Efficient Sorting and Searching in Rendering Algorithms

Eurographics 2006 Tutorial T4

Organizers and Presenters

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Version 1.0, June 15, 2006

Abstract

In the proposed tutorial we would like to highlight the connection between rendering algorithms and sorting and searching as classical problems studied in computer science. We will provide both theoretical and empirical evidence that for many rendering techniques most time is spent by sorting and searching. In particular we will discuss problems and solutions for visibility computation, density estimation, and importance sampling. For each problem we mention its specific issues such as dimensionality of the search domain or online versus offline searching. We will present the underlying data structures and their enhancements in the context of specific rendering algorithms such as ray shooting, photon mapping, and hidden surface removal.

Organizers bibliographies

Vlastimil Havran is an assistant professor at the Czech Technical University in Prague since February 2006. He defended his Ph.D. dissertation on ray shooting algorithms in 2001 at the Czech Technical University in Prague. Later he joined the computer graphics group at Max-Planck-Institute for Informatics in Saarbruecken. He became a research associate at the same institute in 2003. He has contributed to the topic of sorting and searching by his dissertation on ray shooting algorithms which started the area of interactive ray tracing. In addition to sorting and searching he worked on various other topics in rendering.

Jiří Bittner holds a Ph.D. in Computer Science from the Czech Technical University in Prague. His main research interests include visibility preprocessing, occlusion culling, real-time rendering, and computational geometry. He has also been active in development of two commercial products dealing with real-time rendering of large scenes. He is currently affiliated with the Vienna University of Technology and the Czech Technical University in Prague.

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Tutorial Contents

Tutorial Web Page

The updated version of this tutorial presented at Eurographics 2006 can be found under the following URL:

`http://www.cgg.cvut.cz/~havran/eg2006tut/`

Acknowledgements

We would like to thank Robert Herzog, Jaroslav Křivánek, Michael Wimmer, Peter Wonka, Tommer Leyvand, David Luebke, and Hansong Zhang for providing us various materials used in the tutorial. This work has been partially supported by the EU under the project no. IST-2-004363 (GameTools) and by the Ministry of Education, Youth and Sports of the Czech Republic under the research program LC-06008 (Center for Computer Graphics).

Sorting and Searching in Image Synthesis

Eurographics 2006 Tutorial T4

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Content

Part One

- Introduction to Rendering
- Sorting and Searching
- Hierarchical Data Structures
- Ray Shooting
- Questions and Answers (5 minutes)

Overview 2

Content

Part Two

- Hidden Surface Removal
- Visibility Culling
- Photon Maps and Irradiance Caching
- Ray Maps
- Other Algorithms
- Questions and Answers (10 minutes)

Overview 3

Tutorial Goals

- Recall that we often use sorting and searching in rendering
- Highlight connections between different problems in rendering
- Briefly show efficient solutions
- Show unusual solutions resulting from twisting searching queries and domains

Overview 4

What is Not Covered Here

- Collision detection algorithms (EG'05 Tutorial)
- Image based rendering
- Radiosity
- Non-photo realistic rendering
- Clustering techniques
- Graph theory and other related problems

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Introduction to Rendering

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Introduction to Rendering

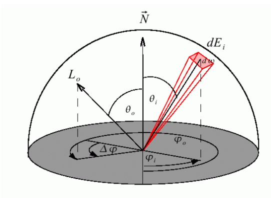
- Rendering equation
- Rendering algorithms
- Sorting and searching in rendering

Introduction to Rendering 2

Rendering Equation

$$L(\vec{\omega_o},x) = L^e(\vec{\omega_o},x) + \int_\Omega L_i(x,\vec{\omega_i}) \cdot f_r(x,\vec{\omega_i},\vec{\omega_o}) \cdot (\vec{n}\cdot\vec{\omega_i}) d\omega_i,$$

- Convolving incoming light with surface reflectance properties



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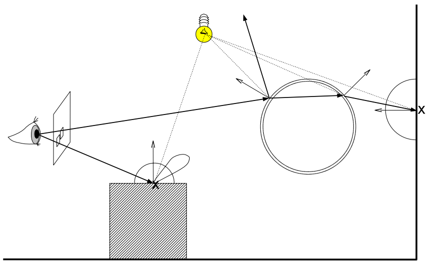
Ray Tracing



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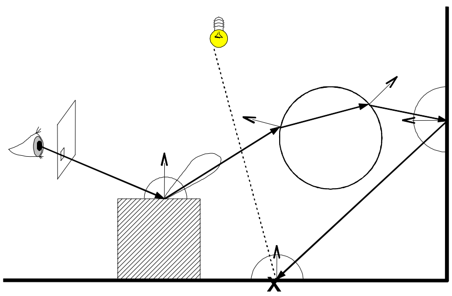
Path Tracing



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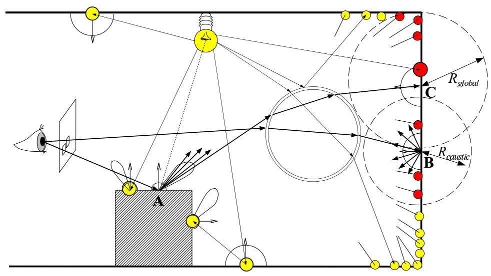
Photon Mapping



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Tutorial Motivation

- Sorting and searching takes usually more than 90% of the rendering time!
- Efficiency of sorting and searching is crucial for the performance
- Examples
  - Ray Tracing
  - Photon Density Estimation

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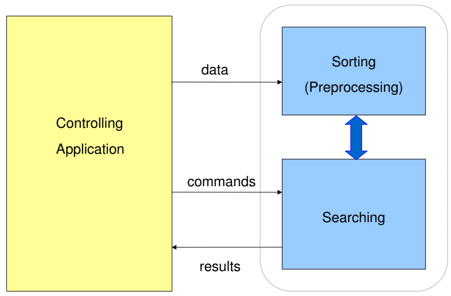
Sorting and Searching General Concept



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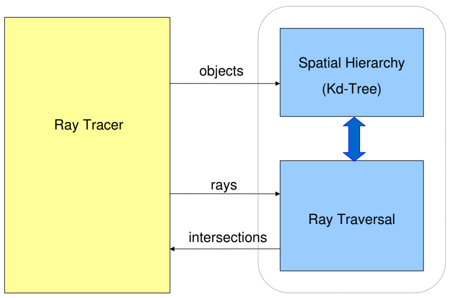
Example 1 Ray Tracing



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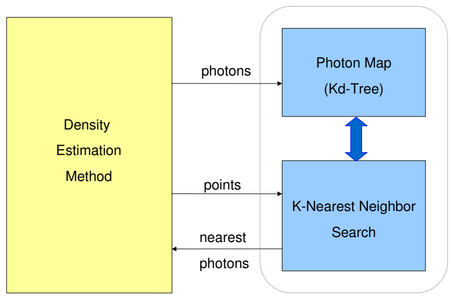
Example 2 Photon Density Estimation



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Introduction to

Sorting and Searching

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Searching

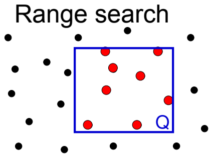
- Searching query definition

$$Q \times S \rightarrow A$$

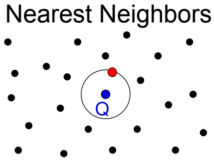
Q set of queries
S search space
A set of answers

Geometric search problems

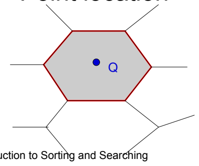
Range search



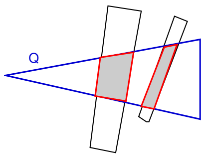
Nearest Neighbors



Point location



Intersection detection

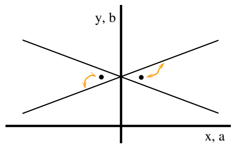


Geometric search problems

- Dimension of S

primitive	R^2	R^3
points	2	3
lines	2	4
spheres	3	4

- Example
  - Lines-points duality in 2D



Geometric search

- Exact vs. Approximate
  - Approximate: finds solution “close to” optimum
- Static vs. dynamic
  - Dynamic: S may change
- Online vs. offline
  - Offline: applied for entire sequence of Q

Search Complexity

- Single result for query
  - O(log n)
- Multiple results for query
  - Reporting O(log n + k)
  - Counting O(log n)

Typical Problems in CG

- Search space
  - Set of points, set of objects, set of oriented discs, set of hemispheres, set of rays
- Queries
  - Point, ray, hemisphere, set of points (polygon, bounding box, bounding sphere)

Introduction to Sorting and Searching 7

Typical Problems in CG

Problem	Q	S	A
Ray shooting	ray	{objects}	{point}
HSR	{rays}	{objects}	{points}
VFC	frustum	{objects}	{objects}
Occlusion culling	{rays}	{objects}	{objects}
Photon maps	point	{points}	{points}
Ray maps	hemisphere	{rays}	{rays}

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Search Problem Transformation

- Halfspace intersection in 2D
  - 2D line maps to 2D point (duality)
  - *Convex hull of points*
- Point and spheres intersection in 3D
  - 3D point maps to 4D point
  - 3D spheres maps to 4D hyperplanes
  - Duality → *Halfspace range reporting in 4D*

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Searching methods

- Sorted arrays
  - Binary search
  - Interpolation search
- Search trees
  - Binary search trees
  - kD-trees, R-trees, AVL-trees….
- Hashing
  - Spatial grid hashing

Introduction to Sorting and Searching 10

Sorting

- Motivation: Improve searching performance
  - Naïve search: O(n) time
  - With sorting: O(log n)!
  - In special cases even O(1)
- Assumption
  - Spatial relations among elements of S (objects, points, rays, normals, …)

Introduction to Sorting and Searching 11

Basic Sorting Algorithms

Algorithm	Method	Best	Average	Worst
Heapsort	Selection	O(n log n)	O(n log n)	O(n log n)
Selection sort	Selection	$O(n^2)$	$O(n^2)$	$O(n^2)$
Quicksort	Partitioning	O(n log n)	O(n log n)	$O(n^2)$
Bucket sort	Distribution	O(n)	O(n)	$O(n^2)$
Merge sort	Merging	O(n log n)	O(n log n)	O(n log n)
Bubble sort	Exchanging	O(n)	$O(n^2)$	$O(n^2)$
Insertion sort	Insertion	O(n)	$O(n^2)$	$O(n^2)$

Space complexity: O(n)

Introduction to Sorting and Searching 12

Examples in CG

- Quicksort
  - Top-down construction of spatial hierarchies
- Bucket sort
  - Z-buffer, voxel grid
- Heapsort
  - Priority queues (occlusion culling, k-NN search)

Introduction to Sorting and Searching 13

Hierarchical Data Structures

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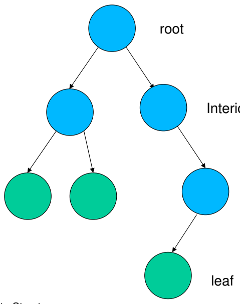
Hierarchical Data Structures (HDS)

- Connection to sorting
- Classification
- Bounding volume hierarchies
- Spatial subdivisions
- Hybrid data structures
- Searching using HDS
- Special techniques on hierarchies

Hierarchical Data Structures 2

Hierarchical Data Structures

= tree or even a graph



Hierarchical Data Structures 3

Connection to Sorting

Hierarchical Data Structures =
implementation of (spatial) sorting

Why ?

