

# Modelling Cloth Buckling and Drape<sup>†</sup>

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## Abstract

*We present a new computational model for plain woven fabrics. The model is able to represent known elastic behaviour in deformation, such as planar extension and shearing and out-of-plane bending, drape and buckling. The buckling behaviour is present both in shear and compression. Visual results of these deformation conditions are shown. The cloth is assumed to be an orthotropic linear elastic continuum, discretized by a mesh of triangles. For the planar deformation, we assume the hypothesis of the plate under plane stress, of the classical theory of Elasticity and each triangle corresponds to a Strain-Rosette. For the out-of plane deformation, we allow linear elasticity and non-linear displacement in bending, as expressed by the Bernoulli-Euler equation. Dynamic equilibrium is formulated using Newton's 2nd law. We model non-linear elastic material behaviour, by piecewise linear approximation of measured data.*

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## 1. Modelling Fabric Deformation

In this work, we represent the free-form deformation of plain-woven cloth surfaces, by modelling separately the in-plane tensile behaviour (extension/compression and shearing) and the out-of-plane curvature deformation. According to the Principle of Superposition, which is valid for linear elasticity, we can sum up the displacements evaluated in each of the two models, to find the total displacement of the surface.

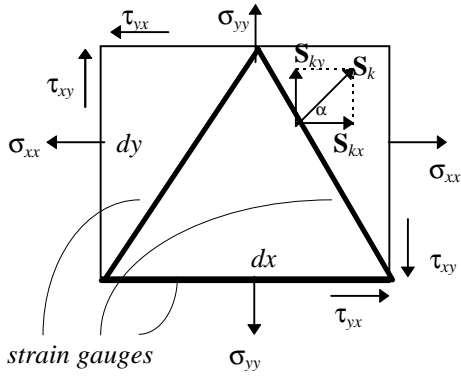
The surface of the cloth is modelled with a triangular mesh. Each irregular triangle of the discrete topology, corresponds physically to a set of tree linked strain gauges, referred as a *Strain Rosette*<sup>1</sup>,

(Fig. 1) each one making a different angle with the warp direction. Each strain gauge is able to evaluate (or, in physical terms, measure) the extensional deformation of the underlying continuum, along that specific angle. If we assume a condition of plane stress for each triangle, then the stress tensor takes the following expression:

$$\left\{ \begin{array}{l} \sigma_{zz} = \tau_{zx} = \tau_{xz} = \tau_{zy} = \tau_{yz} = 0 \\ \sigma_{xx} = \frac{E_x}{1 - \nu_{xy}\nu_{yx}} \epsilon_{xx} + \frac{E_x \nu_{yx}}{1 - \nu_{xy}\nu_{yx}} \epsilon_{yy} - E_x \epsilon_{0xx} \\ \sigma_{yy} = \frac{E_y \nu_{xy}}{1 - \nu_{xy}\nu_{yx}} \epsilon_{xx} + \frac{E_y}{1 - \nu_{xy}\nu_{yx}} \epsilon_{yy} - E_y \epsilon_{0yy} \end{array} \right. \quad (1)$$

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<sup>†</sup> This work was funded by the European Community ESPRIT Project Nr. 22038 "AITEAR- Advanced IT Infrastructure for Accurate Response in the Extended Enterprise" - Applied Research in the Textile-Garment-Retail Supply Chain



**Fig. 1:** Principal stresses on a  $dx dy$  element of a plate under plane stress, with 3 strain gauges attached and organised in a triangle.

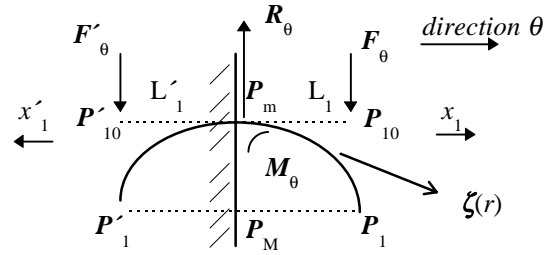
The previous expression is suitable for piecewise linear approximation of non-linear experimental stress-strain relation. Cloth, as an orthotropic material, has 4 independent constants:  $E_x$  and  $E_y$ , the Young modulus along the warp and weft directions, respectively;  $G_{xy}$ , the transversal elasticity modulus; and,  $\nu_{xy}$  and  $\nu_{yx}$ , the Poisson coefficients (the symmetry of the elastic tensor imposes that  $E_y \nu_{xy} = E_x \nu_{yx}$ ).  $E_x$ ,  $E_y$  and  $G_{xy}$  can be obtained from experimental data, developing linear interpolations of non-linear data<sup>2,3</sup>.  $\nu_{yx}$  can be obtained from the following expression<sup>2</sup>:

$$\frac{1}{G_{xy}} = \frac{4}{E_{45}} - \frac{1}{E_x} (1 - 2\nu_{yx}) - \frac{1}{E_y}. \quad (2)$$

