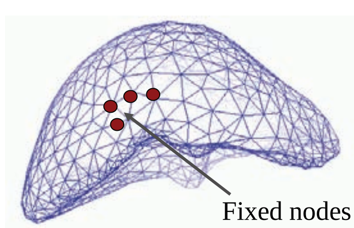
Step1 : Choose

- Finite Element (e.g. linear tetrahedron)
- Mesh discretizing the domain of computation
- Hyperelastic Material with its parameters
- Boundary Conditions



Tetrahedron

4 nodes



$$w(X) = \frac{\lambda}{2}(tr\,E)^2 + \mu\, tr\, E^2$$

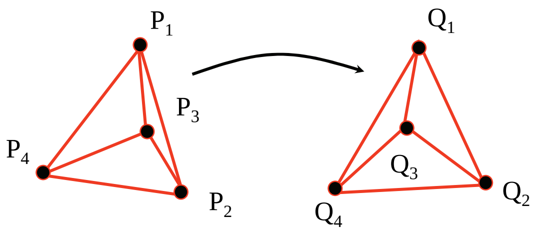
Young Modulus

Poisson Coefficient

Rayleigh-Ritz Method

Step2

- Write the elastic energy required to deform a single element



$$u(P_i) = Q_i - P_i = U_i$$

$$u(X) = \sum_{i=1}^{4} \lambda_i(X)\, u(P_i)$$

$$\nabla \lambda_i(X) = -\frac{M_i}{6V(T)}$$

$$trE = -\sum_i \frac{M_i \cdot U_i}{6V(T)}$$

$$W_{T_i} = \sum_{jk} U_j^t \left[\mathsf{K}_{jk}^{T_i}\right] U_k$$

$$[\mathbf{K}_{jk}^{T_i}] = \frac{1}{36.V(T_i)}\left(\lambda_i \mathbf{M}_k \mathbf{M}_j^T + \mu_i \mathbf{M}_j \mathbf{M}_k^T + \mu_i (\mathbf{M}_j \cdot \mathbf{M}_k)[\mathbf{Id}_{3x3}]\right)$$

Rayleigh-Ritz Method

Step3

- Sum to get the total elastic energy

$$W(U) = \int_{\Omega_h} w(I_1, I_2, I_3)\, dX = \sum_{T_i} W_{T_i} = U^T K U$$

- Write the conservation of energy

$$\underbrace{W(U)}_{\text{Internal Energy}} = \underbrace{F^T U}_{\text{Nodal Forces}} + \underbrace{\int_{\Omega} \rho(X)(X \cdot g)\, dX}_{\text{Gravity Potential Energy}}$$

Rayleigh-Ritz Method

Step3

- Write first variation of the energy :

Linear Elasticity

$$KU = R$$ Static case

$$M\ddot{U} + C\dot{U} + KU = R(t)$$ Dynamic case

HyperElasticity=NonLinear Elasticity

$$K(U) = R$$ Static case

$$M\ddot{U} + C\dot{U} + K(U) = R(t)$$ Dynamic case

Surface-Based Methods

Possible approaches :

- Boundary Element Models (BEM)
  - Based on the Green Function of the linear elastic operator
  - Requires homogeneous material
- Matrix Condensation
  - Full Matrix inversion
- Iterative Precomputed Generation
  - Solve 3*Ns equations F=KU

Other Methods

Meshless Methods

- Use only points inside and specific shape functions
- Can better optimize location of DOFs
- Can cope with large deformations
- Deformation accuracy unknown

Particles

- Smooth Particles Hydrodynamics that interact based on a state equation

Time Integration Scheme

Explicit Schemes :

- Euler, Runge Kutta
- Conditionally Stable : time step must be lower than a critical time step
- Fast update but not suitable for stiff materials

Implicit Schemes :

