A Halfedge Refinement Rule for Parallel Loop Subdivision

Supplemental Material: Implementation Cheat Sheet

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In this document, we provide "cheat sheets" to visually follow a step of the algorithm and facilitate implementation.

In Sec. 1, we provide illustrations that help follow the explanations of the main paper's Section 2.1. That is, we compile each crease configuration for edges and vertices. Each such configuration leads to a different vertex point computation during refinement.

In Sec. 2 we provide our halfedge and crease refinement algebraic rules exhaustively. To make things more intuitive, we provide an example for each rule based on the mesh illustrated above the rules.

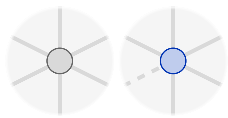
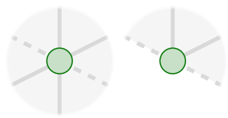
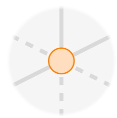
1 Crease configurations

Semi-sharp creases provide a useful design tool for artists, as motivated in seminal papers [HDD*94, DKT98] and the OpenSubdiv documentation[1]. Sec. 2.1 of our main paper discusses how we compute the vertex points of a mesh under Loop subdivision. Similarly to OpenSubdiv, the exact rule we apply depends on local crease configurations.

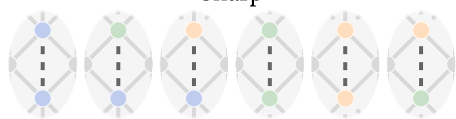
In Section 1.1, we provide a visual classification of all possible crease configurations for edges and vertices. In Section 1.2, we illustrate the letter notations used in describing the Vertex points calculations.

1.1 Crease configurations

Vertex Crease Configurations. The following figure provides the *vertex crease type* classification and associated vertex point rule to apply for each case. Note that we depict creases as dashed lines.

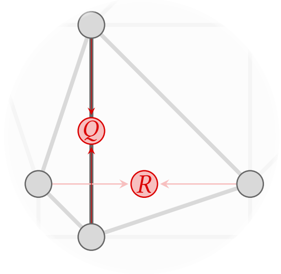
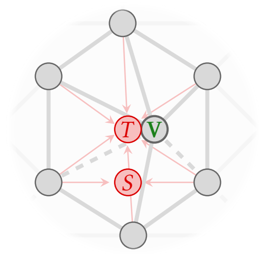
Vertex Crease Type	Smooth 	Crease 	Corner 
Number of adjacent creases	0 or 1	2	> 2
Vertex point subdivision rule to apply	(V.3)	$\begin{cases} (V.2) & \text{if } \bar{\sigma} > 1.0 \\ (V.4) & \text{otherwise} \end{cases}$	(V.1)

Edge Crease Configurations. The following figure provides the *edge crease type* classification and associated edge point rule to apply for each case. Note that we draw creases as dashed lines, and vertex colors refer to their crease type.

Edge Crease Type	Smooth	Sharp 
Edge point subdivision rule to apply	(E.2)	$\begin{cases} (E.1) & \text{if } \sigma > 1.0 \\ (E.3) & \text{otherwise} \end{cases}$

1.2 Illustration of letter notations

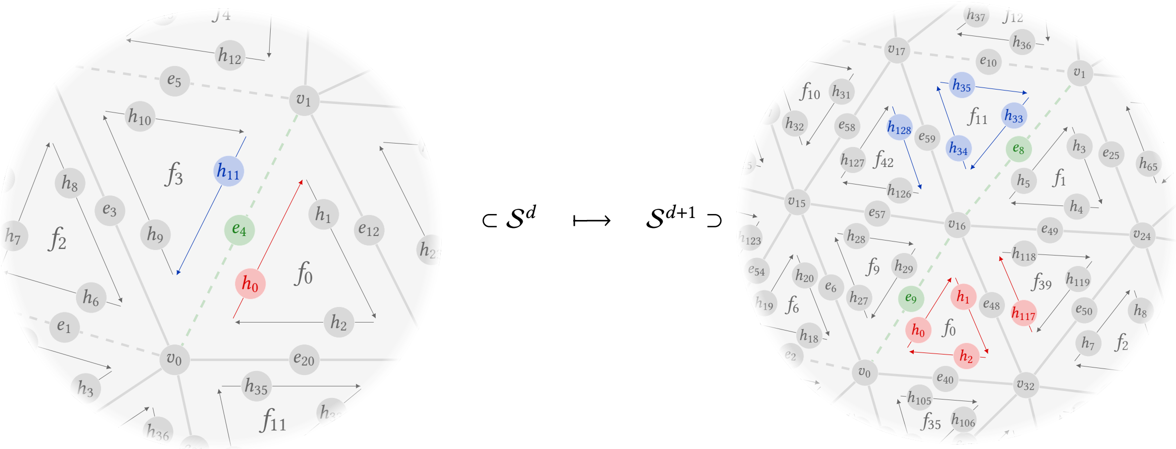
In Section 2.1 of the main paper, we present *vertex point calculation* for the two types of refined points: *edge points* and *vertex points*. These are the rules (E.x) and (V.x), respectively. To facilitate the explanation, we resorted to the letter notations Q and R for rules (E.x) and V, S and T for rules (V.x). Below, we illustrate them.

