Liquid Diffusion Model that Accounts for a Variety of Dyeing Parameters

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Abstract

This paper describes a method for simulating and visualizing dyeing based on weave patterns and the physical parameters of the threads and the dye. We apply Fick's second law with a variable diffusion coefficient, which is calculated using the porosity, tortuosity, and the dye concentration based on the physical chemistry of dyeing. The tortuosity of the channel was incorporated in order to consider the effect of weave patterns on diffusion. The total mass is conserved in this model. We describe the cloth model using a two-layered cellular model that includes the minimum factors required for representing the weft and warp. Our model also includes a simple dyeing technique that produces dyeing patterns by interrupting the diffusion of the dye in a cloth using a press. The results obtained using our model demonstrate that it is capable of modeling many of the characteristics of dyeing.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Picture/Image Generation I.3.5Physically based modeling

1. Introduction

Dyeing is a traditional way to impart color to a cloth. Visual simulation of dyeing is important for representing garments in computer graphics images and it is also important as a drawing simulation. Liquid penetration into a cloth is a complicated phenomenon from both a scientific and a physical perspective. This paper presents a system that employs physical parameters to simulate dyeing on cloth. The pattern and color of dyed cloth is a function of the physical properties of the dye and the fabric. It is a different phenomenon from liquid diffusion on paper. Some characteristic features of liquid diffusion on a cloth that are influenced by weave patterns, such as thin spots and mottles are shown in Fig. 1. In actual dyeing, a wet cloth is used to prevent color heterogeneity, so this paper presents a system that uses physical parameters to represent dyeing on a wet cloth.

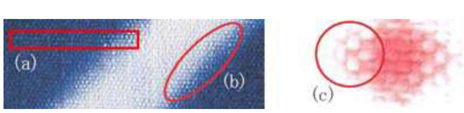

Figure 1: *Some characteristic features of dyeing: (a) thin spots, (b) bleeding ("nijimi" in Japanese), and (c) mottles.*

2. Related works

Methods for simulating painting implements and drawing strokes are being developed in the field of non-photorealistic rendering (NPR). Some studies have been done in this area into watercolor painting and Chinese ink painting, both of which involve diffusion of pigment in paper. Curtis et al. [CAS*97] presented a method for simulating watercolor effects such as dry-brushing, intentional backruns, and flow patterns. Chu and Tai [CT05] provided a novel method that simulates drawing of a fluid on absorbent paper by solving the lattice Boltzmann equation in real time. Kunii et al. [KNV01] used partial differential equations (PDE) with a variable diffusion coefficient to describe this phenomenon. Their PDE's for water spreading in dry paper and pigment movement within water are essentially Fick's law of diffusion. Using a variable coefficient is also important for describing the dyeing phenomenon [SH97]. These methods do not describe diffusion in compressed cloth and paper observed in dyeing techniques and they mainly focus on dry 2D paper. The 2D cloth model is inadequate for representing dye diffusion in woven cloths that have intentional 3D structures consisting of threads. Our model focused on dip dyeing. A variable diffusion coefficient is calculated on the basis of the theory of dyeing with our two-layered cloth model.

Several studies have investigated methods for simulating of painting techniques (including the batik technique) on cloth. Wyvill et al. [WvOC04] presented an algorithm for simulating the cracks that occur in batik. Their method is capable of producing convincing patterns that capture many of the characteristics of the crack patterns found in real batik cloth. In addition, Drago et al. [DC04] performed simulations of canvases used for easel paintings. Their study simulated real woven canvases that have weaving patterns and canvas aging. However, the texture of the cloth was not incorporated in Wyvill's algorithm and diffusion in cloth was not considered in either Drago's or Wyvill's research.

There have also been some studies on visualization methods of cloth. These studies attempted to represent interwoven threads and to model the interaction of light with the threads of the cloth in detail using the bidirectional reflectance distribution function (BRDF) or other representations. In this study, however, we do not use such a light reflection model; instead, we use the procedural thread texture model by Adabala et al. [AMT03] to represent the cloth.