We are interested in evaluating the combined stress  $S_k$ , applied in an edge  $k$  of each triangle (Fig. 1). This problem has a simple solution, known for long, from the macroscopic Mechanics of Materials<sup>1</sup>. From the three local strains ( $\epsilon_\alpha$ ,  $\epsilon_\beta$ ,  $\epsilon_\gamma$ ) evaluated by the strain gauges, we derive the principal strains ( $\epsilon_{xx}$ ,  $\epsilon_{yy}$ ,  $\gamma_{xy} = \gamma_{yx}$ ), by solving a linear system of three equations. The principal strains are then replaced in (1) to find the stresses acting along each of the triangles edges. The stresses, in turn, can be converted to forces simply by multiplication with the length of each edge and these are the forces that will drive the dynamic simulation of the planar elasticity.

The out-of-plane deformation model, aims to represent the bending deformation of cloth at large, that is to say, including single and double curvature that occurs in draping conditions, as well as the reversible elastic buckling phenomena that shows evidence in compression and shear.

Curvature can be handled, along many possible directions, by considering the angles between triangles that share a common edge. The basic deflection model behaves like a cantilever beam bending across a given edge of the mesh. This hypothesis is illustrated in Fig. 2, where we consider, as an example, deflection along a direction that makes an angle  $\theta$  with the warp. A cantilever beam is fixed in a point  $P_m$  belonging to the edge shared between the two triangles (Fig. 3) and is supporting a concentrated load  $F_\theta$  at the position of particle  $P_1$ . Another cantilever beam is defined in a similar way for the opposite vertex  $P'_1$ , supporting a concentrated load  $F'_\theta$ .



**Fig. 2:** Static equilibrium situation of two cantilever beams fixed in position  $P_m$  and supporting concentrated loads  $F_\theta$  and  $F'_\theta$  in their end points.  $P_1$  and  $P'_1$  are the deflected end positions of the beams.

The linear elasticity formulation for the stress-strain relation in this type of deformation, corresponds to the well known Bernoulli-Euler law<sup>1</sup>. Considering bending along cloth's principal directions, we have:  $\mathbf{M}_{x,y} = B_{x,y} k \mathbf{u}_{x,y}$ , where  $\mathbf{M}_{x,y}$  is the bending moment,  $B_{x,y}$ , the flexural rigidity and  $k$  the curvature.  $B_x$  and  $B_y$  can be evaluated theoretically, using the thin plate theory<sup>2</sup> of orthotropic materials or, obtained experimentally for the textile case, using Kawabata or FAST data. If we are looking at the curvature along an arbitrary direction, making an angle  $\theta$  with the warp direction, the following expression for the flexural rigidity<sup>2</sup>, should be used instead:

$$B_\theta = B_x \cos^4 \theta + \left( 2v_{xy} B_x + 4 \frac{G_{xy} h^3}{12} \right) \cos^2 \theta \sin^2 \theta + B_y \sin^4 \theta \quad (3)$$

The curvature at any point along the beam can be calculated straightforwardly if the deflection curve of the bent beam is represented by a parametric spline curve  $\zeta(r)$ . The bending moment  $M_\theta$  is maximum, where the curvature reaches also a maximum, which happens in point  $P_m$ . We can easily shown<sup>3</sup>, from the static equilibrium conditions in Fig. 2, that the bending force at the right extremity is:

$$F_\theta = \frac{M_{\theta \max}(\alpha_0)}{L_1} = \frac{B_\theta k_{\theta \max}(\alpha_0)}{L_1} \Big|_{\zeta(\alpha_0) = P_m} \quad (4).$$

A similar expression can be found for  $F'_\theta$ .

To resist such bending deformation, we apply two restoration forces, along the normal directions at each of the extremities of the beams. Each force is equal in modulus to the respective bending force. These pairs create bending moments that elastically resist the curvature deformation of cloth.

Two additional restoration forces are applied at the vertices of the edge along which the bending is taking place (Fig. 3). These two later forces are calculated in such a way as to cancel the total force acting on the two triangles, thus maintaining the linear momentum of the system.

This linear elasticity formulation, can also be generalised to a piecewise linear approximation of non-linear elastic stress-strain experimental relations<sup>3</sup>.

