Physically Based Modeling of Ice with Bubbles

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Abstract

Bubbles are an important characteristic that determine the appearance of ice. In this paper the authors propose a strategy in order to construct geometric representations of ice that integrates bubbles in an ice cube and visualize them. Bubble characteristics depend on the velocity of ice formation and levels of air concentration of water that in this paper are determined by simulating ice and bubble formation processes together. Simplified physics of heat transfer and a level set method are used in order to evolve the ice-water interface and a simplified model of bubbles as spheres is discussed. Experimental result shows that the shape of ice during formation resembles the one of actual ice. The algorithm has a potential to include more complex physics for better accuracy.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Physically based modeling

1. Introduction

One of the goals of Computer Graphics (CG) is to visually recreate elements in nature. Ice has proven to be a popular element in CG animated films because of its visual appeal. However, actual ice or even clay or plastic is still preferred in actual scenes because photorealistic CG images are difficult to achieve. Ice is a complex material that is currently been researched in different areas. In CG, ice is usually represented as totally white or totally transparent. However, bubbles (in this case, globules of air trapped in a solid) are an important characteristic of ice appearance since they determine its level of transparency and scattering of light. An ice cube that is produced by filling a tray with water is a good example of ice that is either completely transparent or completely white, and its final look depends on the distribution and size of bubbles. In this research the authors aim to create geometric models to represent ice with bubbles, which characteristics depend on the ice formation process (solidification) by a physically based approach. The authors focus in the specific case of home-made ice cubes, created by a cubic mold containing tap water (containing dissolved air), inside a freezer with constant temperature (Fig. 1). Based on simplified physics in ideal conditions, a strategy divided into five steps for simulating ice growth, determine bubble characteristics and to visualize the resulting model is presented.

A physically based method to accurately integrate bubbles into ice for CG has not been proposed before. Melting

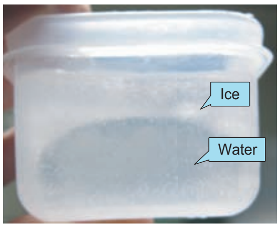


Figure 1: *Actual ice formation*

of ice considering natural convection has been studied by Matsumura *et al.* [MT05]. However, this method is not suitable to simulate ice formation, since the merging of surfaces is not considered. On the other hand, solidification has been seldom studied in the area of CG. As an exception, Kim *et al.* [KDAL06] proposed a method for simulating icicle formation. The equations for determining the velocity of ice growth for the cases of a cylinder and a parabola shape were solved by considering a thin film of water. However, bubbles are not considered at all. In the case of a growth by a thin film, bubbles have a high probability to escape, resulting in clear ice. In contrast, our method focus in the case of ice growth where there is a water supply that contains dissolved air with no chance to escape. Physically based animation of bubbles in a liquid were discussed by Hong *et. al.* [HK03]. In this research we are only dealing with fixed bubbles, so no animation for bubbles is considered.

2. Physics of Ice and Bubble Formation

Ice formation is seen as a moving interface (surface) between ice and water. When the interface moves, it can merge and change topology. Level set methods [OF03] can be used to naturally track this kind of interfaces that move and merge. The conditions that determine the velocity of the interface are referred in physics and mathematics as the Stefan problem [KDAL06] and there are different approaches for solving it. In a one-sided Stefan problem, only the heat field of one of the phases (for example in the liquid) is considered, and simplifies the calculations. Considering pressure to be constant, the Stefan problem states that the velocity of the interface is proportional to the heat that passes through it in its normal direction.