- Time complexity is O(N log N)
- For 1D hierarchy over points the construction of HDS is clearly equivalent to quicksort

Hierarchical Data Structures 4

Recall Quicksort

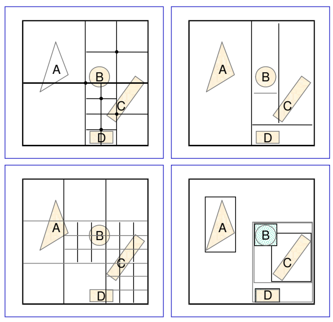
- Pick up a pivot Q
- Organize the data into two subarrays: the left part smaller than pivot Q, the right part larger or equal than pivot Q
- Recurse in both subarrays

Hierarchical Data Structures 5

Examples of HDS in 2D

octree

kd-tree



hierarchy of grids

bounding volume hierarchy

Hierarchical Data Structures 6

HDS Classification

- Data domain organization
- Dimensionality
- Data layout

Hierarchical Data Structures 7

HDS Classification

1) Data domain organization of HDS

- Spatial subdivisions – primarily organizing space (non-overlapping)
- Object hierarchies – primarily organizing objects (overlapping)
- Hybrid data structures
- Transformations and mappings

Hierarchical Data Structures 8

HDS Classification

2) Dimensionality of HDS

- Necessary to represent data entities: 1D, 2D, 3D, 4D, or 5D
- Data entities: points, lines, oriented half-lines, disks, oriented hemispheres, etc.
- Possibility to extend many problems to time domain (so plus one dimension)

Hierarchical Data Structures 9

HDS Classification

3) HDS data layout

- Internal data structures
- External data structures (out of core)
- Cache-aware data structures
- Cache oblivious data structures

Hierarchical Data Structures 10

Types of Nodes in HDS

- An interior node represents a *"pivot"* – according to the data entities are sorted
- Typical content is a subdivision plane or a set of planes plus references to child nodes
- The way of interior node representation with respect to the task is crucial for searching performance

Hierarchical Data Structures 11

Spatial Subdivisions

- **Non-overlapping *regions of child nodes***
- Space is organized by some (cutting) entities , typically by planes, constructed top-down
- Fully covering an original spatial region, every point can be located in some (empty or non-empty) leaf
- They are often called space partitionings

Hierarchical Data Structures 12

Spatial Subdivision Examples

- Kd-trees
- BSP-trees
- Octrees
- Uniform grids
- Recursive grids

Hierarchical Data Structures 13

Object Hierarchies

- Possibly **overlapping** extents of child nodes
- Often called bounding volume hierarchies
- Possibly some spatial regions are not covered
- Constructed top-down or bottom-up
- The shape represented by interior nodes typically a box, but other shapes possible

Hierarchical Data Structures 14

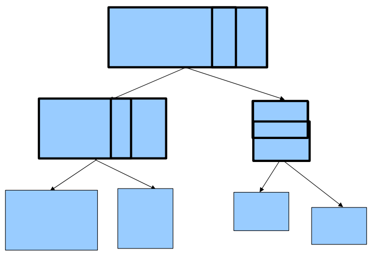
Names used for Object Hierarchies

- Bounding Volume Hierarchies (BVHs)
- R-trees and their many variants
- Box-trees
- Several others (special sort of bounding volumes... sphere trees etc.)

Hierarchical Data Structures 15

Bounding Volume Hierarchies

Constructed Top-Down



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Hybrid Data Structures

- Combining between various interior nodes
- Possibly combining between spatial subdivisions and object hierarchies
- Sharing pros and cons of both types
- They can be tuned to compromise of some properties, for example efficiency and memory requirements

Hierarchical Data Structures 17

Other HDS

- *Content of the node* – a single splitting plane, more splitting planes, a box, additional information.
- Arity of a node (branching factor)
- A way of constructing a tree (height, weight balancing) + postprocessing
- Data only in leaves or also in interior nodes
- Augmenting data

Hierarchical Data Structures 18

Example of Other HDS

- Cell trees (polyhedral shapes for splitting)
- SKD-trees (two splitting planes at once)
- hB-trees (holey brick B-trees)
- LSD-tree (height balanced kd-tree)
- P-trees (polytope trees)
- BBD-trees (bounding box decomposition trees)
- And many others

(See the surveys listed in tutorial notes)

Hierarchical Data Structures 19

Transformation Approach

- *Input:* A spatial object in 2D or 3D domain, for example a box
- *Output:* A point in 4D or 6D domain
- More complicated mapping is possible, for example a sphere in 3D -> point in 4D
- The transform often changes the searching algorithm used completely

Hierarchical Data Structures 20

HDS Construction Algorithm

Initial Phase: create a node with all elements and put it to the auxiliary structure AS (stack or priority queue).

Top-Down, Divide and Conquer:

(1) Take a node from an auxiliary structure AS. If AS is empty, then we are finished.

(2) Take a set of elements in the node and decide if to subdivide or not. If not, create leaf, go to (1).

(3) Decide how to split the set into two (N) subsets and create new nodes.

(4) Put the new nodes to AS. Recurse.

Hierarchical Data Structures 21

Search Algorithms using HDS

- Start from a root node
- Typically down traversal phase (location phase) + some other phase
- During visiting an interior node use either a stack (LIFO) or priority queue to record the nodes to be visited in future
- Compute incidence computation when visiting a leaf

Note: auxiliary structure implements another sorting phase during searching

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Search Algorithms using HDS

- *Range queries* – given a range X, find all the incidences of X with data
- *Nearest neighbour* – find a nearest neighbor
- *k-nearest neighbour*
- *Reverse nearest neighbors – given a point Q, find all the points to which Q is nearest neighbor*
- *Ranking* – given a query object Q, report on all the objects in order of distance from Q
- *Intersection search* – given a point Q, find all the objects that contain Q

Hierarchical Data Structures 23

Search Performance Model

- Result = the cost of computation ... C
- Performance is inverse proportional to the quality of the data structures for given problem
- The two uses of performance model
  - *a posteriori:* documenting and testing performance
  - *a priori:* constructing data structures with higher expected performance

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Search Performance Model

Typical cost model:

C = C_T + C_L + C_R

C = C_TS * N_TS + C_LO * N_LO + C_Access * N_Access

- C_T … cost of traversing the nodes of HDS
- C_L … cost of incidence operation in leaves
- C_R … Cost of accessing the data from internal or external memory

Hierarchical Data Structures 25

HDS Dynamization

- Given changes, only update data structures to reflect these changes
- It is assumed that the performance of searching remains acceptable after update
- Dynamization can require additional bookkeeping data to monitor the cost/quality of a HDS node and its associated subtree
- Techniques known for 1D trees (rotation, balancing) are often not applicable
- It is usually required to update larger amount of data at once (bulk updating)

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Ray Shooting

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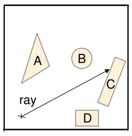
Ray Shooting

- Ray shooting versus ray tracing
- Connection to sorting and searching
- Ray shooting with kd-trees
- Performance studies
- Octrees, uniform grids, recursive grids
- Bounding volume hierarchies
- Offline ray shooting

Ray Shooting 2

Ray Shooting Algorithm (RSA)

Task: Given a ray, find out the first object intersected.



Input: a *scene* and a *ray*

Output: the *object C*

Ray Shooting 3

Ray Tracing *versus* Ray Shooting

- *Ray shooting* – only a single ray
- *Ray tracing* in computer graphics can be:
  – Ray shooting
  – Ray casting – only primary rays from camera
  – Recursive ray tracing
  – Distribution ray tracing and others

Ray Shooting 4

Some Complexity Issues

Computational Geometry
- aimed at worst-case complexity
- restriction to certain class of object shape (polygons, triangles)
- unacceptable memory requirements O(log N) query time induces Omega (N^4) space

Computer Graphics
- aimed at average-case complexity
- practical feasibility and robustness
- implementation issues important for performance

Ray Shooting 5

Some Complexity Results

Lower bound for worst-case complexity: 1997/98

Laszlo Szirmay-Kalos + Gabor Marton – lower bound for space complexity is Omega(N^4) for O(log N) search

Applicability of Computational Geometry techniques in CG for ray tracing
– CGE techniques are not general
– limited to small number of primitives
– no implementations available

Ray Shooting 6

Computer Graphics Techniques Overview

Techniques developed: average-case complexity, no complexity guarantees, many “tricks”

Basic techniques: bounding volumes, spatial subdivision, ray classification

Augmented techniques: macro regions, pyramid clipping, proximity clouds, directed safe zones

Special tricks: ray boxing, mailbox, handling CSG primitives, other types of coherence

Ray Shooting 7

RSA Techniques Classification

A) Subdivision techniques (top down)
- -- binary space partitioning (kd-trees)
- -- octrees
- -- uniform and hierarchical grids
- -- bounding volume hierarchy

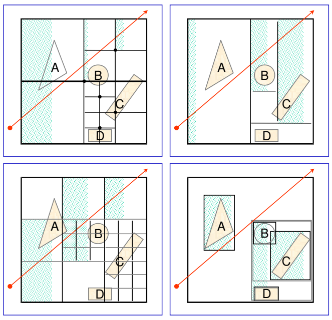
B) Clustering (bottom up)
- -- bounding volume hierarchy