## 2. Results and Discussion

Our model was tested in several different application environments, ranging from standard Textile fabric deformation tests, used in fabrics quality control, up to deformable surface modelling in Computer Graphics, including the deployment of these type of soft objects in Virtual Environments, based in Open Inventor. In the simulations, we have used non-linear stress-strain relations for extensional deformations and linear stress-strain for shearing and bending, since the particular fabric<sup>‡</sup> used, presented almost a linear behaviour in these last two modes and a very low degree of hysteresis. However, as it was mentioned, our model can use non-linear

<sup>‡</sup> The fabric was kindly provided by Maconde, a large Portuguese garment manufacturer

stress-strain relation data in all deformation modes. All simulations were carried in a Silicon Graphics Indy computer, with a 200 MHz MIPS R4400 CPU.

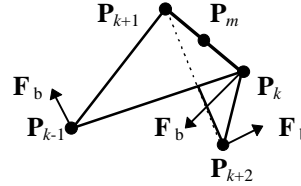


Fig. 3: Restoration forces, that resist bending, applied in vertices of two adjacent triangles.

The rendering tools used where developed by our group, and are based in Open Inventor.

To test the planar deformation model, we have simulated a real uniaxial tensile experiment, as a component of a computational framework that enables the simulation of several Instron, FAST and Kawabata Textile fabrics tests. The real test was performed in a national textile laboratory, CIVEC, using an Instron extensometer tester and following the ISO 5081Textile standard. The cloth sample had the following characteristics: rectangle of 0,05 x 0,10 m; composition: 100% wool; mass density: 0,137 kg/m<sup>2</sup>; tickness: 3,14 x 10<sup>-4</sup> m.

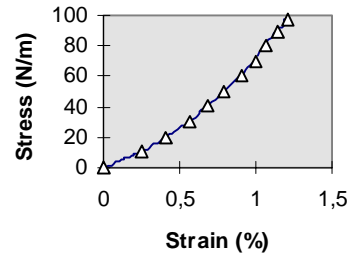


Fig. 4: Comparison between the experimental curve  $\sigma = f(\epsilon)$  obtained in the Instron Tensile Tester, warp direction (triangles) with the one simulated by our method (line), using a piecewise linear approximation .

As we can see from Fig. 4, we have achieved very good agreement between the simulated and the experimental stress-strain curve. Similar comparisons where made with tests carried along the weft and bias directions, for the same cloth material and provided the same good agreements<sup>3</sup>.

In Fig. 5, we visualise the test results of our in-plane *Strain Rosette* deformation model. The figure depicts two frames of a dynamic simulation sequence, showing the deformation of a rectangular piece of fabric, produced by the same Instron extensometer. The traction was exerted along the fabric bias direction. Frame (a) is the undeformed state and frame (b) is the deformed one, after 3 secs of simulated time. The ‘‘Poisson’’ effect is visible.



(a)



(b)

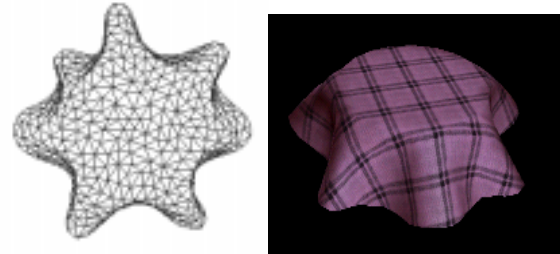
**Fig. 5.** Simulation of a tensile deformation experiment along the bias direction of a fabric, as produced by an Instron extensometer.

In Fig. 6 we have simulated a standard and simple test of cloth draping, traditionally used in the Apparel industry. A circular piece of fabric with 0.36 m of diameter was placed on top of a cylinder with 0.18 m of diameter. The free surface of the fabric could bend under the gravity field. Our main purpose with this test, was to provide a graceful image of cloth draping. In (a) we see a top view in wire-frame and in (b), a rendered view.

Our bending model incorporates the differential equation of the planar elastica, which is evaluated in the neighbourhood of every two adjacent triangles. This is the same formulation, as the one used in the Theory of Elastic Buckling or Euler Buckling<sup>1</sup>. Given this consideration, we have tested our model in a pure compression situation, where we have disabled gravity, making an analogy of the cloth surface with a column under a compressive load, with articulated end points. This test was implemented using a similar framework of the case in Fig. 5: a simulated extensometer compresses the cloth along one of its principal directions, at the same constant speed. During the first time step of the simulation, we have added a small random displacement (between 0 and 0,01 cm) to the surface particles, in the normal direction of surface. This small disturbance, has effectively activated the bending model in buckling. Fig. 7 shows the side

view of tree instances of the dynamic deformation, which took 8 secs of simulated time.