The physics of air bubble formation in ice were studied by Carte [Car61] and by Zhekamukhov [Zhe77]. When the ice forms it will either take a transparent look or it will be opaque, containing trapped bubbles and this depends on the growth rate of ice [Car61]. For a low rate, less and bigger bubbles are formed that can be easily visible, usually with an egg shape. For a high rate, more and smaller bubbles are formed that may have the shape of a cylinder, or a thread, that form in parallel direction of ice growth. The bubble formation also depends on the amount of dissolved air and other impurities (only dissolved air will be considered). In the case of an ice cube, as the interface of ice continues moving, air is dragged into the inside because ice can only dissolve about one third of gas than water. In a low rate formation, a big bubble may continue growing absorbing other micro bubbles until it gets trapped into the ice. This dragging makes the remaining water to become more saturated of air. When a supersaturation of water reaches around 10% to 40% the ice will grow opaque [Car61], in the case of ice cubes will form a cloud of micro bubbles. Other variables on which bubble formation depends are movement of the liquid, change on pressure, or even escape of the bubbles by buoyancy. However, they are not considered in this paper.

3. Algorithm

The five steps of the proposed strategy are:

1. Determining a heat transfer rate.
2. Determining the velocity of ice growth.
3. Moving the ice-water interface.
4. Determining bubble formation.
5. Visualizing the volume.

These steps repeat on every time step until all the water becomes ice. A fixed 3D Cartesian grid is used for most of calculations. As initial inputs, a grid size, an ambient temperature (below 273 K), a mold description for collision, and values for initial air concentration, and bubble formation threshold are required. In the first three steps the ice formation process is simulated by a level set method. In the fourth step, bubble formation characteristics are determined and its output is an implicit surface that represent both ice and bubbles. The last step, visualizes the geometry.

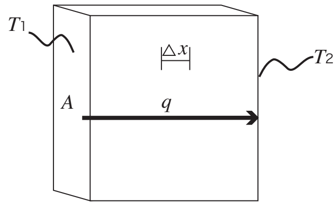


Figure 2: *One dimensional heat transfer*

3.1. Determining a heat transfer rate

Heat can be transported by conduction, convection and radiation. In this research, the authors propose a simplified model of heat transfer that only considers conduction. Analytical solutions exist for cylinders and spheres and usually numerical methods are used for more complicated shapes. However, in our simplified model an analytical solution considering one dimensional heat flow for a shape of a cube is considered.

A temperature gradient is calculated on each point $\mathbf{p}$ of a regular Cartesian grid that will be later used to calculate the velocity of the interface. The Fourier's law defines that the heat flow in a homogeneous material as

$$\mathbf{q} = -k\nabla T \tag{1}$$

where $\mathbf{q}$ is the heat flux vector (in [W/m^2]) and it is proportional to the constant thermal conductivity k (in [W/mK]) and the temperature gradient ∇T (in [K/m]) [Hol96].

For a simple linear approximation as in Fig. 2, it can be integrated to:

$$\frac{\partial \mathbf{q}}{\partial t} \approx kA\frac{(T_1 - T_2)}{\Delta x} \tag{2}$$

where $\frac{\partial \mathbf{q}}{\partial t}$ is the time rate of heat conduction (in [W]), A is the area of the object perpendicular to the heat conduction (in [m^2]), T_1 and T_2 are the temperatures on each side respectively, and Δx is the distance (in [m]).

In our method, ∇T is considered constant, and it is calculated on each point of a fixed grid by considering its distance to each wall where heat escapes. Then T_1 becomes the ambient temperature T_{env}, and T_2 a temperature on the interface which is considered to be constant equal to the freezing temperature $T_{\mathrm{m}} = 273$ K.

$$\frac{\partial T}{\partial x} = \left(\frac{T_{\mathrm{m}} - T_{\mathrm{env}}}{L_1}\right) - \left(\frac{T_{\mathrm{m}} - T_{\mathrm{env}}}{L_3}\right) \tag{3}$$

where L_1 and L_3 are the respective distances to each wall. In the case of y and z directions, the calculations are done in the same manner (Fig. 3).