Ray Shooting 8

Some RSA Techniques

octree

kd-tree

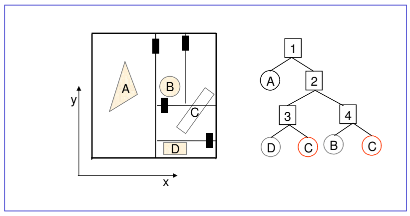


hierarchy of grids

bounding volume hierarchy

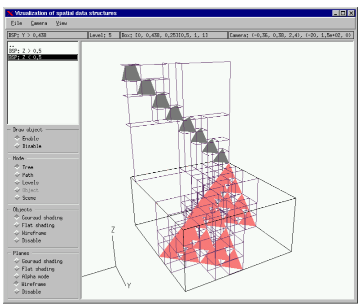
Ray Shooting 9

Kd-tree Construction



Ray Shooting 10

Visualisation of Kd-tree

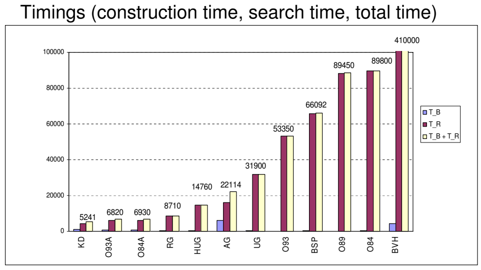


Ray Shooting 11

RSA Techniques Comparison

30 scenes *times* 12 RSAs *times* 4 ray distribution methods
= 1440 measurements, year 2000-2001

Timings (construction time, search time, total time)

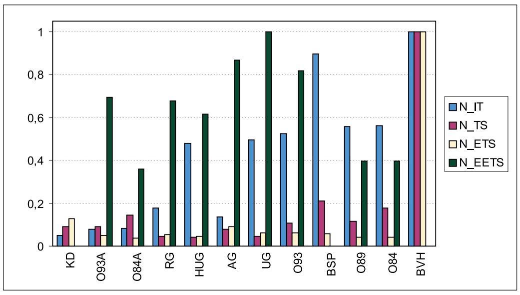


Note: In test BVH constructed in bottom-up fashion !

Ray Shooting 12

RSA Techniques Comparison

Number of operations (ray-object intersection, traversal steps)



Note: values normalized to the worst value.

Ray Shooting 13

RSA based on Kd-tree

Quite an efficient solution used in practice

Construction (in O(N log N) time)

- based on cost function and geometric probability
- automatic termination criteria algorithm
- various efficiency improvements:
  - construction of kd-tree with empty spatial regions
  - reducing objects' axis-aligned bounding boxes
  - preferred ray sets

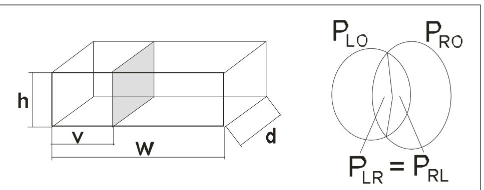
Ray traversal

– in practice achieves expected O(log N) time
– robust recursive ray traversal algorithm

Ray Shooting 14

Geometric Probability of Ray Intersecting a Subdivided Box

$probability_{LEFT} = P_{LO} + P_{LR} + P_{RL}$
$probability_{RIGHT} = P_{RO} + P_{LR} + P_{RL}$

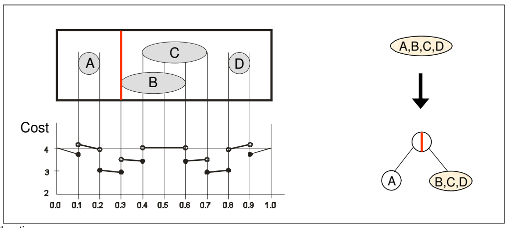


Condition: uniform ray distribution

Ray Shooting 15

Kd-tree Construction Based on Cost Function with Greedy Heuristics

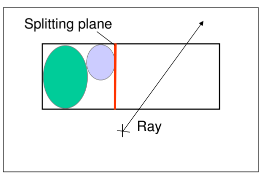
Cost = $probability_{LEFT}$ * $cost\_subtree_{LEFT}$ + $probability_{RIGHT}$ * $cost\_objects_{RIGHT}$



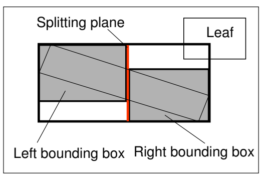
Ray Shooting 16

Kd-tree Efficiency Improvements

Cutting off empty space



Reducing objects' axis-aligned bounding boxes



Ray Shooting 17

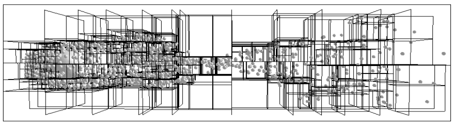
Termination Criteria for Construction

- *Local: using a stack*
  - *Simple local*: maximum depth + number of objects
  - *More complicated local*: a maximum number of cost improvement failures + maximum estimated depth + number of objects
- *Global: using a priority queue*
  - maximum memory used
  - maximum memory used + maximum leaf cost

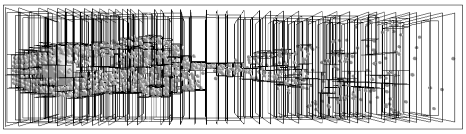
Ray Shooting 18

Kd-tree Construction for Preferred Ray Sets

Idea: using different than uniform distribution of rays



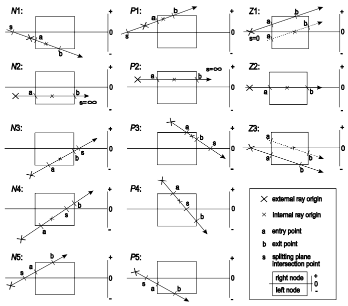
Uniform



Preferred

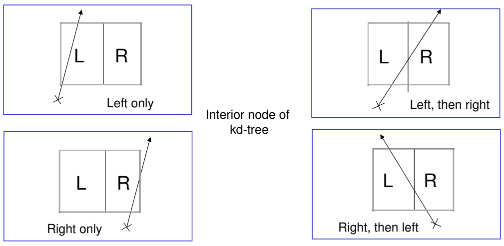
Ray Shooting 19

Recursive Ray Traversal Algorithm for Kd-tree



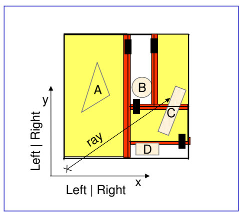
Ray Shooting 20

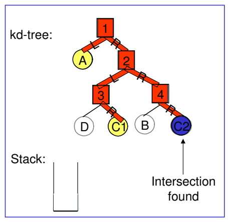
Recursive Ray Traversal Basic Cases Classification



Ray Shooting 21

Recursive Ray Traversal Algorithm

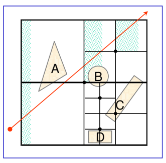




Ray Shooting 22

Ray Shooting with Octrees

- Interior node arity is eight
- Up to four child nodes can be traversed in an interior node
- Traversal algorithm necessarily more complicated than for kd-tree
- About 26 papers about ray tracing with octrees were published
- The octree is less adaptive to the scene geometry than kd-tree
- Geometric probability can be used in the same way as for kd-trees (Octree-R)
- According to the experiments, octrees are less efficient than kd-trees

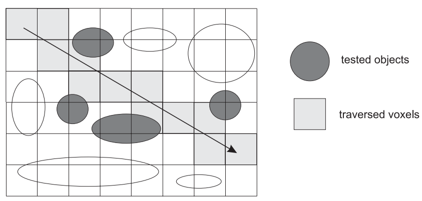


Note: octrees can be simulated by kd-trees

Ray Shooting 23

Ray Shooting with Uniform Grids

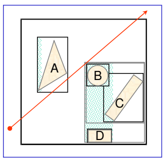
- Arity of a node proportional to the number of objects
- Special traversal method based on 3DDA
- For skewed distributions of objects in the scene is slow
- For highly uniform distributions of objects it is slightly faster than kd-trees



Ray Shooting 24

Ray Shooting with Bounding Volume Hierarchy (BVH)

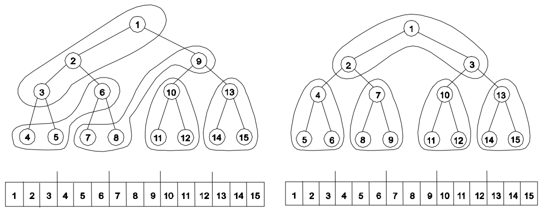
- Each interior node is fully described by a bounding box
- The number of child nodes is usually two for top-down construction (more for bottom-up construction)
- The nodes can overlap
- Each object is referenced only once
- The memory consumption is higher
- Traversal algorithm similar to kd-trees
- Kd-trees can be emulated by BVHs. The other way round is also feasible.



Ray Shooting 25

Data Layout in Memory

Inorder, preorder (depth-first-search), heap (bread-first-search), van Emde Boas



Ray Shooting 26

Performance Model of Ray Shooting

Total cost for RSA =
cost for ray-object intersection tests +
cost for ray traversal of kd-tree +
cost for data move from memory to CPU

- Faster ray-object intersection tests
- Decreasing number of ray-object intersection tests
- Faster traversal step
- Decreasing number of traversal steps
- Reducing CPU-memory traffic

Ray Shooting 27

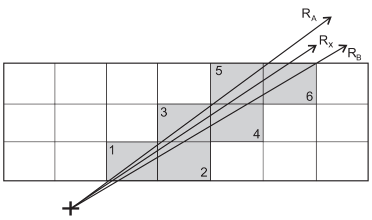
Offline Ray Shooting

- Shooting more rays at once
- Rays are formed by camera, by viewing frustum or by point light sources
- Rays are coherent = similar in direction and origin
- Problem can be formulated as offline setting of searching
- We can amortize the cost of traversal operations though the data structure … the number of traversal steps is decreased typically by 60-70%

Ray Shooting 28

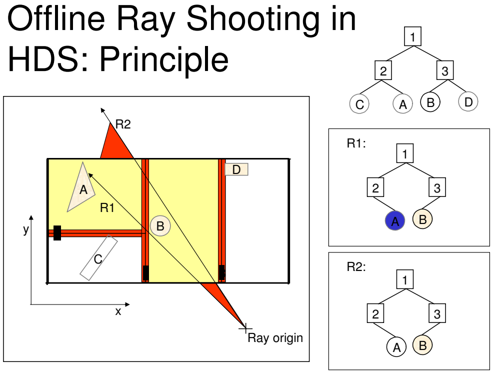
Offline Ray Shooting: Coherence

- If boundary rays traverse the same sequence S of leaves, then all rays in between also traverse the same sequence.
- Proof by convexity