The fabric evolves through 3 buckling modes. The leftmost mode (rendered also in Fig. 8), with 3 lobes, was produced after 0,26 secs. The central



(a)

(b)

**Fig. 6:** Cloth draping. simulation.

mode has appeared after 1.86 secs and finally, the rightmost mode, is where the surface stabilises after 7 secs of simulated time. This result is in agreement with the Euler-Buckling theory.



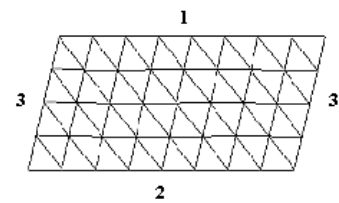
The buckling behaviour of fabrics in shear has been studied by Ly<sup>4</sup> from

**Fig. 7:** Three modes of Euler-Buckling as simulated by our method.



**Fig. 8:** Rendered view of the three lobe mode in Euler-Buckling.

the analytical point of view, using the theory of thin plates in bending, of the Mechanics of continuum. This theory, is a generalisation for 3D surfaces, of



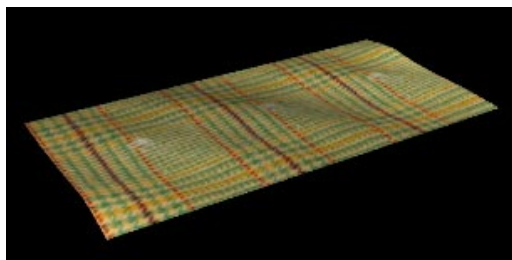
**Fig. 9:** Boundary conditions for fabric buckling in shear.

the theoretical framework of the planar cantilever beam used in this text.

However, the authors recognise, that the simulation of this type of deformation has not been yet addressed by the Computer Graphics community.

In Fig. 9 we show how we have set-up the boundary conditions of the simple shear deformation. In edge (2) the fabric was clamped. Edge (1) was subjected to a longitudinal motion at constant speed, in the direction of the warp, until the shear angle reaches  $13^\circ$ . This figure is a typical “large” value, observed in some situations of “daily” usage of fabrics. Edges (3) were free to deform. In the first iteration of the simulation, a small random displacement in the normal direction of the surface was added to the particles positions.

The obtained “buckled” pattern, can be visualised in Fig. 10. Patterns with the same geometrical configuration were analytically obtained by Ly, that has used in his theoretical framework, a Fourier Series approximation to fabric deflection.



**Fig. 10:** Rendered view of elastic buckling in shear.

Table 1 lists the achieved CPU times per 10 ms of simulated time in each of the mentioned experiments, which corresponds to the time that each frame simulates.

We can compare the results of the specific test illustrated in Fig. 6, with the ones reported for a different structural model for cloth, based on a particle based system<sup>5</sup> and running on a faster SGI machine (R8000 CPU). In that paper, to simulate a circular piece of fabric (with 841 triangles) draping over a round table, it took between 180 secs up to 480 secs per frame (each frame was simulating 40 ms).

We can average this numbers to a range of 45 secs up to 120 secs, simulating 10 ms per frame, as in our case. The authors claim<sup>5</sup> that the collision detection between the cloth and the table was only responsible

for 6% to 15% of the reported computation time. In our test we have performed a similar simulation, achieving CPU times per frame (in seconds), that reduces by one order of magnitude the reported

Test	Time (secs)	Nr. Triang	CPU/frame (secs)
Fig. 5	3	64	8.06
Fig. 6	4	796	9.36
Fig. 8	8	64	1.39
Fig. 10	2.2	256	9.58

**Table 1.** CPU times per 10 ms of simulated time.

computation times.

### Conclusions

We have presented a new computational framework for modelling cloth deformation, based in the macroscopic theory of Elasticity and in principles from the Mechanics of Materials. We have extended previous cloth models of in-plane tensile deformation<sup>6</sup>, with the same theoretical framework, by allowing orthotropic material behaviour and piecewise linear approximation of non-linear stress-strain experimental curves. Our bending model, follows previous strategies<sup>6</sup>, but enhances them with the support to model Euler buckling in compression and in shear. Future developments could be in the direction of improving the out-of-plane deformation model, including differential geometry surface concepts that would enable the representation of both surface bending and torsion, as well as in the area of new numerical algorithms to compute faster dynamic equilibrium solutions, using parallel machines and high performance computing.

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