A Fresnel Model for Coated Materials

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Figure 1: *A composition of various conductors with and without coatings, rendered in our in-house renderer using our proposed model.*

Abstract

We propose a novel analytical RGB model for rendering coated conductors, which provides improved accuracy of Fresnel reflectance in BRDFs. Our model targets real-time path tracing and approximates the Fresnel reflectance curves with noticeably more accuracy than Schlick's approximation using Lazanyi's error compensation term and the external media adjustment. We propose an analytical function with coefficients fitted to measured spectral datasets describing the complex index of refraction for conductors. We utilize second-order polynomials to fit the model, subsequently compressing the fitted coefficients to optimize memory requirements while maintaining quality. Both quantitative and visual results affirm the efficacy of our model in representing the Fresnel reflectance of the tested conductors.

CCS Concepts

• ***Computing methodologies*** → ***Reflectance modeling;***

1. Introduction

The accurate representation of the reflection of light is crucial for achieving realistic rendering in real-time applications. The Fresnel term in Bidirectional Reflectance Distribution Functions (BRDF) plays a pivotal role in simulating this complex behavior at the surface of objects. In recent times, there has been a notable shift towards coated materials, which aim to emulate the appearance of materials that have been treated with a thin layer of another material [WW07, Bel18, dDB22]. The coating alters the Fresnel reflectance curves based on its index of refraction (IOR), a change that is particularly evident in conductors due to their spectrally variable and complex IOR. Currently Schlick's approximation is a widely adopted approximation for the Fresnel equations [Sch94]; it can be used with an adjustment to account for external media like coatings [Hof20], but this only modifies F0 and still leads to a noticeable error on most conductors with higher incident IOR values. In this paper, we propose an accurate, empirically derived analytical RGB model that approximates the spectral Fresnel functions for conductors, incorporating the incident Index of Refraction. Our model is designed for use in real-time path tracing.

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2. Previous Work

To date, numerous studies have explored various aspects of BRDF models to simulate the reflectance of light more realistically. A fundamental aspect of this research is the Fresnel effect. Practical approximations are often employed in the film and game industry to strike a balance between accuracy and real-time performance. These approximations simplify the complex spectral dependencies of the Fresnel equations.

One of the earliest approximations of Fresnel reflection was introduced by Blinn [Bli77]. Blinn utilized the Fresnel equations to obtain the angular variation in the intensity of reflected light; however, this approach proved less effective at accurately rendering conductors. Later, Cook [CT82] introduced a model that evaluates the Fresnel equations using a single IOR value. The resulting scalar value is used to interpolate between the color at an incident angle of 0 degrees and white, providing an approximation that is improved compared to Blinn's model. Schlick's approximation [Sch94], inspired by Cook's approach, aims to enhance the practicality of the analytical approximation by substituting the IOR value with a power term. However, both Schlick and Cook's model fail to account for a decrease in reflectivity near an incident angle of 90 degrees, which is important for the appearance of conductors like aluminum and chromium. Karis explored an empirical model aimed at efficiently approximating Fresnel reflectance using Schlick's approximation to pre-calculate the reflectance into a look up table (LUT) that can be sampled using roughness and $\cos\theta$ as texture coordinates [Kar13]. To provide more artistic control, Gulbrandsen [Gul14] developed a parameterization that is directly based on Fresnel equations. It maps RGB values to η and κ values to evaluate the Fresnel equations. This RGB-based approximation does not consider the perceptual and physical meaning of the quantities, which can lead to significant inaccuracies [Hof19]. Belcour et al. use pre-integrable decomposition to fit Fresnel reflectance values, which enables pre-integrating Gulbrandsen's proposed parameterization [BBB20]. This technique, however, does not provide a more accurate parameterization for coated materials.

Lazányi and Szirmay-Kalos propose an improvement to Schlick's approximation, which involves incorporating an error compensation term to achieve a closer match with the spectral reference for metals [LSK05]. Hoffman reparameterized Lazányi's model to make artistic control more intuitive. The coefficients needed for Lazányi's error compensation term are replaced by a color value at approximately 82 degrees [Hof19]. Hoffman also showed a term to adjust F0 based on the index of refraction of the external medium but even the more accurate form only adjusts F0 and still leads to a clearly visible difference to ground truth for many conductors [Hof20]. Kutz et al. modified Hoffman's parameterization to an edge tint that is applied multiplicatively and reduces to Schlick's reflectivity when a white edge tint is specified [KHE21]. Hoffman has further generalized this parameterization to accommodate the widely accepted extensions of Schlick's model. [Hof23].

3. Method

Previous works in analytical approximations of the Fresnel term show noticeable errors for many tested conductors on larger incident IOR values. To address this limitation, we propose an analytical approximation that more accurately matches the spectral reference data for coated conductors with varying incident IORs. In this section we outline our method.