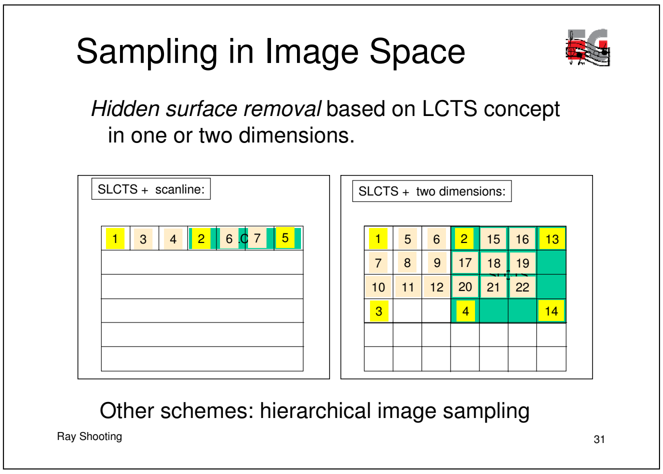


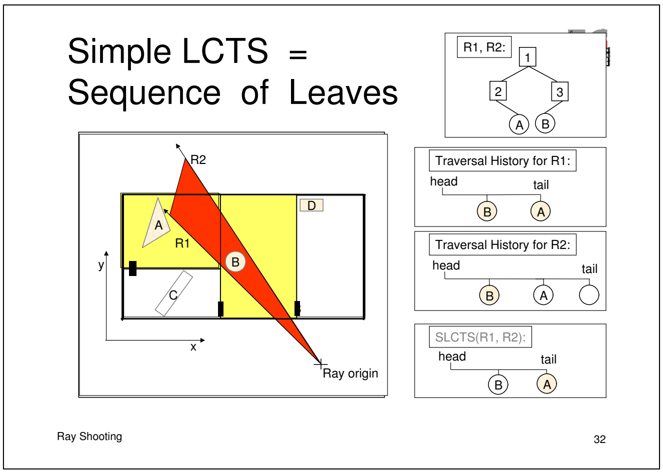
Ray Shooting 29

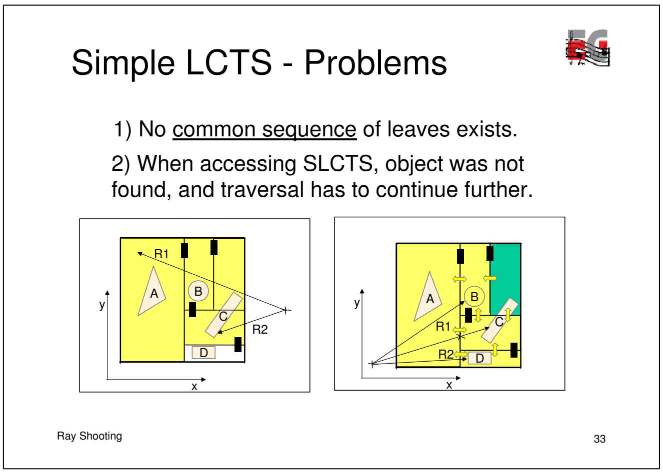
Offline Ray Shooting in HDS: Principle

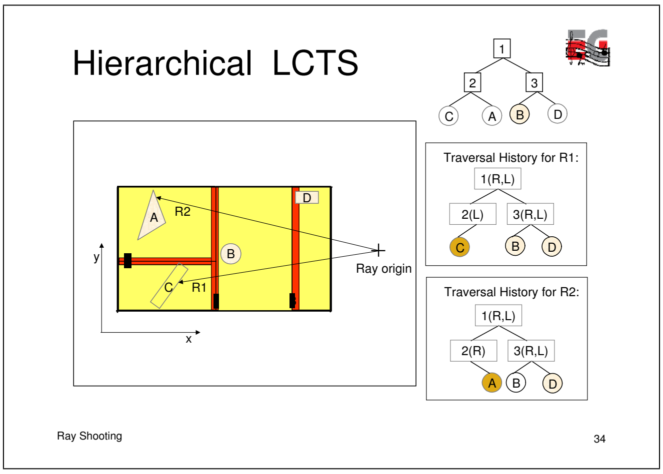


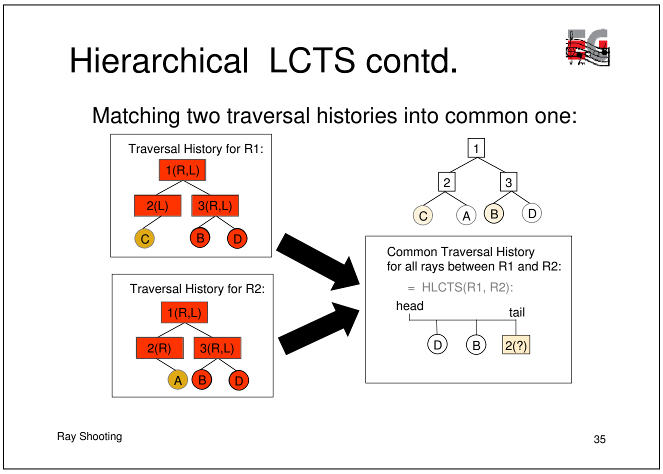
Ray Shooting 30

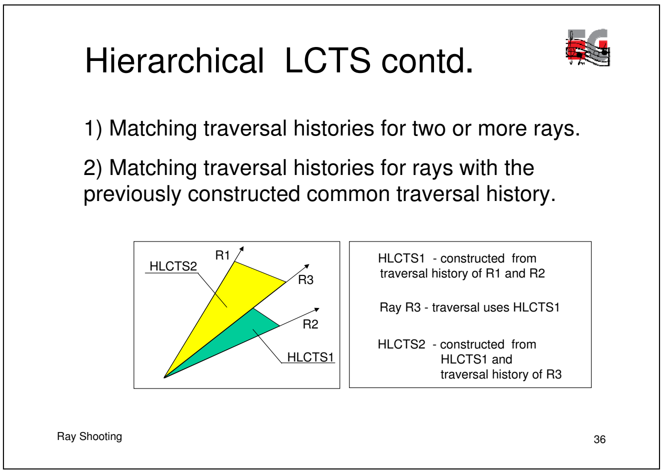

Sampling in Image Space
Hidden surface removal based on LCTS concept in one or two dimensions.
SLCTS + scanline:
1 3 4 2 6 C 7 5
SLCTS + two dimensions:
1 5 6 2 15 16 13
7 8 9 17 18 19
10 11 12 20 21 22
3 4 14
Other schemes: hierarchical image sampling
Ray Shooting
31


Simple LCTS = Sequence of Leaves
R1, R2:
1
2
3
A
B
R2
D
A
R1
B
C
y
x
Ray origin
Traversal History for R1:
head
tail
B
A
Traversal History for R2:
head
tail
B
A
SLCTS(R1, R2):
head
tail
B
A
Ray Shooting
32


Simple LCTS - Problems
1) No common sequence of leaves exists.
2) When accessing SLCTS, object was not found, and traversal has to continue further.
R1
A
B
C
R2
D
y
x
A
B
C
R1
R2
D
y
x
Ray Shooting
33

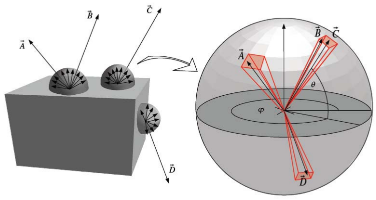

Hierarchical LCTS
1
2
3
C
A
B
D
A
R2
D
B
C
R1
y
x
Ray origin
Traversal History for R1:
1(R,L)
2(L)
3(R,L)
C
B
D
Traversal History for R2:
1(R,L)
2(R)
3(R,L)
A
B
D
Ray Shooting
34


Hierarchical LCTS contd.
Matching two traversal histories into common one:
Traversal History for R1:
1(R,L)
2(L)
3(R,L)
C
B
D
Traversal History for R2:
1(R,L)
2(R)
3(R,L)
A
B
D
1
2
3
C
A
B
D
Common Traversal History for all rays between R1 and R2:
= HLCTS(R1, R2):
head
tail
D
B
2(?)
Ray Shooting
35


Hierarchical LCTS contd.
1) Matching traversal histories for two or more rays.
2) Matching traversal histories for rays with the previously constructed common traversal history.
HLCTS2
R1
R3
R2
HLCTS1
HLCTS1 - constructed from traversal history of R1 and R2
Ray R3 - traversal uses HLCTS1
HLCTS2 - constructed from HLCTS1 and traversal history of R3
Ray Shooting
36

Ray Cache in Final Gathering

- Store the rays into cache according to direction
- When a bucket is filled in, shoot all of them at once
- Improves access pattern for incoherent queries
- Speedup up to 30%



Ray Shooting 37

Surveys on Ray Shooting and Ray Tracing

- G. Simiakakis: Accelerating Ray Tracing with Directional Subdivision and Parallel Processing, 1995
- V. Havran: Heuristics Ray Shooting Algorithms, 2001
- I. Wald: Real Time Ray Tracing and Global Illumination, 2004
- A. Y-H. Chang: Theoretical and Experimental Aspects of Ray Shooting, 2005

Ray Shooting 38

Questions and Answers for Part One

Ray Shooting 39

Part 2 - Content

- Hidden Surface Removal
- Visibility Culling
- Photon Maps and Irradiance Caching
- Ray Maps
- Other Algorithms
- Questions and Answers (10 minutes)

1

Hidden Surface Removal

Jiří Bittner

Czech Technical University in Prague
Vienna University of Technology

2

Hidden Surface Removal

- List priority algorithms
- Area subdivision algorithms
- Scan-line
- Z-buffer
- Ray casting

Hidden Surface Removal 3

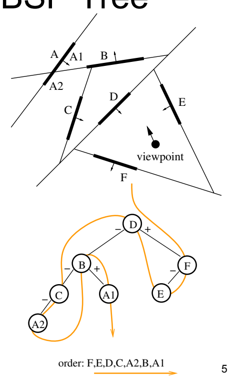
Depth Sort

- Draw faces back to front [Newell72]
- Overwrite the farther ones (painter's alg.)
- Determine strict depth order
  - Resolve cycles of overlapping polygons
- Step 1: depth sort (Z)
  - Quick sort, bubble-sort (temporal coherence)
- Step 2: rasterization (YX)
  - Bucket sort

Hidden Surface Removal 4

Depth Sort with BSP Tree

- BSP build in preprocess
  - Select a plane
  - Partition the polygons in front/back fragments
  - If >1 polygon → recurse
- Quick-sort like
- Heuristics for splitting plane selection



Hidden Surface Removal 5

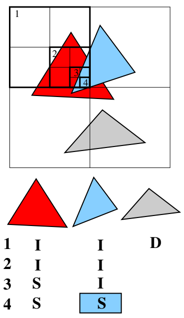
Depth Sort with BSP Tree

- Tree size: $O(n^2)$
- BSP need not be autopartition!
- For manifolds depth order can be predetermined → coarser BSP
- Generalization to all BSP nodes 'Feudal priority tree' [Chen96]

