Multidimensional Shape Modeling for Animation

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Abstract

An approach is proposed for generating animation sequences from functionally defined multidimensional shapes. An animation sequence can be considered as a particular case of a multimedia object and can be described using 2D/3D world coordinates, time, color, texture, audio, or other "multimedia coordinates". We introduce a space mapping between geometric coordinates and multimedia coordinates. An example is provided for modeling a multidimensional shape, mapping it onto multimedia space, and subsequently generating the animation sequence "Homotopic Fun in 5D space".

Keywords: Implicit Surface, Metamorphosis, Multidimensional Space, Animation, Multimedia.

1. Outline of the approach

Modern modeling, rendering, and animation systems provide a variety of tools for modeling time-dependent 3D shapes and generating still images and animation sequences. However, time dependence is usually introduced after the shape has been defined. Direct modeling of a 4D object with time as one of its coordinates is not widely supported. An important example of a 4D operation is metamorphosis (transformation of one 3D shape into another), which can not be easily applied with current animation systems. The main problem with such a metamorphosis is the necessity to manually establish a one-to-one correspondence between points on the two shapes. Moreover, robust methods of metamorphosis between surface models of different genus are not known [3].

On the other hand, modeling of multidimensional point sets (shapes) has attracted much attention in different fields such as mathematics, the natural sciences, data mining, and aesthetic and industrial design. One of the models used for multidimensional shape modeling is the *function representation* (*F-rep*) [3]. In general, F-rep defines a geometric object by a single continuous real function of several variables $F(X) \geq 0$, where X is a vector of point coordinates in n-dimensional Euclidean space. A set of operations closed on the representation is provided (set-theoretic, blending, sweeping, etc.). The following are special operations for modeling in multidimensional space: Cartesian product (increasing an object's dimension) and projection (decreasing an object's dimension). Metamorphosis and non-linear deformations of different types are generalized by so-called extended space mappings [5]. Objects modeled in the following different representational schemes can be converted to F-rep: implicit surfaces, CSG, sweeping, voxel data, and closed parametric surfaces. Shapes can be modeled directly in multidimensional space and treated uniformly in spite of their dimension. Time dependent shapes can be modeled directly as 4D objects to be followed by the generation of time cross-sections (or 3D frames). The metamorphosis is performed automatically and handles objects of different topology (e.g., genus change and generation of disjoint components).

An animation sequence can be considered as a particular type of a multimedia object and can be described using 2D/3D world coordinates, time, color, texture, audio, or other "multimedia coordinates". To generate animation sequences, we propose to map multidimensional F-rep shapes onto multidimensional spaces of multimedia coordinates.

To operate with multimedia coordinates, one can introduce a system of normalized numerical coordinates (a unit cube) and its one-to-one correspondence to the multimedia space. By selecting a real normalized value, one can select the corresponding value of the multimedia coordinate.

The following types of multimedia coordinates with their respective variation intervals can be considered:

- *World* coordinates (types **x**, **y**, **z**) of 2D and 3D "real life" geometry. They can be Cartesian, cylindrical or any other coordinates. The selection of these types and their variation

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intervals defines the "elementary" image or 2D/3D shape within a bounding box in the selected geometric space. An "elementary" shape (i.e., curve, surface, and isosurface) is a projection of a cross-section of the initial multidimensional shape.

- *Dynamic* coordinates (type **t**) represent continuous values that can be linearly or non-linearly mapped onto physical time. This type can be assigned to one or several geometric coordinate variables. The user can actually provide a path in the space of variables of the "t" type by filling out a table specifying their discrete values. Each row of the table corresponds to a certain time value. Then, frames of the animation sequence in the form of the model's cross-sections can be produced. This procedure can serve as the basis for implementing a quite complex metamorphosis.
- *Spreadsheet* coordinates (types **u** and **v**) take discrete values in the given bounding box. This type allows for a spreadsheet-like spatial organization of elementary images or shapes in regularly or irregularly placed 1D, 2D, or 3D nodes.

Other types of multimedia coordinates (*photometric, transformation, audio, haptic*, etc.) are considered elsewhere [1].

Each geometric coordinate variable takes values within a given interval. On the other hand, multimedia coordinates also have their own intervals of variation. To define the mapping, one has to establish correspondence between these intervals. Generally, more than one multimedia coordinate can correspond to one geometric coordinate. For instance, one can map a certain geometric coordinate simultaneously onto a dynamic coordinate and a color coordinate. This results in associating the geometric coordinate with "dynamic color" allowing for the production of differently colored frames in an animation sequence.

To support this approach, we have developed software tools on the Windows NT platform. These tools provide the user a means to:

- Specify a functionally based model in C or in HyperFun (a specialized high-level language for specification F-rep shapes) [2]. One can use the F-rep library of geometric objects and transformations as well as the user's own library of geometric objects written in HyperFun or C.
- Define mappings of geometric space to multimedia space by assigning "multimedia types" to geometric coordinates. At least one geometric coordinate has to be assigned the dynamic type to generate an animation sequence.
- Generate images of polygonized or ray-traced elementary shapes, animation sequences, or spreadsheets in accordance with assigned multimedia types.