3. Computer generated dyeing

3.1. Cloth model

We represent the cloth by cells (see Fig. 2). We define two kinds of cells, namely a cloth cell, and a diffusion cell. In cloth cells, parameters such as the weft or warp, the vertical position relative to the weave patterns, the size of threads and gaps, and other physical diffusion factors specific for each thread are defined. Each cloth cell is subdivided into several diffusion cells and these diffusion cells are then used to calculate the diffusion. Initially, cloth cells are defined by the size of the warps and wefts, which can be assigned arbitrarily. They are then arranged in the cloth at intervals equal to the spacing of gaps. Two layers are prepared as the weft and warp for cloth cells. Next, a defined weave pattern determines whether each cloth cell is orientated up or down. Diffusion cells are also arranged in the cloth size and two layers are formed. Their properties are defined in reference to the cloth cell in the layer that they are located in. Each diffusion cell can have only one of two possible orientations (parallel to the x and y-axes) as the fiber's orientation in its layer.

3.2. Dye diffusion model

In our method, Fick's second law [Fic55] is used to simulate dye diffusion in a wet cloth:

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial \phi}{\partial x}\right) + \frac{\partial}{\partial y}\left(D\frac{\partial \phi}{\partial y}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial \phi}{\partial z}\right) \quad (1)$$

where Φ is the diffusion density, t is the time, D is the diffusion coefficient, x is the displacement along the x-axis, and

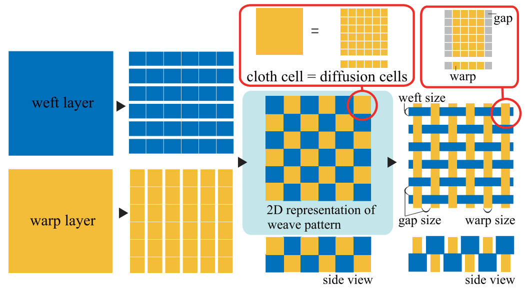

Figure 2: *The plain woven cloth with cloth cells.*

y is the displacement along the y-axis. Generally, a simplified diffusion equation is obtained when Eq. 1 is descretized since $\delta D/\delta x$ is 0. However, if D is variable, $\delta D/\delta x$ is nonzero. So we discretize Eq. 1 to obtain

$$\frac{\Delta\phi_{i,j}}{\Delta t} = \frac{1}{\Delta d}\{D_{i+1/2,j,e}\frac{\phi_{i+1,j,e}-\phi_{i,j,e}}{\Delta d} + D_{i-1/2,j,e}\frac{\phi_{i-1,j,e}-\phi_{i,j,e}}{\Delta d} + D_{i,j+1/2,e}\frac{\phi_{i,j+1,e}-\phi_{i,j,e}}{\Delta d} + D_{i,j-1/2,e}\frac{\phi_{i,j-1,e}-\phi_{i,j,e}}{\Delta d} + D_{i,j,(e+a)/2}\frac{\phi_{i,j,a}-\phi_{i,j,e}}{\Delta d}\} \quad (2)$$

where i and j are the x-axis and y-axis positions of the diffusion cells respectively, and e, a indicate which layer a diffusion cell belongs to, where e is a weft layer and a is a warp layer. Δd is the distance interval. D is defined as the diffusion coefficient between diffusion cells. The total mass is conserved theoretically. We define the diffusion cell size to be the same as the pixel size. Each diffusion cell has various parameters associated with it; these parameters include its weft layer or warp layer, fiber or gap (i.e., no fiber), up- or down-orientation, position, porosity, and tortuosity. Porosity is defined as the ratio of void volume in a thread fiber. Tortuosity denotes the degree of twist, such that the smaller the value of the tortuosity is, the larger the twist is. We define three kinds of tortuosities in our method: one is the twist of the thread (τ_1), another is the position of the thread, including its orientation in the weave pattern (τ_2), the other is from the different orientations of fibers in neighboring diffusion cells (τ_3). Each of these tortuosities has values in the range (0, 1). τ_1 is defined in each thread. τ_2 is determined by the connection between neighboring diffusion cells whether they are in the same layer, contain fibers, and whether their orientation is up or down. τ_3 has five different conditions (I: different layer, II: two fibers are in the same layer, and are connected to each other perpendicularly, III: fiber and gap, I V: gap and gap, V: fiber and fiber in the same layer, and are connected to each other in same direction as the fiber axis) which depend on properties such as the layer neighboring diffusion cells are in and their porosity (see Fig. 3). Finally, the tortuosity of a cell T between diffusion cells is defined as $T = \tau_1\tau_2\tau_3$. The depth of the cloth should be represented using two or more cloth cells to simulate diffusion in an accurate and physical manner. However, we used a two-layered cloth model since our goal is to represent the visual appearance of diffusion by taking the cloth structure into consideration. It is thus possible to use a different discretized distance for the z-axis from those for the x- and y-axes. However, we do not employ such a method since we can represent the visual features of diffusion using a two-layered structure of the cloth by conditioning $\tau_3 I$. The diffusion coefficient is calculated between diffusion cells using the following equation that is based on the Weisz-Zollinger model [SH97] in accordance with our definition for T;