Hidden Surface Removal 6

Area Subdivision

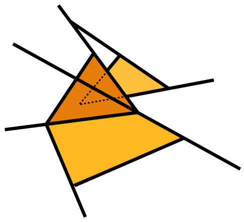
- Subdivide screen space [Warnock69]
- Classify polygons with respect to the area
- Terminate if trivial solution
- Step 1: octree subdivision (XY)
  - Quick sort like
- Step 2: list for octree nodes (Z)
  - Insertion sort



Hidden Surface Removal 7

Naylor’s BSP projection

- Draw polygons front to back
- Clip polygons by 2D BSP of projected polygons
- Step 1: depth sort (Z)
  - 3D BSP built in preprocess
- Step 2: 2D BSP (XY)
  - Quick sort like subdivision of the projection plane



Hidden Surface Removal 8

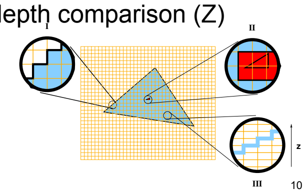
Scan-Line

- Sort by scan-lines (Y)
- Sort spans within a scanline (X)
- Search for closest span (Z)
- [Watkins70]
  - Bubble sort in X and Y
  - O(log n) search in Z

Hidden Surface Removal 9

Z-buffer

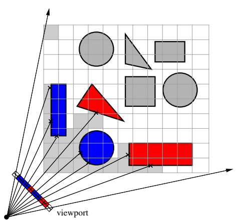
- Rasterize polygons in arbitrary order
- Maintain per pixel depths
- Step 1: rasterization (YZ)
  - Bucket sort
- Step 2: per pixel depth comparison (Z)
  - Min selection



Hidden Surface Removal 10

Ray Casting

- Cast ray for each pixel
- Step 1: spatial data structure (XYZ)
  - Preprocess
  - Trees ~ quick sort
  - Grid ~ distribution sort
- Step 2: search for nearest intersection
  - Min selection with early termination



Hidden Surface Removal 11

Z-buffer vs. Ray Casting

	scan-line coherence	output sensitive	presorting
Z-buffer	+	-	+ (no)
Ray casting	-	+	- (yes)

- Z-buffer better in simple sparsely occluded dynamic scenes
- Ray casting better in complex densely occluded static scenes

Hidden Surface Removal 12

Summary

- HSR algorithms sort in
  - Directions (XY)
  - Depth (Z)
  - Differ in sorting order and methods [SSS74]
- Current winners: z-buffer, ray casting

Hidden Surface Removal 13

Visibility Culling

Jiří Bittner

Czech Technical University in Prague
Vienna University of Technology

1

Visibility Culling - Introduction

- Main goal: reduce linear complexity of z-buffer
  - Render only potentially visible polygons
- Online
  - Applied for every view point at runtime
- Offline
  - Partition view space into view cells
  - Compute Potentially Visible Sets (PVS)

Visibility Culling 2

Visibility Culling – Motivation

- Q: Why visibility culling, when:
  - Object outside screen culled by HW clipping
  - Occluded objects culled by z-buffer
- A: Linear complexity not sufficient!
  - Processing too many invisible polygons
- Goal
  - Render only what can be seen!
  - Make z-buffer output sensitive

Visibility Culling 3

Online Visibility Culling

- For every frame cull whole groups of invisible polygons
- Conservative solution
  - Trading accuracy for speed
  - Leaves a superset of visible polygons
  - Precise visibility solved by z-buffer

Visibility Culling 4

Online Visibility Culling

- Backface culling
- View-frustum culling
- Occlusion culling
  - CPU techniques
  - GPU based (HW occlussion queries)

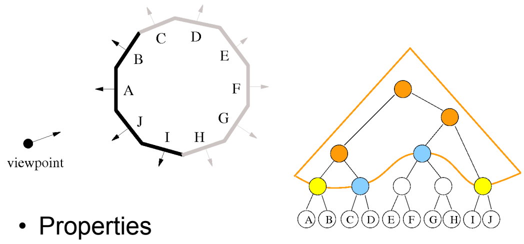
Visibility Culling 5

Backface Culling

- Culls about 50% polygons
- Supported by the GPU
- Alternative: Hierarchical back-face culling [Kummar96]
  - Sort polygons based on their normals into a tree
  - Skip whole groups of backfacing polygons

Visibility Culling 6

Hierarchical Backface Culling



- Properties
  + Skips whole groups of polygons
  - Regroups the scene (discards objects)
  - Limited gain

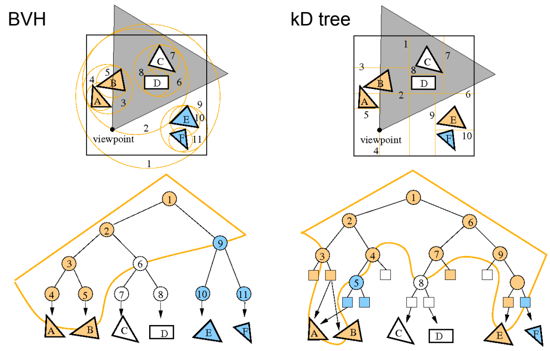
Visibility Culling 7

View Frustum Culling

- Culls 0-100% polygons
- Conservative algorithm
  – Spatial hierarchy: kD-tree, BSP tree, octree, BVH
  – Intersection test with the view frustum
- Optimizations
  – Temporal coherence
  – Efficient intersection test [Assarson00]

Visibility Culling 8

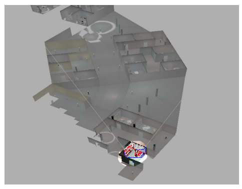
View Frustum Culling - Example



Visibility Culling 9

Occlusion Culling

- Previous methods disregard occlusion
- 99% of scene can be occluded!

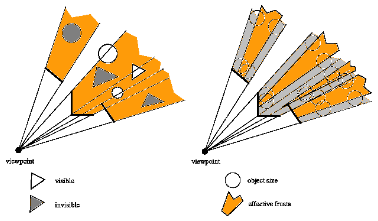


- Solution: Detect and cull also occluded objects

Visibility Culling 10

Shadow Frusta

- Construct shadow frusta for several occluders [Hudson97]
- Object is invisible if inside a shadow frustum
- Queries on the spatial hierarchy



Visibility Culling 11

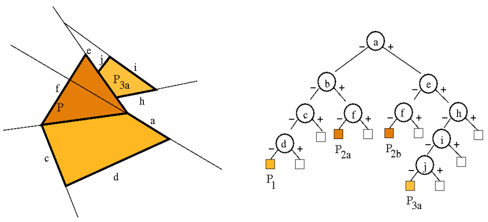
Shadow Frusta - Properties

- Properties
  + Easy implementation
  - No occluder fusion!
  - No occluder sorting - O(n) query time
  - Small number of occluders (~10)

Visibility Culling 12

Occlusion Trees

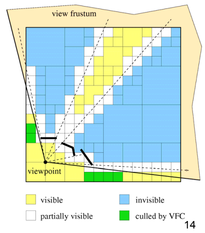
- Occluders sorted into a 2D BSP tree [Bitt98]
- The tree represents fused occlusion
- Example: occlusion tree for 3 occluders



Visibility Culling 13

Occlusion Tree - Traversal

- Visibility test of a node
  - Depth-first-search
  - Found empty leaf → tested object is visible
  - Depth test in filled leaves
- Example of final visibility classification of kD-tree



Visibility Culling 14

Occlusion Tree - Properties

- Presorting occluders
  - Tree size: worst case $O(n^2)$
  - O(log n) visibility test

\+ Allows to use more occluders (~100)

\- Not usable for scenes with small polygons

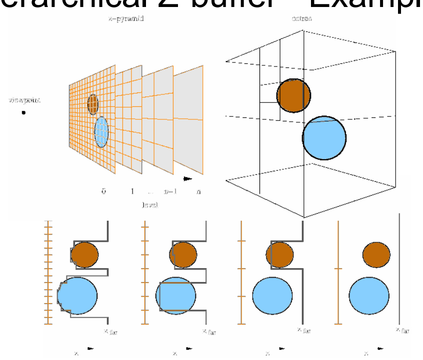
Visibility Culling 15

Hierarchical Z-buffer

- Extension of z-buffer to quickly cull larger objects [Greene 96]
- Ideas
  - octree for spatial scene sorting
  - z-pyramid for accelerated depth test

Visibility Culling 16

Hierarchical Z-buffer - Example



Visibility Culling 17

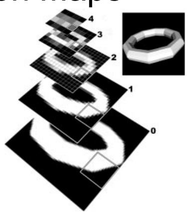
Hierarchical Z-buffer - Usage

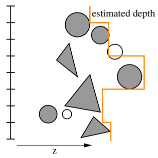
- Hierarchical test for octree nodes
- Find smallest node of z-pyramid, which contains the tested box
- Box depth > node depth → cull
- Otherwise: recurse to lower z-pyramid level
- Optimization: use temporal coherence
  - z-pyramid constructed from polygons visible in the last frame

Visibility Culling 18

Hierarchal Occlusion Maps

- Hierarchical occlusion maps [Zhang97]
- Pyramid of occlusion maps
- Separate occlusion and depth representation
  - Hierarchical occlusion
  - Coarse depth
- Queries on spatial hierarchy





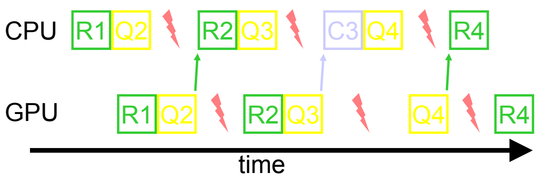
Visibility Culling 19

HW Occlusion Queries

- ARB_occlusion_query, NV_occlusion_query
- Return #pixels passing the depth test
- Feature which has been missing!
- Issues
  1. Latency – the result not readily available
  2. The query costs time

Visibility Culling 20

CPU Stalls GPU Starvation

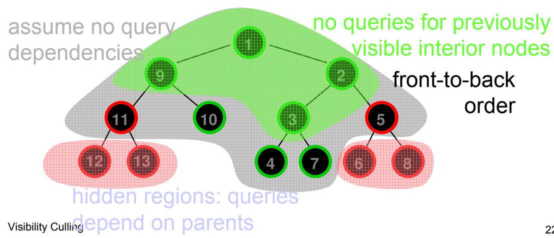


Rx Render object x
Qx Query object x
Cx Cull object x
Waiting time

Visibility Culling 21

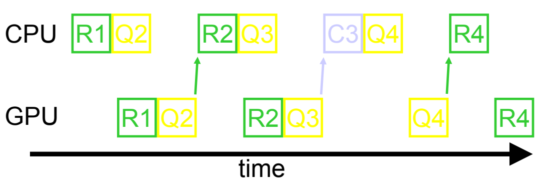
Coherent Hierarchical Culling

- CHC [Bitt04]
  - Interleave queries and rendering
  - Schedule queries based on temporal coherence