2. Example: bi-directional metamorphosis as a 5D object

We present a bi-directional metamorphosis, which is not a traditional transformation in computer graphics and animation. Modeling and visualization tools that have already been implemented allow us to produce quite intriguing shapes by mixing four key shapes. The selected key-shapes (see Fig. 1) are, to some extent, "cultural key signs" in Japan:

- A cat (upper left in Fig. 1) resembles the cult character of children's animation. Its complete model in HyperFun can be found at [2].
- "NiHon" (lower left) is a 3D puzzle representing the word for "Japan" in Japanese. First, the two 3D Chinese characters "*Ni*" and "*Hon*" are constructed independently as unions of blocks. Then, the solids are oriented along Z and X axes respectively and combined as $NiHon = Ni \cap Hon$, where $\cap$ represents the intersection operation. The idea of this puzzle construction is that the resulting 3D solid looks like a single initial 2D character "*Ni*" or "*Hon*" when projected along Z and X axes respectively onto a plane.
- A robot (upper right) and 3D word "robot" (lower right) are also modeled using set-theoretic operations on primitive shapes implemented with R-functions.

The transformation applied to the key shapes can be described by the following expression:

Meta5D = (Cat*(1.-x[4])+Robot*x[4])*(1.-x[5])
+(NiHon*(1.-x[4])+Rob_let*x[4])*x[5];

where **Meta5D** stands for the defining function of the bi-directional metamorphosis; **Cat**, **Robot**, **NiHon**, and **Rob_let** each represent the individual defining functions $f(x_1, x_2, x_3)$ of the key-shapes (where $f = 0$ defines the object surface); **x[4]** and **x[5]** are coordinates of 5D space and parameters of the interpolation. Algebraically, the model of the metamorphosis is the bilinear interpolation between four real-valued functions by coordinates **x[4]** and **x[5]**. Geometrically, it is a 5D object defined by the real function as $\phi(x_1, x_2, x_3, x_4, x_5) \geq 0$.

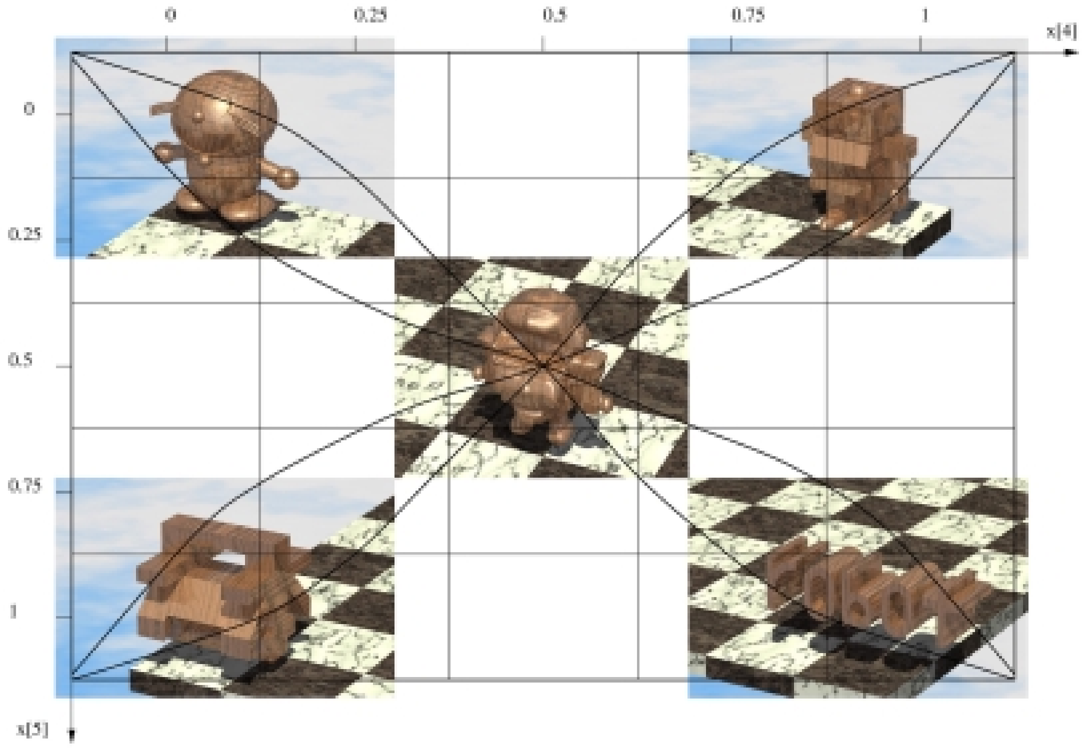

Figure 1: *Bi-directional metamorphosis: spreadsheet and animation path design*

Another way that the metamorphosis can be viewed is as homotopy in the functional space, which is reflected in the title of the final animation [6].

x[1] → x	x[1] → x
x[2] → y	x[2] → y
x[3] → z	x[3] → z
x[4] → u	x[4] → t1
x[5] → v	x[5] → t2

Figure 2: *Mappings for spreadsheet (left) and animation (right)*

A 2D spreadsheet (Fig. 1) is the most adequate visual representation of this 5D object. The mapping shown in Fig. 2 (left) defines such a spreadsheet. The four key-shapes are placed in the corners of the 5×5 spreadsheet pattern with the **u** and **v** coordinates varying in the [0,1] interval using a step value of 0.25. For example, (0.5,0.5) cell contains the image of the equal-weighted mixture of all four shapes. The spreadsheet can be considered as a set of frames of an animation with two dynamic variables. It can serve as a reference for constructing an animation sequence containing particularly interesting shape transformations. To create an animation, we used the mapping shown in Fig. 2 (right). This mapping assigns *dynamic* types to coordinates x[4] and x[5]. Each frame of the animation is a rendered 3D object corresponding to a specific point in the (x_4, x_5) plane. The animation sequence corresponds to a curve in (**t1**,**t2**) space. The spreadsheet helps to introduce such a curve (see Fig. 1) and to select the points on the curve for the animation frames. The frames of the resulting animation [6] were produced using POVRay 3.0 with the isosurface patch by R. Suzuki.

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