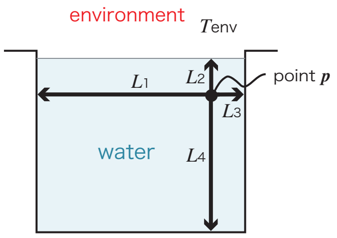


Figure 3: *Determination of local heat*

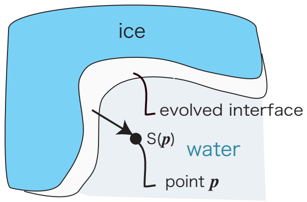


Figure 4: *Air saturation level displacement*

3.2. Determining the velocity of ice growth

The velocity is defined by the Stefan condition. At constant pressure is defined as:

$$\mathbf{v_n} = D\nabla T \cdot \mathbf{n} \tag{4}$$

where $\mathbf{v_n}$ is the local velocity of the interface in direction of its normal vector $\mathbf{n}$ (in [m/s]) and D is a thermal diffusion constant (in [m^2/s]) [OF03, KDAL06].

3.3. Moving the ice-water interface

Using the velocity calculated in the previous step, the interface is moved to its new location by using level set methods. Each time the interface is evolved, numerical dissipation occurs and different schemes have been proposed in recent years for reinitializing. A particle level set (PLS) method [EFFM02] is used for implementation. This method is suitable for CG applications because particles are efficient to correct the numeric dissipation that occur when evolving the level set, instead of higher order accuracy methods that need more computational power [MF06]. However, the other steps of the method should remain without changes if other level set approach is used for evolving this interface.

3.4. Determining bubble formation

For simplification, bubbles are defined to be spherical (considering a high surface tension) and with a center on a point of the fixed grid. However, more complex shapes can be achieved by joining the spheres together. The force that dissolved air produces on both ice and water is neglected since the weight of air is much lower. Only fixed bubbles in ice are considered (not the ones in water). An air concentration level in water $S(\mathbf{p})$ is defined on each point $\mathbf{p}$ of the Cartesian grid represented in saturation percentage. For example, boiled water has a less initial air concentration than water that was shaken. A bubble formation threshold value is also defined and determines the quantity of air concentration required for bubble creation.

a) Rise concentration level on points in water: As the interface moves, most of the dissolved air is pushed into the water, and creates an air supersaturated zone near the interface. We simulate this process by adding the air concentration of each point of the grid that became part of the ice and moving $2/3$ of it to its nearest point $\mathbf{p}$ (in the grid as well) that is on water (to simulate the fact that micro bubbles join together and also that ice can only dissolve about $1/3$ of the air than water), as shown in Fig. 4. $S(\mathbf{p})$ in points that become ice are cleared (become zero), since the concentration of air in ice is not taken into account.

b) Determine if a new bubble is created: Air concentration in every point in the grid is compared to the established bubble formation threshold value. If it is higher a bubble will form in this point. The velocity of the interface is not considered directly. However, the interface determines in which points air concentration raise more quickly.

c) Bubble Creation: If a bubble is created, its center position is assigned to the point $\mathbf{p}$ and its radius assigned proportionally to its air concentration level $S(\mathbf{p})$. The air concentration in this point is cleared and air concentration of some neighbors is moved to this point. This is done in order to simulate the fact that bubbles form aligned, some times forming egg shape bubbles or cylinders, and it is accomplished by combining two or more spherical bubbles.

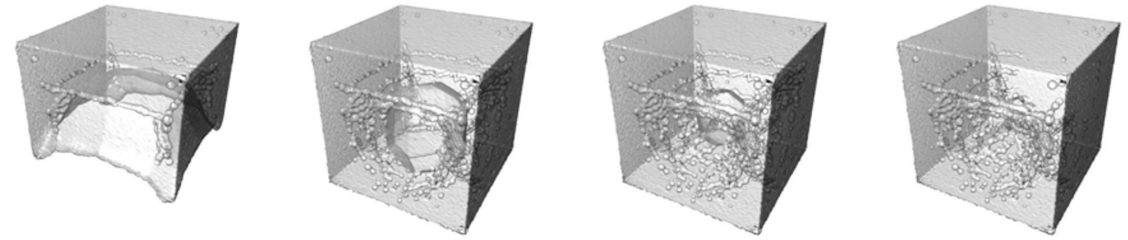
3.5. Visualizing the volume

A CSG difference operation is done in order to merge the implicit surfaces that represent ice and bubbles without modifying the originals. In order to visualize this volumetric data, it is first polygonized by the marching cubes [LC87] technique and then it is rendered using an ice material with ray-tracing.