Visibility Culling 22

CHC



Rx Render object x
Qx Query object x
Cx Cull object x

Visibility Culling 23

CHC

- Video
- UNC power plant, 12.7M polygons

Visibility Culling 24

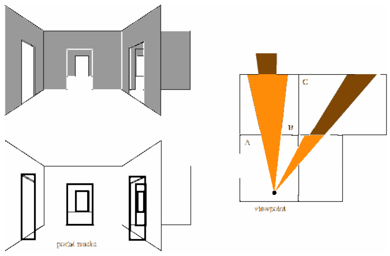
Cells and Portals

- Partition the scene in cells and portals
  - Cells ~rooms
  - Portals ~ doors&windows
- Cell adjacency graph
- Constrained DFS
  - Portal visibility test [Luebke 96]

Visibility Culling 25

Portal Visibility Test

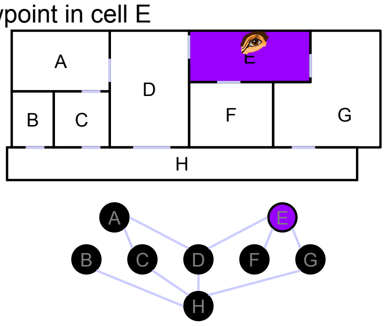
- Intersection of bounding rectangles of portals



Visibility Culling 26

Cells and Portals Example

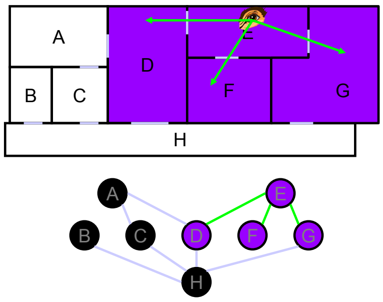
- Viewpoint in cell E



Visibility Culling 27

Cells and Portals - Example

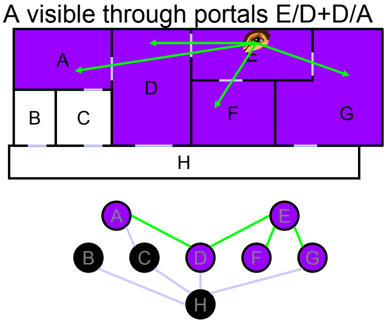
- Adjacent cells DFG



Visibility Culling 28

Cells and Portals - Example

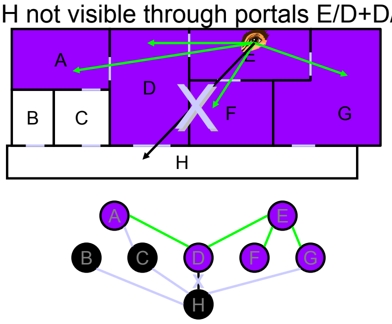
- Cell A visible through portals E/D+D/A



Visibility Culling 29

Cells and Portals - Example

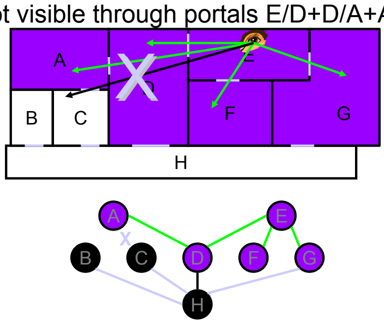
- Cell H not visible through portals E/D+D/H



Visibility Culling 30

Cells and Portals - Example

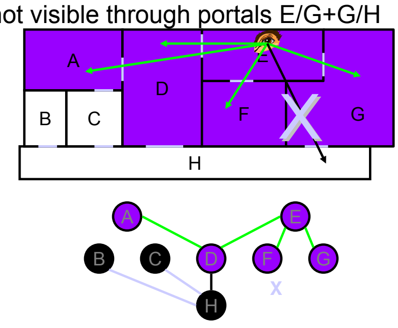
- C not visible through portals E/D+D/A+A/C



Visibility Culling 31

Cells and Portals - Example

- H not visible through portals E/G+G/H



Visibility Culling 32

Visibility Preprocessing

- Preprocessing
  - Subdivide view space into view cells
  - Compute Potentially Visible Sets (PVS)
  - Solves visibility “offline” for all possible view points
- Usage
  1. Find the view cell (point location)
  2. Render the associated PVS

Visibility Culling 33

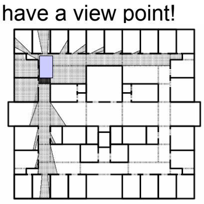
Visibility Preprocessing

- Other benefits
  - Prefetching for out-of-core/network walkthroughs
  - Communication in multi-user environments
- Problems
  - Costly (treats all view points and view directions)
  - 4D domain (online methods only 2D)

Visibility Culling 34

Interiors – Cells and Portals

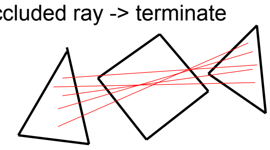
- Subdivide the scene into cells and portals
- Constrained DFS on the adjacency graph
  - Portal visibility test
- More complex than the online algorithm
  - We do not have a view point!



Visibility Culling 35

Interiors – Cells and Portals

- Sampling [Airey90]
  - Random rays
  - Non-occluded ray -> terminate

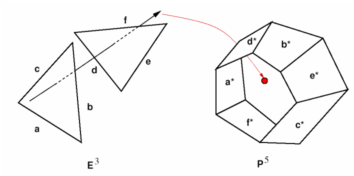


+ Simple implementation
- Approximate solution

Visibility Culling 36

Interiors – Cells and Portals

- Exact computation [Teller 92]
  - Mapping to 5D (Plücker coordinates of lines)
- Portal edges → hyperplanes H_i in 5D
- Halfspace intersection in 5D



Visibility Culling 37

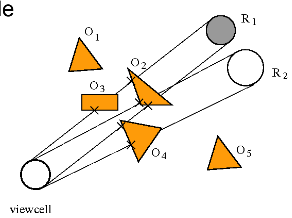
General Scenes - Strong Occlusion

- Occlusion by single object [CohenOr98]
- For each cell and object
  - Cast rays defining convex hull of the cell and object
  - If a convex occluder intersects all rays → invisible

Visibility Culling 38

General Scenes - Strong Occlusion

- Example



- Properties
  - +Simple
  - - No occluder fusion
  - - Too conservative for larger view cells and small objects

Visibility Culling 39

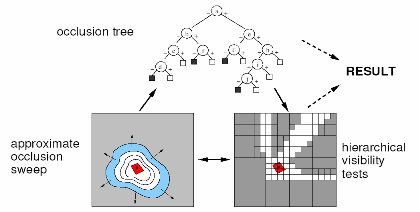
General Scenes
Occlusion Tree

- Extension of the 2D occlusion tree
- 5D BSP tree
  - Plücker coordinates of lines
- The tree represents union of all occluded rays for a given view cell

Visibility Culling 40

General Scenes
Occlusion Tree

- Process polygons in front-to-back order
- Polygon visible → enlarge tree by visible rays
- Polygon invisible → tree not modified



Visibility Culling 41

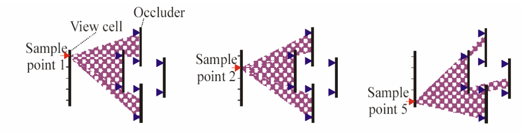
General Scenes
Occlusion Tree

- Properties
  - + Exact solution
  - + Uses visibility coherence
  - - Difficult implementation

Visibility Culling 42

2.5D Scenes Occluder Shadows

- Footprint of occluded volume [Wonka00]
  - Agrregates the shadow polygons using z-buffer
  - Represents intersection of all 'shadows'



Visibility Culling 43

2.5D Scenes Occluder Shadows

- Conservative solution
  - Shrinking occluder polygons
- Properties
  - + Relatively easy implementation
  - + Uses GPU
  - - Large view cells → more conservative solution
  - - Needs high resolution cull map

Visibility Culling 44

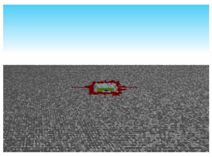
2.5D Scenes Ray Space Factorization

- Main ideas [Leyvand et al. 2003]
  - Occluder in 2.5D → 3D polygon in ray space
  - Polygon shape: defined by 2D footprint
  - Polygon depth: defined by heights

Visibility Culling 45

2.5D Scenes Ray Space Factorization

- Render polygons using z-buffer
- Visible polygons in ray space → visible objects in primal space

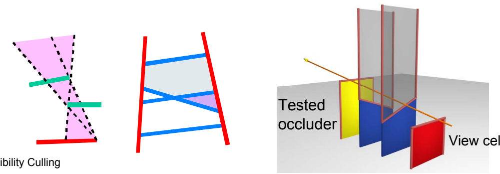



- Properties
  - - Conservative solution
  - + GPU implementation

Visibility Culling 46

2.5D Scenes Occlusion Tree + Virtual Portals

- Occlusion tree for visibility in 2D footprint
- Identifies sequencies of occluders
- Construct virtual portals over the occluders
- Portal visibility test in 5D [Teller 92]



Visibility Culling 47

2.5D Scenes Occlusion Tree + Virtual Portals

- Properties
  - + exact solution for 2.5D scenes
  - + computation time comparable with conservative methods
  - - difficult implementation

Visibility Culling 48

Summary

- Online occlusion culling
  - Computation for every frame
  - No global information
  + Dynamic scenes
  + Easy implementation
- Visibility preprocessing
  + Gives global information
  + Almost no computation at runtime
  - Static scenes
  - Significant preprocessing time
  - Difficult implementation

Visibility Culling 49

Surveys on Visibility

- F. Durand. 3D Visibility: Analytical Study and Applications
- D. Cohen-Or et al.: A survey of visibility for walkthrough applications
- J. Bittner and P. Wonka: Visibility in computer graphics

Visibility Culling 50

Photon Maps and Irradiance Caching

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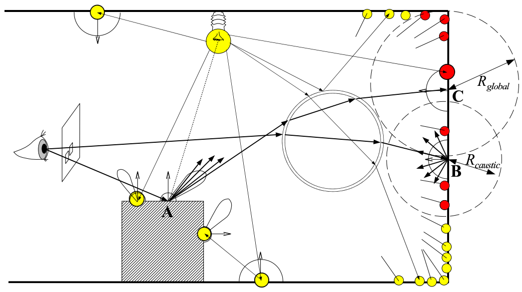
1