In an offline step, we evaluate the Fresnel equations using measured spectral η and κ values per conductor and subsequently convert the results to the ACEScg color space (Section 3.1). The resulting dataset has two input parameters: the cosine of the angle between the normal and incident direction ($\cos\theta$) and the incident IOR that map to a result per color channel. We then employ non-linear least squares using the Trust Region Reflective objective function to fit coefficients of second-order polynomial functions to represent the three-dimensional R, G, and B curves. The resulting quadratic functions are employed in our real-time rendering model to approximate conductors. Our model is designed to be both accurate and efficient in evaluation, while operating on RGB components of the chosen color space.

3.1. Generating reference data

To fit our conductors, we produce three-dimensional curves from measured spectral reference data. Measured data sets consist of η and κ values and we tested our model on 22 different conductors [Pol08, Boi00, JC74, QWH*74, BJCP91, LH85, WGAD09]. Using this data we evaluate the Fresnel equations at wavelengths from 360 to 830 nanometers. We then integrate and convert the resulting SPDs to the ACEScg color space. We chose to use this color space since the gamut of sRGB cannot accurately represent the Fresnel reflectance of gold.

3.2. Representation of conductors

Our model is based on Schlick's approximation using the error compensation term proposed by Lazányi and Szirmay-Kalos:

$$F_0 + (1-F_0)(1-\cos\theta)^5 - a\cos\theta(1-\cos\theta)^\alpha \quad (1)$$

Using Schlick's method with this error term, a first order polynomial would perform notably worse when the incident IOR is 1.0, which is a common case. Therefore, our model utilizes a second order polynomial. We fit coefficients for the variables a, α, and F_0 to the dataset per color channel for each conductor. The general form of the function is as follows:

$$f(\cdot; c_0, c_1, c_2) = c_0 + c_1\eta_i + c_2\eta_i^2 \quad (2)$$

This results in 9 coefficients per color channel, with each variable having its unique set of coefficients c_0, c_1, and c_2.

One of our objectives is to provide an efficient and compact representation of the coefficients needed to represent a conductor to make it suitable for real-time path tracing. We achieve this by representing each coefficient as 16 bit float. The resulting analytical model allows us to render conductors with varying incident IOR in real-time, and can be evaluated using the coefficients, $\cos\theta$ and η_i.

4. Results

In this section we present the results of our experiments, focusing on two key aspects. First, we assess the accuracy of our fitting code

Conductor	Δ E-Ours	Δ Schlick
Al	0.09	0.62
Au	0.14	3.36
Cu	0.10	2.32
Ta	0.47	3.54
Cr	0.31	1.73

Table 1: *The average Δ E2000 error of several conductors.*

by comparing it to Fresnel reflectance values derived from reference data sets (Section 4.1). Second, we validate the visual output of our method, rendered in our in-house renderer, by comparing it to the Pbrt-v4 spectral rendering system (Section 4.2). All validation tests are conducted on a system equipped with an RTX 2080TI. We find that evaluating our model in full screen 2560p x 1440p takes about 199 μs while Schlick's approximation with the error compensation term proposed by Lazányi and Szirmay-Kalos using Hoffman's notation with the external media adjustment [Hof20] (hereafter referred to as 'adjusted Lazányi approximation') takes about 143 μs. Visual and quantitative results of all tested conductors can be found in our supplemental material.

When generating reference data for visual validation we specifically chose the *ConductorMaterial* and set the incident IOR manually. It is important to note that we intentionally excluded the *CoatedConductorMaterial* from Pbrt-v4 for validation purposes. This decision is made to isolate the effect of the Fresnel equations and eliminate potential bias from light scattering between layers. We render a shader ball illuminated by a white sky environment to reduce errors from unrelated factors. Reference images are generated using Monte Carlo simulation, converged over 1024 frames. Figure 2 shows an example for chromium and tantalum, results are shown at various incident IOR values for comparison. On the left side of each image, we showcase our model, while on the right, we feature the adjusted Lazányi approximation.

4.1. Quantitative results

We quantitatively evaluated the accuracy of our model by comparing 10,000 sampled reflectance values from our model, evenly distributed across all incident angles and IOR values, to samples from generated three-dimensional reference curves. We conducted a comparison of average error values between our model and the adjusted Lazányi approximation. Differences are expressed using the ΔE2000 metric since it is a well-established standard in color measurement, it aims to make the perceived color difference consistent regardless of the colors being compared. The error metric is calculated by averaging the errors across all values of $\cos\theta$ and η_i (Eq. 3).

$$\mathrm{RMSE} = \sqrt{\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\left(f(i,j)-\hat{f}(i,j)\right)^2} \tag{3}$$

Differences in color values are shown for various conductors in Table 1.