- Euler, Newmark
- Require solving a linear system of equations

Some Bibliography References

[Hauth03]	Michael Hauth, Olaf Etzmuß, Wolfgang Straßer: **Analysis of numerical methods for the simulation of deformable models**. The Visual Computer 19(7-8): 581-600 (2003)
[Nealen06]	Andrew Nealen, Matthias Mueller, Richard Keiser, Eddy Boxerman, Mark Carlson, **Physically Based Deformable Models in Computer Graphics ,** *Computer Graphics Forum*, Vol. 25, No. 4. (December 2006), pp. 809-836.
[Deling04]	H. Delingette and N. Ayache. **Soft Tissue Modeling for Surgery Simulation**. In N. Ayache, editor, *Computational Models for the Human Body*, Handbook of Numerical Analysis, pages 453-550. Elsevier, 2004
[Fung94]	Y. C. Fung, **A First Course in Continuum Mechanics: For Physical and Biological Engineers and Scientists** **Book**, Prentice Hall (January 1994)
[Bathe95]	Bathe, K.J., ***Finite Element Procedures*** Prentice-Hall, Englewood Cliffs, 1995, 1037 pp.

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Towards Realistic Interactive Simulation

Surgery Simulation must cope with several difficult technical issues :

- Soft Tissue Deformation
- Collision Detection
- Collision Response
- Haptics Rendering

Real-time Constraints :

- 25Hz for visual rendering
- 300-1000 Hz for haptic rendering

SOFA :: Objectives

Provide a common software framework for the medical simulation community

Enable component exchange to reduce development time

Promote collaboration among research groups

Enable validation and comparison of new algorithms

- www.sofa-framework.org

SOFA :: Targeted Users

Non-Technical end users

- Rapid prototyping with XML scene descriptions
- Text editing – no compiling necessary
- Plug n' play interface (Maya plug-in)

Researchers and developers

- Develop new application procedurally
- Add functionalities by writing new modules in C++

SOFA :: a flexible and efficient framework

Component Abstraction

- Minimize inter-dependencies between components

Objects have multi-modal representation

- Visual, Behavior, Collision, Haptic, etc.

Physics-based objects can be further decomposed

- Degrees of Freedom
- Force Fields
- Integration Schemes
- Solvers

SOFA :: a flexible and efficient framework

Scene graph representation

- Common in computer graphics
- Dynamic hierarchy is useful for collision management
- New objects or complete scenes can be added easily

Transparent support for parallel computing

- GPU optimized computation
- Cluster-based computing

SOFA :: Current Results

Create complex and evolving simulations by combining new algorithms with algorithms already included in SOFA

Modify most parameters of the simulation by simply editing an XML file

Efficiently simulate the dynamics of interacting objects using abstract equation solvers