Photon Maps and Irradiance Caching

- Final gathering versus direct visualization
- Photon maps
- Irradiance caching
- Offline techniques

Photon Maps and Irradiance Caching 2

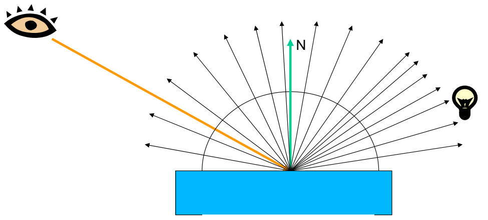
Photon Mapping



Photon Maps and Irradiance Caching 3

Final Gathering

- Shooting many secondary rays (possibly according to BRDF), gathering radiances from the rays
- Integrating the radiances properly to render image

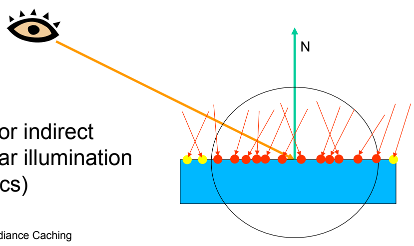


- Used for indirect diffuse illumination

Photon Maps and Irradiance Caching 4

Direct Visualization

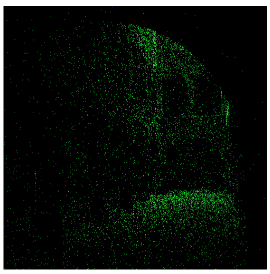
- Do not shoot final gather rays, use directly visible photons from camera
- It is prone to artifacts on object boundaries referred to as bias



- Used for indirect specular illumination (caustics)

Photon Maps and Irradiance Caching 5

Example of Direct Visualization



Photon Hits

Direct Visualization

Photon Maps and Irradiance Caching 6

Estimating Radiance along Final Gather Ray

- Using the density estimation, from the photon hits estimating *PDF*
- It requires K nearest neighbor searching for each final gather ray
- The number of final gather rays (the number of searches) is enormous
- Typically we shoot 200-4000 final gather rays per pixel
- The number of pixels per image 1-6 x 10^6

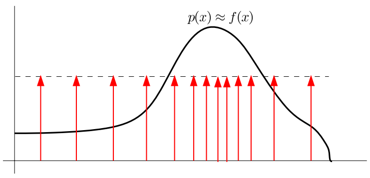
Photon Maps and Irradiance Caching 7

Intro to Density Estimation

- Histogram method – take hits into buckets
- Kernel density estimation
- K-Nearest neighbors estimator
- Variable kernel density estimator
- Multiple pass methods
  - First pass – pilot estimate
  - Second pass – final estimate

Photon Maps and Irradiance Caching 8

Example: Density Estimation in1D

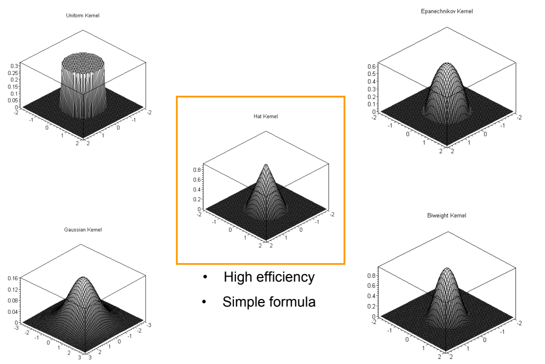


Note: **Importance Sampling**: from given *p(x)* to samples

Density Estimation: from samples to *p(x)* … more complicated

Photon Maps and Irradiance Caching 9

Kernel Types



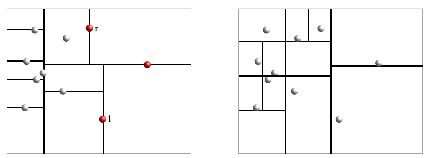
Relation to Searching

- *Range search* – given a fixed range query (sphere, ellipsoid), find all the photons in the range
- *K nearest neighbor search* – given a center of the expanding shape X (sphere, ellipsoid), find K nearest photons
  - Without considering the direction of incoming photons
  - With considering only valid photons with respect to the normal at point X

Photon Maps and Irradiance Caching 11

Search Techniques

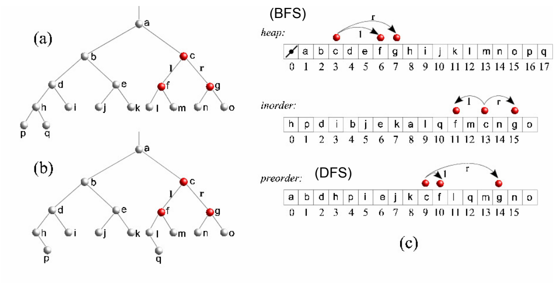
- Use any data structures described in the section "*Hierarchical Data Structures*"
- Typically kd-trees or kd-B-trees are used



Kd-tree Kd-B-tree

Photon Maps and Irradiance Caching 12

KD-tree Layout in Memory



+ van Emde Boas layout (see ray tracing)

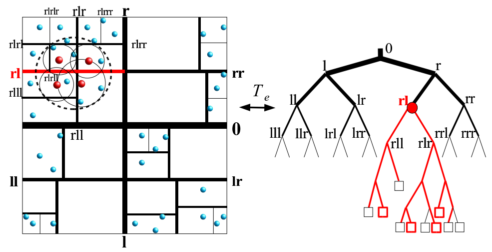
Photon Maps and Irradiance Caching 13

Practical yet Efficient Solution

- Use Kd-B-trees
- Construct a tree over an array of photons
- Use 8 Bytes nodes – good packing
- DFS or van Emde Boas Layout
- Sliding mid-point rule = spatial median + shift to nearest photon if one side empty
- One leaf contains a range of 30-70 photons (two indices to photon array)
- Properties:
  - fast construction time
  - fast search (complexity proved to be optimal)

Photon Maps and Irradiance Caching 14

Aggregate Searching (= Offline Search)



- splitting planes of the spatial kd-tree
- first intersected plane <=> start node
- density estimation point (ep)
- photon hit point
- leaf node containing ep
- searched leaf
- start node for individual queries
- l left child , r right child

Photon Maps and Irradiance Caching 15

Searching Tricks for k-NN Search

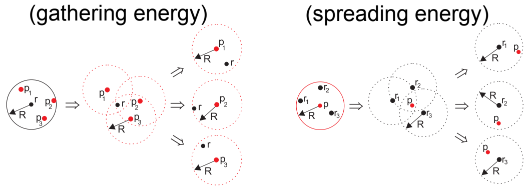
- Do not use uniform grids, they do not work efficiently for skewed distributions
- Try to avoid a priority queue
- Use a fixed radius search where a radius is estimated for given N
- Radius estimate can be based on the properties of the data structure over photons (diagonal of a leaf box) or taken from already computed query
- Use offline search if possible
- Try to change the role of input data to be queried and queries

Photon Maps and Irradiance Caching 16

Reverse Photon Mapping

Normal Photon Mapping (gathering energy)

Reverse Photon Mapping (spreading energy)



r – ends of final gather rays (in black)

p – photons (in red)

Photon Maps and Irradiance Caching 17

Why Does It Work Faster ?

Assume that the number of interactions among photons and final gather rays is the same !

Traditional Photon Mapping – a single tree

- Many searches (~10^9) in a small tree over photons (~10^6)
- kNN search based on the photon density

Reverse Photon Mapping – more involved *(two trees)*

- Smaller number of searches (~10^6) in a larger tree over the ends of final gather rays (~up to 10^9)
- k-NN search is also based on the photon density

Properties

- Search in a tree is logarithmic, reverse photon mapping then faster
- Reverse photon mapping takes more memory

Photon Maps and Irradiance Caching 18

Time Complexity Formulas

- F … number of final gather rays
- K … number of neighbors for kNN search
- V … number of photons
- F * K … number of interactions photon-final gather ray

Traditional Photon Mapping Time

C_PT = C_1 * F * K + C_2 * F * log V

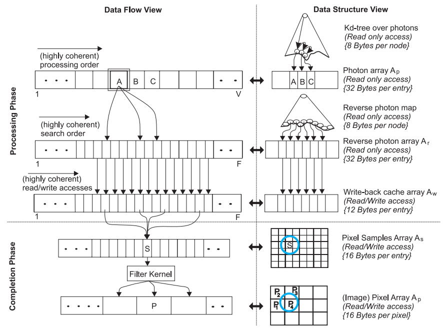
Reverse Photon Mapping Time

C_RPT = C_1 * F * K + C_2 * V * log F

For **F >> V** it is easy to show that **F * log V > V * log F**

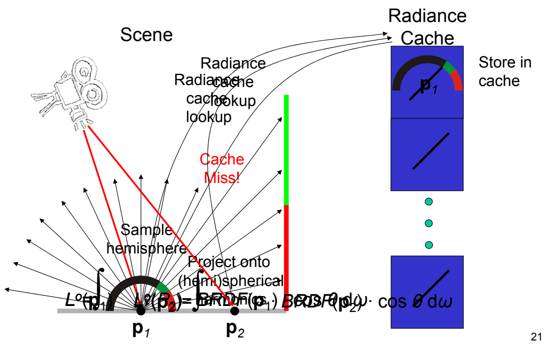
Photon Maps and Irradiance Caching 19

Data Flow + Data Structure View



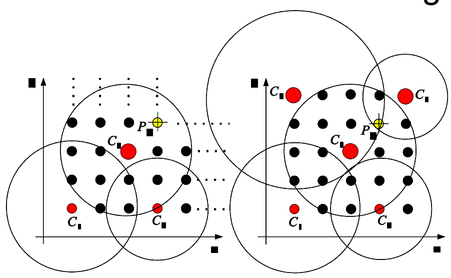
Photon Maps and Irradiance Caching 20

Radiance and Irradiance Caching



21

Data Structures for Caching



- Red – data in the cache
- Black – queries

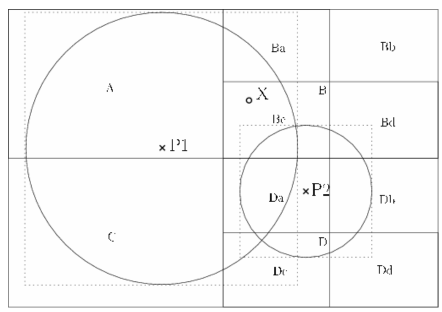
Photon Maps and Irradiance Caching 22

Search Specification

- Records – the irradiance specified by a point and radius of influence
- Query: given a point, find all the sphere in which the point is contained
- Problem is *intersection search*
- Data structures should be dynamic – insertion and deletion is possible