4.2. Visual results

To visually validate the accuracy of our method, we compare the results from our model to the reference images generated using Pbrt-v4. The outcomes of this experiment are compared using FLIP [ANAM*20], a difference evaluator that focuses on the perceived difference by the human eye. Figure 2 provides a detailed view of the visual results for chromium, tantalum and cobalt rendered using η_i values 1.5 and 2.5.

4.3. Data reduction impact

The coefficients representing the conductors in our model consume a substantial amount of data. We discovered that this can be effectively minimized with negligible loss in quality by encoding them as 16-bit values. To assess the impact of this data reduction, we calculated the Root Mean Square Error (RMSE) of our model across 22 measured datasets of distinct conductors, comparing the results between 32-bit and 16-bit coefficients using the ΔE2000 metric. The analysis of the results was conducted across four intervals of η_i, starting at 1 and increasing in increments of 0.5, up to 2.5. The findings revealed a slight increase in error at larger IOR values. Specifically, all variations remained below 0.03 at an incident IOR of 1. However, at an η_i of 2.0 only cobalt exhibited a divergence exceeding 0.03, reaching approximately 0.0328, which is well below the threshold at which this is perceivable. At an η_i of 2.5, five of the tested conductors reached a peak increase in ΔE2000 exceeding 0.03, with niobium showing a maximum increase of approximately 0.0738.

5. Discussion and Limitations

Both quantitative and visual results affirm the efficacy of our model in representing the Fresnel reflectance of all tested conductors. In all tested cases, our model outperforms the adjusted Lazányi approximation. As indicated in Figure 2, the error of our model is difficult to discern visually when compared to the converged images from the Pbrt-v4 spectral renderer. This holds true for all tested conductors, with η_i ranging from 1.0 to 2.5 η_i. However, it is important to note that our model requires more data to represent conductors. Additionally, the increased dimensionality of our model renders it incompatible with the split-sum approximation used in certain real-time rendering engines [Kar13]. Addressing potential performance concerns due to increased data usage, we propose to represent coefficients as 16-bit floats and find that there is an imperceivable loss in quality. Due to the increase in data usage, our method is expected to be slower than the adjusted Lazányi approximation.

In the context of our research, we experimentally extend our exploration to assess the capability of our model to render spectral datasets with optical constants of semiconductors, metalloids, and dielectrics, even though our study predominantly concentrates on conductors. Our model accurately represents the majority of tested datasets. However, we encounter representational challenges with certain elements. Specifically, we identify notable discrepancies in rendering some narrow-gap semiconductors; they exhibit significant alterations in shape that our model is unable to accurately

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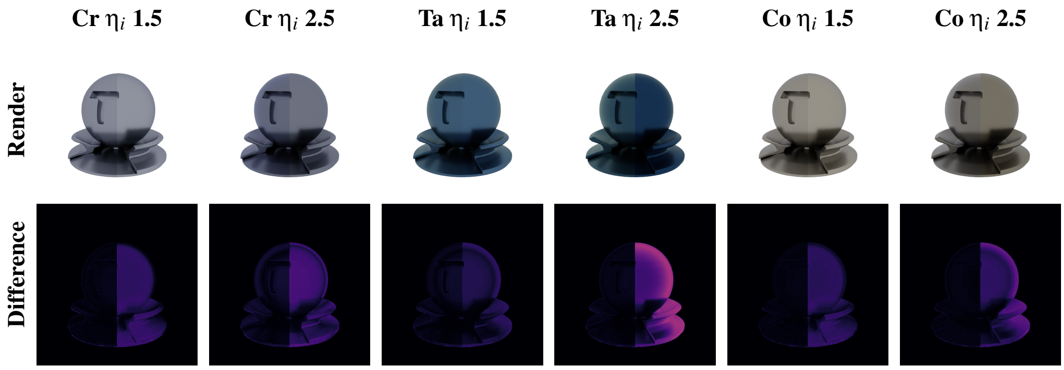

Figure 2: *Visual results of Cr, Ta and Co at various incident IOR values. Left is our model, right is Schlick's approximation.*

replicate as η_i increases. So far, we have only found these significant alterations in inherently incomplete datasets, where the extinction coefficient, κ, is zero across all wavelengths. While these observations are noted, they are not extensively discussed or elaborated upon, as they diverge from the central theme of our research.

6. Future Work

In this study, we have developed an accurate Fresnel model tailored for real-time path tracing. Future research could explore adapting our model for efficient pre-integration, which would extend its applicability to image-based lighting and area light techniques used in real-time rendering engines.

While our model is successful in accurately representing conductors, it lacks artistic control explore incorporating a curve editor, allowing modifications to the RGB reflectance dataset, and subsequently fitting coefficients to the adjusted data. This would bridge the gap between accuracy and artistic expression, providing users with flexibility to realize their creative vision while maintaining the model's integrity.

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