Better Pasting Via Quasi-Interpolation

Stephen Mann† and Blair Conrad

Department of Computer Science, The University of Waterloo, Waterloo, Ontario, Canada

Abstract

In this poster, we show our latest work on surface pasting*, a hierarchical modeling technique. In surface pasting, we attach a feature surface to a base surface using an approximation technique. While we can use knot insertion on the feature to get the approximation to any level of tolerance, knot insertion increases the size of the representation of the feature. The work we present in this poster shows how we use quasi-interpolation to improve the approximation, allowing us to use smaller representations of our features.*

1. Introduction

In surface modeling, a typical model will have a basic, smooth shape, with high detail in a few regions. This is the case, for example, for a car body: the side panel is a smooth surface with little variation, except for the door handle, which has higher detail than the rest of the panel.

Tensor product B-splines are commonly used for surface modeling. However, they are poorly suited to building surfaces with varying amounts of detail. Although detail can be added using knot insertion, knot insertion adds detail across wide regions of the surface in addition to the area in which the detail is needed. This extra detail increases the storage required for the surface, and makes it difficult to edit the surface at the lower level of detail.

Several hierarchical methods have been developed for tensor product B-splines. In particular, Hierarchical B-splines[5] provide for local detail, and allow for editing the surface at any resolution. Changes to the coarse representation automatically adjust the detail in a natural manner.

Surface pasting[2, 1, 3] is a method developed at the University of Waterloo that extends the idea of Hierarchical B-splines. Feature surfaces are attached to a base surface by offsetting the feature surface's control points from the base, allowing for rapid placement of the feature as an approximation to displacement mapping. Surface pasting allows a modeler to reuse features and create libraries of features for later use. Further, the orientation of a feature on the base is unrestricted, and unlike Hierarchical B-splines, features can be rotated to any orientation.

The pasting process is inherently an approximation technique. If both the feature and base surfaces are bi-cubic B-splines, when the feature is rotated there is no possibility of the cubic feature boundary exactly reproducing the sextic curve along the base surface. In the original surface pasting method, the boundary control points of the feature were placed on the base surface itself. If the C^0 discontinuity was too high, then the feature surface had to be refined and repasted. This process allows the feature boundary and the base surface to be separated by no more than a user specified tolerance. However, the total number of control points in the feature increases as the square of the number of boundary points, and the cost of pasting the feature can be quite high if an extremely small tolerance is desired.

2. Quasi-interpolation in Surface Pasting

Quasi-interpolation[6] is a method for approximating a curve with a spline curve to high precision. The quasi-interpolation method computes coefficients to weight samplings of the curve to be approximated.

† On sabbatical leave at The University of Amsterdam, Amsterdam, The Netherlands, May 1999—October 1999.

© Stephen Mann, Blair Conrad, 1999.
Published by the Eurographics Association, ISSN 1017-4656.

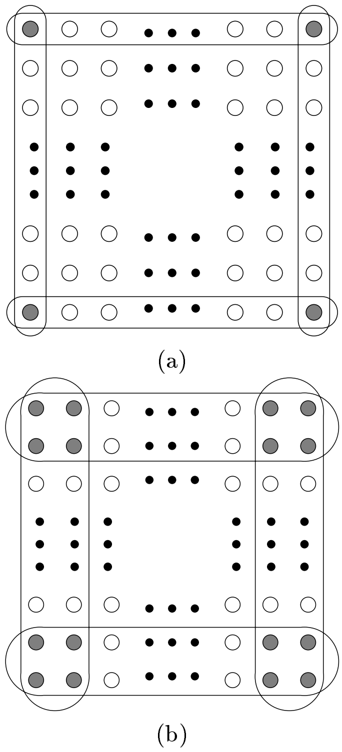

Figure 1: *Quasi-interpolation must reproduce the corner points.*

The Lyche-Schumaker quasi-interpolant uses coefficients that are inexpensive to compute and samplings that are relatively expensive to compute (divided differences of points of the curve to be approximated). For each control point of the quasi-interpolant, their method requires $n + 1$ samples of the curve to be approximated, where n is the degree of the quasi-interpolant.

For surface pasting, we use quasi-interpolation on each of the four boundaries of the feature surface.[4] Since our feature is a tensor product surface, some of the control points are set by multiple quasi-interpolants as shown in Figure 1(a). Further, if we want the feature to meet the base with approximate C^1 continuity, then we use quasi-interpolation on two layers of control points, and larger groups of control points are set by multiple quasi-interpolants as shown in Figure 1(b). To get consistent settings of these control points, we designed a quasi-interpolant that interpolates position and derivatives at the endpoints of the curve.

Further, as we move the feature, we need to recompute the samplings, but not the coefficients. Therefore, we use a variation of the quasi-interpolant whose coefficients are expensive to compute, but whose samplings are relatively inexpensive to compute (points and derivatives of the curve to be approximated). To reduce the cost further, our quasi-interpolant shares samples, reducing the sampling cost to roughly one evaluation of the base surface per control point on the feature boundary.

3. Results

Figures 2–4 show a feature pasted using standard pasting, quasi-interpolation just on the boundaries, and quasi-interpolation on both the boundaries and cross-boundaries. The feature surface has the same number of control points in all three figures.