Photon Maps and Irradiance Caching 23

1) Using Octree (Ward et al. 88)



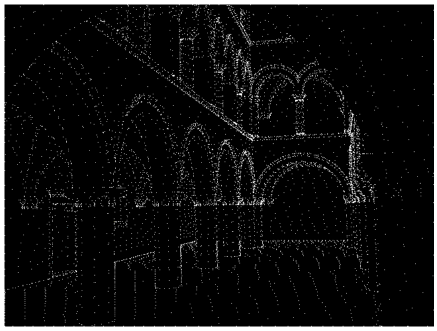
Photon Maps and Irradiance Caching 24

2) Using Mapping to R^4

- A sphere (a,b,c,r) in R^3 as a point in R^4 (t1,t2,t3,t4) by linearization:
  (2.a, 2.b, 2.c, a*a + b*b + c*c - r*r)
- Query: a point (a, b, c) in R^3 as a hyper-plane in R^4 (t1,t2,t3,t4) as follows:
  H: a*t1+b*t2+c*t3-t4 –(a*a+b*b+c*c) > 0
- Compute *half-space range reporting* in R^4 space, it requires a spatial data structure in R^4
- Efficiency depends highly on
  – Position of points with respect to the space origin
  – Efficiency of half-space range reporting

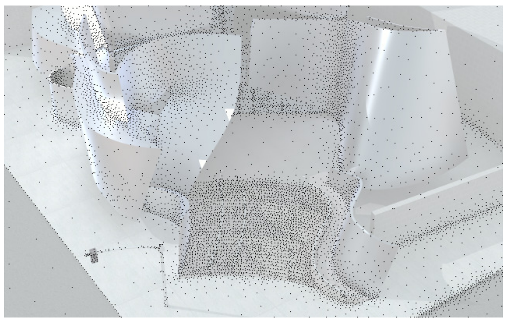
Photon Maps and Irradiance Caching 25

Irradiance Caching Records



Photon Maps and Irradiance Caching 26

Radiance Caching Records



Photon Maps and Irradiance Caching 27

Ray Maps

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1

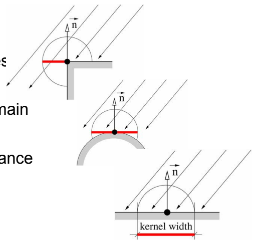
Ray Maps

- Ray map: data structure sorting rays
- Allows efficient searching for rays
  - intersecting a disc/sphere/hemisphere
  - nearest to a point using
  - combination of intersection & NN
- Main application:
  improved density estimation

Ray Maps 2

Density Estimation

- Problems with photon maps
  1. Limited to object surfaces
  2. Boundary bias – darkening on objects' boundaries
  3. Topological bias – assumption on shape of the domain
  4. Proximity bias – tradeoff between noise and variance



- Ray maps

  eliminate 1. and 2., possibly reduce 3.

Ray Maps 3

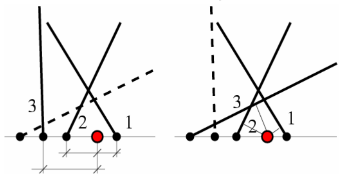
Ray Map Queries

- Queries types
  - intersection search (fixed query domain)
  - k-nearest neighbors (k-NN) search
- Query domains
  - disc
  - sphere
  - hemisphere
  - axis-aligned box
  - \+ possible limitation on ray directions

Ray Maps 4

Metrics for k-NN Search

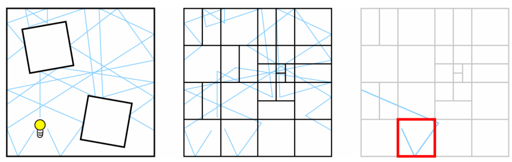
- Distance on the tangent plane
- Distance to the ray segment
- Distance to the supporting line of the ray



Ray Maps 5

Ray Map Implementation

- Kd-tree
- Leaves store references to the rays
- Lazy construction driven by the queries
- Support efficient searching and updating



Ray Maps 6

Construction

- Spatial median split
- Subdivide if #rays > budget
- Classify rays back, front, both
- Termination criteria
  - #ray references per leaf (~32)
  - size of the leaf (~0.1% of the scene box)
  - Max tree depth (~30)

Ray Maps 7

Searching

- Intersection search
  - Locate all leaves containing query domain
  - Gather rays
  - Compute intersections
- k-NN search
  - Priority queue (stack)
  - Locate the leaf containing the query origin
  - If #rays < N get next node from stack

Ray Maps 8

Maintenance

- Deleting a ray
  - Ray cast and remove references
- Adding a ray
  - Ray cast and subdivide if required
- Keeping memory budget
  - Collapsing of unused subtrees nodes
  - LRU strategy

Ray Maps 9

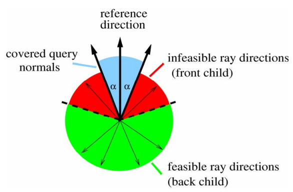
Optimization 1
Exploiting Query Coherence

- Subsequent queries highly coherent
- Store traversed nodes of the previous query
- Initiate the priority queue with the saved nodes
- Top-down traversal is reduced

Ray Maps 10

Optimization 2
Directional Splits

- Queries are oriented
- Many rays in the opposite direction after reflection
- Optimization: inserting directional nodes



Ray Maps 11

k-NN Search with Ray Maps

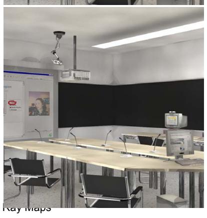
- 1M – 2.5M rays
- Typical memory usage: 16 – 128MB
- Query time: 0.2 – 1.5ms (3.2GHz PC)
- 2.1 to 4.7 times slower than photon maps

Ray Maps 12

Comparison with Photon Maps

Photon maps

Ray maps



Photon maps + convex hull

1,000,000 photons, 500 kNN

Similar Data Structures

- Ray cache [Lastra02]
  - Hierarchy of spheres
- Volumetric ray density estimation [VanHaevre04]
  - Octree
  - Simulation of plant growth

Ray Maps in Line Space

- Idea
  - Ray → 5D point (Plücker coordinates)
  - Query → 5D polyhedron
  - Report all points in the polyhedron
  - Use 5D kD-tree to sort points
- Poor performance
  - Culling only at very bottom of the tree
  - 5D boxes not separate well from the query polyhedron

Ray Maps - Summary

- Sorting rays + efficient searching
- Kd-trees implementation
  - Simple implementation
  - Efficient memory usage control
  - About 2-5x slower than photon maps
- Density estimation
  - New query domains + new metrics
  - Elimination of boundary bias
  - Reduction of topological bias

Other Algorithms

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1

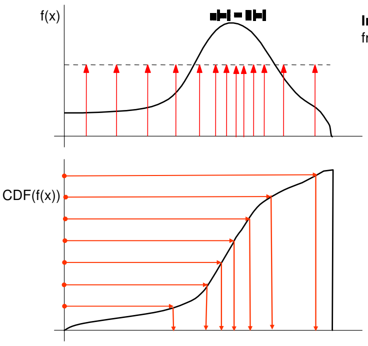
Other Algorithms

- Importance sampling
- Hierarchies over light sources
- Extensions to ray tracing
- Some other techniques

Note: this list on sorting and searching in rendering is definitely not complete !

Other Algorithms 2

Importance Sampling



Importance Sampling: from given *p(x)* to samples

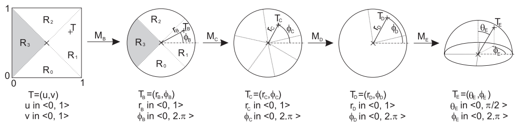
Step 1: compute cumulative distribution function into a table

Step 2: via uniform distribution over CDF generate required distribution f(x)

Other Algorithms 3

Importance Sampling Transforms

- *Results:* samples on the hemisphere (2D domain)
- *Usage:* for BRDF and environment maps
- *Realization:* using four mappings
- *Properties:* bijectivity, continuity in both directions, low distortion
- *Complexity of sampling:* O((log N) * (log M))



Other Algorithms *Havran et al. 03c: Goniometric Diagram Mapping for Hemisphere* 4

Hierarchies over Light Sources

- Another hierarchy (=sorting) if number of light sources is high, approximating or discarding less important light sources
- Papers:
  - Ward92: Adaptive Shadow Testing, discard less important contributions, avoid shadow rays testing
  - Lazanyi and Szirmay-Kalos 04: Speeding up the Virtual Light Sources Algorithm
  - Paquette et al. 98: A Light Hierarchy for Fast Rendering of Scenes with Many Lights
  - Walter et a. 05: Lightcuts: a scalable approach to illumination
  - Walter et al. 06: Multidimensional lightcuts

Other Algorithms 5

Extensions to Ray Tracing

- Spatio-temporal domain
  - Continuous setting (Glassner 88)
  - Multiframe ray tracing (discrete time setting) (Havran et al. 03b)
  - Reprojection for walkthroughs (Havran et al. 03a)
- Approximate ray tracing
  - Szirmay-Kalos et al.: Approximate Ray-Tracing on the GPU with Distance Impostors (2005)
- Fast construction or update for animations
  - Several algorithms proposed in 2005, not yet resolved issue

Other Algorithms 6

Some Other Techniques

- Temporal Photon Mapping and Spatio-Temporal Density Estimation
  - Cammarano and Jensen 02: Time Dependent Photon Mapping
  - Weber et al. 2004: Spatio-Temporal Photon Density Estimation Using Bilateral Filtering
- Reordering the queries for photon mapping
  - Havran et al. 05: Reverse Photon Mapping
  - Steinhurst et al. 05: Reordering for Cache Conscious Photon Mapping
- Changing the role of queries and input data to be queried
  - Havran et al. 05: Reverse Photon Mapping (here in the slides)
  - Laine and Aila 05: Hierarchical Penumbra Casting

Other Algorithms 7

Remainder and Question

- This list on the use of sorting and searching in rendering algorithms is definitely not complete!
- Are you convinced now that sorting and searching is really relevant to rendering?

Other Algorithms 8

Content

Part One

- Introduction to Rendering
- Sorting and Searching
- Hierarchical Data Structures
- Ray Shooting
- Questions and Answers

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Content

Part Two

- Hidden Surface Removal
- Visibility Culling
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Questions and Answers for Part Two

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