Edge point calculation: rules (E.x)	Vertex point calculation: rules (V.x)
	
$\begin{cases} Q & \text{is the midpoint of the old (thick) edge} \\ R & \text{is the midpoint of the two opposite vertex points in both incident triangles} \end{cases}$	$\begin{cases} S & \text{is the midpoint of the two vertex points that share a crease edge (dashed) with the old vertex point } V \\ T & \text{is the average of the old neighboring vertices adjacent to } V \end{cases}$

[1] https://graphics.pixar.com/opensubdiv/docs/subdivision_surfaces.html#semi-sharp-creases

2 Subdivision rules

Here, we provide the exhaustive set of algebraic rules for halfedge and crease refinement. An example for each rule is given, with the help of the illustration provided above. Creases are drawn as dashed lines in the figure. Note that this figure is the same configuration as the teaser figure of the main paper. Notably, we have the constants $H_d = 39$ halfedges, $V_d = 12$ vertices and $E_d = 24$ edges.



Halfedge refinement rules

(a) halfedge's twin rule	(b) halfedge's edge rule (*using* $h' := \textsc{Prev}(h)$)	(c) halfedge's vertex rule (*using* $h' := \textsc{Prev}(h)$)
$\textsc{Twin}(h) \mapsto \textsc{Twin}(3h+0) = 3\,\textsc{Next}(\textsc{Twin}(h)) + 2$ $\mapsto \textsc{Twin}(3h+1) = 3\,H_d + h$ $\mapsto \textsc{Twin}(3h+2) = 3\,\textsc{Twin}(\textsc{Prev}(h))$ $\mapsto \textsc{Twin}(3H_d+h) = 3\,h+1$	$\textsc{Edge}(h) \mapsto \textsc{Edge}(3h+0) = \begin{cases} 2\,\textsc{Edge}(h) & \text{if } h > \textsc{Twin}(h) \\ 2\,\textsc{Edge}(h)+1 & \text{otherwise} \end{cases}$ $\mapsto \textsc{Edge}(3h+1) = 2E_d + h$ $\mapsto \textsc{Edge}(3h+2) = \begin{cases} 2\,\textsc{Edge}(h')+1 & \text{if } h' > \textsc{Twin}(h') \\ 2\,\textsc{Edge}(h') & \text{otherwise} \end{cases}$ $\mapsto \textsc{Edge}(3H_d+h) = 2E_d + h$	$\textsc{Vert}(h) \mapsto \textsc{Vert}(3h+0) = Vert(h)$ $\mapsto \textsc{Vert}(3h+1) = V_d + \textsc{Edge}(h)$ $\mapsto \textsc{Vert}(3h+2) = V_d + \textsc{Edge}(\textsc{Prev}(h))$ $\mapsto \textsc{Vert}(3H_d+h) = V_d + \textsc{Edge}(\textsc{Prev}(h))$
examples: $\textsc{Twin}(0) \mapsto \{29,\ 117,\ 105,\ 1\}$ $\textsc{Twin}(11) \mapsto \{5,\ 128,\ 36,\ 34\}$	*examples:* $\textsc{Edge}(0) \mapsto \{9,\ 48,\ 40,\ 48\}$ $\textsc{Edge}(11) \mapsto \{8,\ 59,\ 10,\ 59\}$	*examples:* $\textsc{Vert}(0) \mapsto \{0,\ 16,\ 32,\ 32\}$ $\textsc{Vert}(11) \mapsto \{1,\ 16,\ 17,\ 17\}$

Crease refinement rules

(g) creases's sharpness rule	(h) crease's next rule (*using* $c' := \textsc{Next}(c)$)	(i) crease's previous rule (*using* $c' := \textsc{Prev}(c)$)
$\sigma(c) \mapsto \sigma(2c+0) = \langle \frac{\sigma(\textsc{Prev}(c)) + 3\,\sigma(c)}{4} - 1 \rangle$ $\mapsto \sigma(2c+1) = \langle \frac{\sigma(\textsc{Next}(c)) + 3\,\sigma(c)}{4} - 1 \rangle$	$\textsc{Next}(c) \mapsto \textsc{Next}(2c+0) = 2c+1$ $\mapsto \textsc{Next}(2c+1) = \begin{cases} 2c' & \text{if } c = \textsc{Prev}(c') \text{ and } c' \neq c \\ 2c'+1 & \text{otherwise} \end{cases}$	$\textsc{Prev}(c) \mapsto \textsc{Prev}(2c+0) = \begin{cases} 2c'+1 & \text{if } c = \textsc{Next}(c') \text{ and } c' \neq c \\ 2c' & \text{otherwise} \end{cases}$ $\mapsto \textsc{Prev}(2c+1) = 2c$
example: $\sigma(4) = 1.8 \mapsto \{0.8,\ 0.8\}$	*example:* $\textsc{Next}(4) = 1 \mapsto \{9,\ 2\}$	*example:* $\textsc{Prev}(4) = 5 \mapsto \{10,\ 8\}$

References

[DKT98] DeRose T., Kass M., Truong T.: Subdivision surfaces in character animation. In *SIGGRAPH '98* (New York, NY, USA, 1998), ACM, pp. 85–94. doi:10.1145/280814.280826.

[HDD*94] Hoppe H., DeRose T., Duchamp T., Halstead M., Jin H., McDonald J., Schweitzer J., Stuetzle W.: Piecewise smooth surface reconstruction. In *SIGGRAPH '94* (New York, NY, USA, 1994), ACM, pp. 295–302. doi:10.1145/192161.192233.