$$D = D_0 PT(\phi_0/\phi) \tag{3}$$

where P is the porosity, which can have any value in the range (0, 1], ϕ_0 denotes the dye concentration in the external solution when equilibrium is achieved and is given arbitrarily. D_0 is the diffusion coefficient in free water and is calculated using the following equation [vd94]

$$D_0 = 3.6\sqrt{76/M} \tag{4}$$

where M denotes the molecular mass of dye. The volume capacity of the dye absorption in the diffusion cell V_d is determined by the proportion of fiber $(1-P)$ in the diffusion cell according to the following expression

$$V_d = (1-P)V_{max} \tag{5}$$

Another parameter B that denotes the volume ratio in the range (0, 1] is used to simulate dyeing patterns. It prevents the pressed area of the cloth being dyed and is used to represent the dyeing patterns of certain dyeing techniques. Curtis et al. [CAS*97] used a similar method in which the paper affects fluid flow (it also affects watercolor to some extent and by doing so generates patterns). However, its effect is not as marked as dyeing patterns. In our method, we use the amount of dye that is not absorbed by the fiber to calculate the total amount of dye in the diffusion cell. The total amount of dye in a diffusion cell V is defined by V_u and V_d, where V_u is the dye capacity minus the dye absorbed in the diffusion cell.

$$V_u = V_{max}P(1-B), \qquad V = V_u + V_d \tag{6}$$

In actual dyeing, the distribution of B is determined by the artist. We calculate the distribution of B based on the normalized average RGB values in an image.

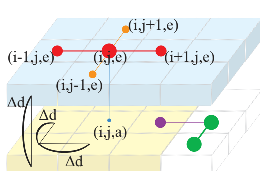

Figure 3: *Connection between diffusion cells. White cells are gaps in the warp layer. When the diffusion cell (i,j,e) is the target, τ_3 is defined by the following colored lines. Red : V. Orange : II. Blue : I. Green : IV. Purple is III in $\tau 3$.*

3.3. Visualization of the dye and cloth

Applying the algorithm described in Section 3.2 gives the dye dispersion. The algorithm can be used to determine the basic color of the diffusion pixels. We decide the basic colors by applying linear interpolation to the highest and lowest color values of a real image of a dyed cloth. In order to represent the dye, other color values can be added to determine the color from a real image of a dyed cloth. In this way, the pigment changes in dye stains are obtained. We use a method that determines the cloth's texture by mapping the texture using the yarn's shadow [AMT03]. The texture of yarn is produced by its tightness or roughness. Finally, the texture of the cloth is multiplied by the color of the dye. To model the fact that a high light reflectance renders the cloth's texture invisible, the brightness of the cloth color depends on the brightness of the color from the dye diffused at each diffusion cell.

4. Results

The results of our simulation are shown in Figs. 4, 5, 6, while Table 5 gives the parameters used. Figure 4d shows mottles similar to those in Fig. 1c. This shows that mottles are formed not just when the value of $\tau_3 I$ is small; they are also produced in the two-layered model. Figures 4f and g are the results for random P, while Fig. 4e shows the result for random τ_1. Some thin colored threads can clearly be seen in Fig. 4f. We find that P has more effect on the final appearance than τ_1. Figure 4b shows the result obtained when a high absorption coefficient is used; in this result, the initial dyeing area seems to contain a lot of dye. Figure 5 shows the effect of different kinds of weave patterns having identical parameters on dye diffusion. In this calculation a small value of $\tau_3 I$ was used, similar to Fig. 4d. There are obvious differences in the patterns obtained for different weave patterns that are caused not by differences in the rendering cloths, but also by the number of diffusion cells having τ_2. However, the effect of τ_2 on dye diffusion is not so large. Figure 6 shows a comparison of simulations for simple dyeing techniques and an image of a real dyed cloth. The image input for B can represent patterns by preventing diffusion that have never been considered in previous studies.