If we perform knot insertion on the feature to reduce the error to below a common threshold for all three methods, we find that the new surface pasting method produces features whose boundaries approximate the base surface to the same tolerance as the original pasting method, but with one third the boundary control points. The resulting feature has one ninth the control points and can be constructed in approximately one tenth the time as the original pasting method.

4. Acknowledgments

Thanks to Tom Lyche for suggesting using quasi-interpolation for our boundary curves. And many thanks to Kirk Haller and Richard Bartels for their helpful dicsussions early in this work.

References

1. C. Barghiel, R. Bartels, and D. Forsey. Pasting spline surfaces. In L. Schumaker M Daehlen, T. Lyche, editor, *Mathematical Methods for Curves and Surfaces*, pages 31–40. Vanderbilt University Press, 1995.
2. R. Bartels and D. Forsey. Spline overlay surfaces. Technical Report CS-92-08, Univeristy of Waterloo, Waterloo, Ontario, Canada N2L 3G1, 1991.
3. L. K. Y. Chan, S. Mann, and R. Bartels. World space surface pasting. In *Proceedings of Graphics Interface*, pages 146–154, Kelowna, May 1997.
4. Blair Conrad. Better pasting through quasi-interpolation. Master's thesis, University of Waterloo, Waterloo, Ontario, Canada N2L 3G1, 1999. Available as Computer Science Department Technical Report CS-99-14, ftp://cs-archive.uwaterloo.ca/cs-archive/CS-99-14/.
5. D. Forsey and R. Bartels. Hierarchical B-spline refinement. *Computer Graphics (SIGGRAPH '88 Proceedings)*, 22(4):205–212, 1988.

© Stephen Mann, Blair Conrad, 1999.

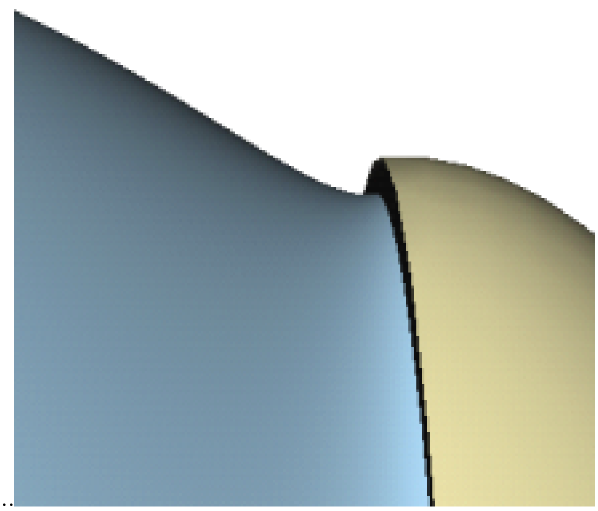

Figure 2: *Standard Pasting*

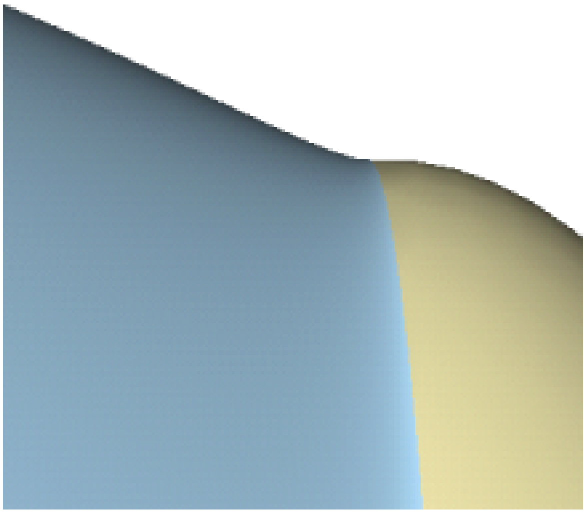

Figure 4: *Quasi-interpolation on the boundaries and cross-boundaries.*

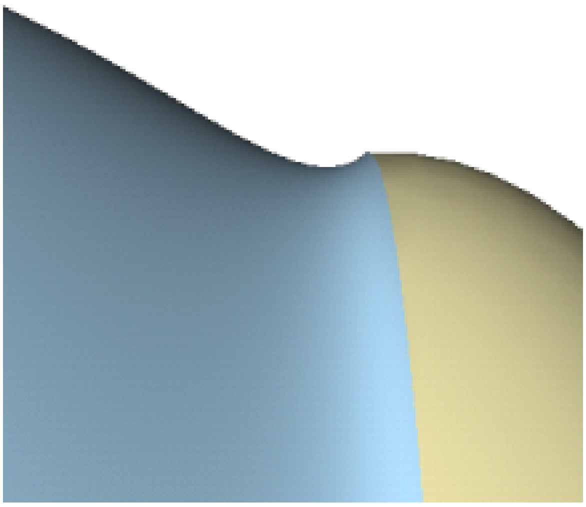

Figure 3: *Quasi-interpolation on the boundaries.*

6. T. Lyche and L. Schumaker. Local spline approximation methods. *Journal of Approximation Theory*, 15:294–325, 1975.

© Stephen Mann, Blair Conrad, 1999.