Reuse and easily compare various deformable models

Overview

Measuring Soft Tissue Deformation

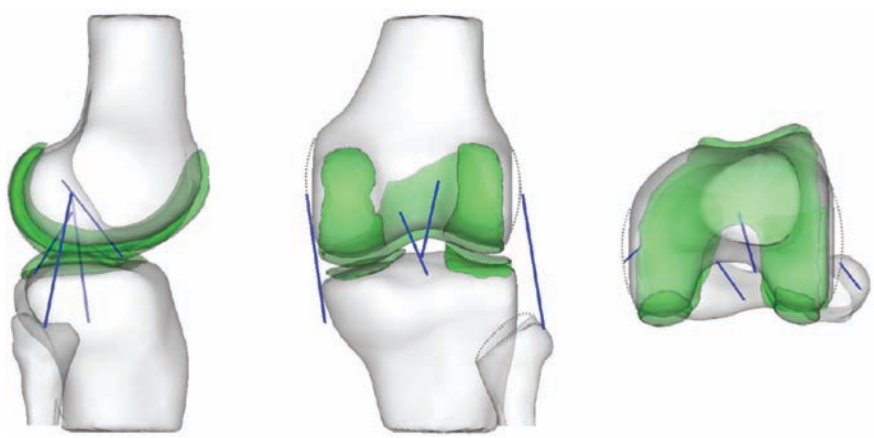
Continuum Models of Soft Tissue

Discretization Methods

Interactive Simulation : SOFA Platform

Examples

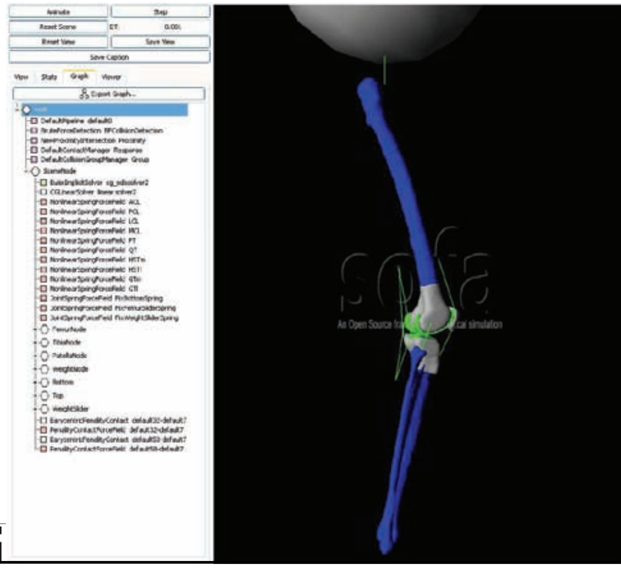
Example 1 : Simulation of knee joint



Cruciate ligaments segmented from MRI

Collateral ligaments determined from geometry

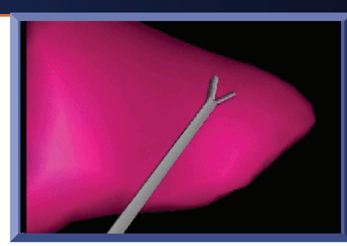
Simulation in SOFA



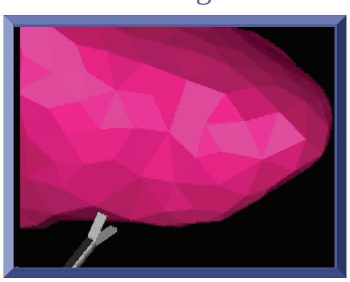
Tutorial 6

Simulation of Liver Surgery

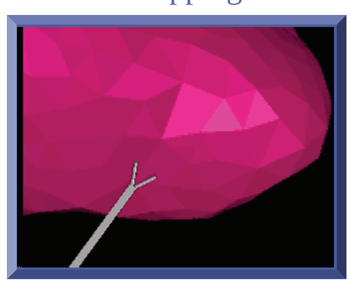
Gliding



Gripping



Cutting (pliers)



Cutting (US)

Eurographics 2009 Tutorial 6

Example 3 : Cardiac Simulation

4 Cardiac Phases:

- Filling
- Isovolumetric Contraction
- Ejection
- Isovolumetric Relaxation

2 Volumetric Conditions:

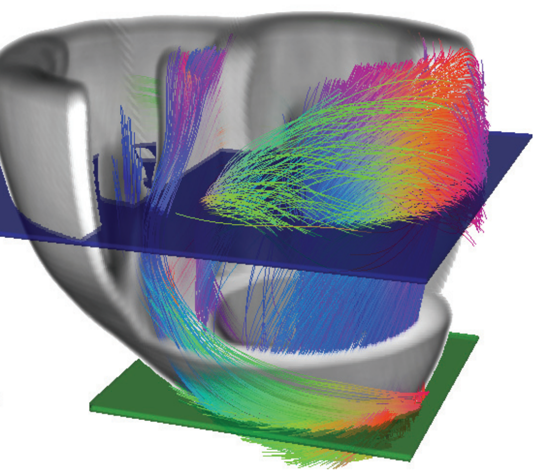
- Pressure Field in the endocardium
- Isovolumetric Constraint of myocardium

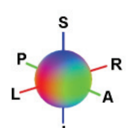
Slowed

6 times

Fiber Tracking on the Average Cardiac DTI

Use cardiac fiber orientation based on Diffusion Tensor MRI





http://www.inria.fr/asclepios/software/MedINRIA