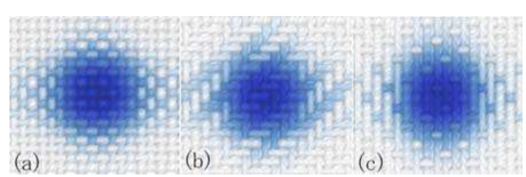

Figure 5: *The differences between dye stains on different weave patterns. (a) Plain weave, (b) diagonal weave, (c) satin weave. Computational time of (a) is 6 min 15 sec.*

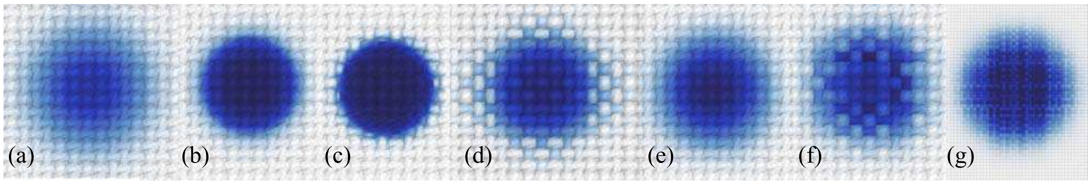

Figure 4: *Computer generated dye stains with various parameters including T, P, D_{ab}.*

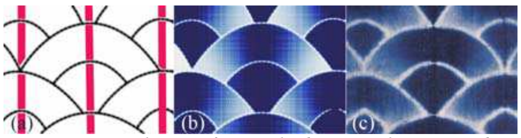

Figure 6: *Simulation of a simple dyeing technique. Red region in (a) is distribution of the dye, while the black region is the pressure distribution. (b) is the result with (a), (c) is a real dyed cloth showing the seikaiha pattern.*

5. Conclusion and future work

We have presented an improved diffusion equation that can take into account different weaving patterns, uneven diffusion coefficients, and different pressures for a tie-dyeing simulation. We used a physical theoretical model to calculate the diffusion coefficients, the diffusion coefficients in the water and the absorption rate. Additionally, we considered the porosity, tortuosity, capacity of the yarns to simulate dyeing in a cloth that can account for many of the features of dyeing.

Our future goals are mixing colors, an interactive simulation of dyeing a 3D cloth topology, modeling fluff on the cloth, including other dyeing techniques, and improving the interface of these systems. One of our goals in simulating dyeing on a dry cloth is to develop a diffusion system that can take capillary flow into account. The rendering of cloth could be enhanced by modeling the interaction of light with the threads and the 3D cloth topology.

Common factors	
size of threads, gaps	8, 2 (pixels) (2, 1 in 4g)
$D_0, \Delta t$	1.93 (mm^2/h), 0.0005 (h)
M	262.26288 (g/mol)
$\Delta d, \tau_2, IV$	0.05 (mm), 0.47, 1 (ratio)
initial dye destribution	circle with 30 (pixels) "(1
initial amount of dye	1.0V "(2
total number of time step	5000 "(3
the number of diffusion cells	200*200*2 (800*600*2 in 6b)

Others	I	II	III	V	τ_1	P	D_{ab}
4a	1	1	1	1	1	.5	0
4b	1	1	1	1	1	.5	$.01D_0$
4c	0	.05	1	0	1	.5	0
4d	.005	.005	.005	1	1	.5	0
4e	1	1	0	1	$.5r$	.5	0
4fg, 6b	1	1	0	1	1	$.5r$	0
5abc	.01	.01	.01	1	1	.5	0

Table 1: Parameters from Figs. 4 to 6. r is a random value in (.5, 1) for each yarn. "(1: In Fig. 6, the initial amount of dye is kept constant during the simulation. "(2: First, we set the dye capacity (1.0V) in cells inside the area of a circle with a radius of 30 pixels. "(3: This is true for all the result images shown in this paper, with the exception of Fig. 6.

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