4. Experiment and Discussion

The result of the experiment is as shown in Fig. 5. A $100 \times 100 \times 100$ grid is used to represent a 4.5^3 cm^3 ice cube. The rate for the calculation of the volumetric data is 40 seconds per frame. The rendering frame rate is around one minute per frame at a 640×480 resolution. The computational environment is an iMac G5, 1.8 GHz CPU, Mac OSX 10.4.10, and an NVIDIA GeForce FX5200 Ultra GPU.

In step 1, an initial supercooling condition is applied. In other words, water is considered to be cooled uniformly below its freezing point $T_m = 273$ K. The temperature at the interface remains constant at T_m. T_{env} was set to 258 K because it is a common temperature inside a freezer. The initial interface was set to the center of the upper wall in the shape of a half-sphere.



(a) frame 100 ($t = 12.5$ s) (b) frame 150 ($t = 18.75$ s) (c) frame 200 ($t = 25$ s) (d) frame 300 ($t = 37.5$ s)

Figure 5: *CG ice cube formation sequence with bubbles (t indicates physical time)*

In step 2, a time step of $\Delta t = 1/8$ s is used in order to achieve an animation 3 times faster than actual ice at 24 fps. The total physical simulation time is only 37.5 s because rapid crystallization takes place at supercooling.

In step 3, a PLS library by Mokbery *et al*. [MF06] was used as a base code to implement the evolution of the ice-water interface. Particles are seeded every frame. Collision of the interface with the mold is achieved by setting the velocity to zero if the interface is not inside the mold.

In step 4, The initial air concentration level $S(\mathbf{p})$ on each point was defined to be 15% saturation. Bubble formation threshold was set to 130% saturation.

In step 5, the volumetric data generated by the previous step is polygonized by the marching cubes algorithm that is written into a file in OBJ format. The OBJ file sequence is imported into Autodesk Maya by a MEL script and rendered by Maya Software.

In the simulation, ice formation resembles the shape of actual ice (Fig. 1), were first formed in the top, follows growing in the walls and encloses water. However, only ideal conditions were considered and the physical model should be extended to include other forces such as variable heat flows and convection in order to obtain better results. Ice growth velocity can become exponential, and can lead to a dissipation of the interface. In the current method, the number and size of bubbles is limited to the size of the grid. Although the current method can represent micro bubbles with finer grids, it will be more practical to use other representation that does not limit one bubble per grid point (micro bubbles look like a cloud from the distance). When polygonizing the volume by marching cubes, information of smaller bubbles is lost, and a direct visualization scheme will be more suitable. Furthermore, in order to make better comparisons with real ice, other ice shapes different than cubes need to be accomplished, as well rendering water and placing it on a virtual set that matches an actual illumination.

5. Conclusion

For CG, the addition of bubbles during formation will be necessary for a realistic ice modeling, something that has been difficult to achieve. A method for simulating ice and bubble formation was proposed and implemented. Ice is a complex material and further considerations have to be taken into account in order to match the actual ice and bubble formation processes. Comparison with real ice is needed to evaluate the proposed method. Future works include improving the physics for determining the temperatures that drives the velocity of the interface, that determines the bubble formation, and improving the algorithm for determining the bubble characteristics. Furthermore, information generated during the previous steps (such as crystal orientation and temperatures) will be considered for visualization.

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