Shape Analysis for Augmented Topological Shape Descriptor

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Abstract

In this work we propose a scheme for analysis of the shape of a 3D model. We use Extended Reeb Graph to describe the topological structure of the model which we further enrich with extracted geometrical features. The nodes of the graph represent different components of the model, and we propose to analyze their shape separately. Shape characteristics such as taper/enlargement, average curvature, bending are revealed through tracing changes in cross sections of each shape component. Finally the topological structure together with the associated geometrical characteristics represents the shape descriptor of the 3D model.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computational Geometry and Object Modeling]: Curve, surface, solid, and object representations. Shape analysis, shape descriptor

1. Introduction and Related Work

Representation and recognition of a 3D shape are two challenging research topics in computer graphics and vision. A shape can be roughly represented by its topological structure, and shape details can be characterized by local geometrical features. Depending on a requested accuracy of a retrieval system topological and/or geometrical shape descriptors can be used for representation of a 3D model.

In this paper we propose a scheme for analysis of geometrical features of the shape of a 3D model which can be further integrated into topological graph. We use the Extended Reeb Graph (ERG) proposed in [Bia04], [ABS03] for rough shape description and we will augment it with local geometrical characteristics.

The authors of [Bli87] and [SKK91] were the first who proposed to use a topological graph for shape representation. This approach was extended in [HSKK01], [ABS03], [BM06], [TS04], [BRS03] and [BKZK05]. The authors of the latter works calculate several geometrical characteristics of parts of a 3D model represented by the nodes of the graph.

A Reeb graph is defined as the quotient space of the manifold representing a model. If we define a Morse measuring function on a model then topological changes of the manifold correspond to critical points of the function. The nodes of the Reeb graph code the evolution of level sets, whereas the nodes of Extended Reeb graph encode the areas of the manifold where the topological changes occur [Bia04]. Consequently, at the preliminary step of construction of the ERG we decompose a shape into segments corresponding to minimal, maximal and saddle values of the measuring function. Each of these critical regions is represented by a node in the graph. Further the connectivity between the regions is expressed through the edges of the ERG.

In [PSF04] and [MGS*04] the authors decompose a shape into segments and classify them as the components with one, two and more boundaries. In this work we will use the names cone-like, cylinder-like and branching segments for the components having correspondingly one, two and more boundaries. Our segmentation approach [SDA07] is similar to the one proposed in [PSF04] with the extension that it maximizes cone- and cylinder-like parts by minimizing branching ones. Other segmentation techniques were proposed in [KT03] and [AFS06]. However, the target of shape segmentation in these works is different from decomposition of a model into critical regions. Precisely, segmentation in [KT03] subdivides a model into meaningful parts, and in [AFS06] the authors decompose a model into parts which can be approximated by different primitives.

In [MGS*04] several geometrical shape characteristics are proposed for the cone and cylinder-like segments while providing just a few for the branching components. In

[BKZK05] the authors assign the weight to each node of the topological graph which encodes the shape of level curves defined on a topologically homogeneous part of a 3D model. However, the aim of such encoding is shape reconstruction and not shape retrieval. In this work we propose to use several non-dimensional measures to describe the overall geometry of a segment, and also to perform detailed shape analysis for the cone- and cylinder like parts. These shape characteristics together with the topological Reeb graph can be further used as the shape descriptor in the retrieval process.

2. Reeb Graph for Shape Description

In this section we first give a brief overview of the segmentation process used for the construction of ERG. Further, we give the list of non-dimensional shape characteristics which can be used for all three types of segments, and provide the description of more detailed analysis for the components with one and two boundaries.

2.1. Shape Segmentation

As was mentioned in the previous section the construction of the ERG includes shape segmentation on its preliminary step. In [SDA07] we described in details the algorithm for shape segmentation which aims at maximizing cylinder- and cone-like parts and minimizing branching areas. The shape segmentation is performed through iterative bisection of the surface of a 3D model and is represented in the tree structure "left child - right sibling". Figure 1 illustrates this process. In the beginning the whole model represent the root of the tree. Then we define the initial partition of a model M into K intervals

$$M = (M_{min}, M_1)\bigcup(M_1, M_2)\bigcup\ldots\bigcup(M_{K-1}, M_{max})$$

by inserting $K-1$ contours. The difference of the value of the mapping function f on the boundaries of each interval in this case is $\triangle f = \frac{f_{max}-f_{min}}{K}$. We decompose the model by the inserted contours into connected components and we store each component as a child node of the root. Then, the process of the segmentation continues automatically. We analyze all components stored as child nodes. If a component has the non-zero genus or the number of the boundary components is more than two then the component is bisected, decomposed, and new components will be stored as its child nodes. The iteration terminates when the difference of the mapping function on the boundaries of a branching component $\triangle f$ reaches a predefined threshold. In this way the segmentation increases the number of components but at the same time reduces the size of branching areas. Figure 1 shows that the segmentation can generate several adjacent components with less then three boundaries. These components can be merged on the post-processing step. As result, the model is subdivided into large cone- and cylinder-like and small branching components. In [SDA07] we used height, distance from the barycenter and integral geodesic distance functions for the segmentation.The results of our

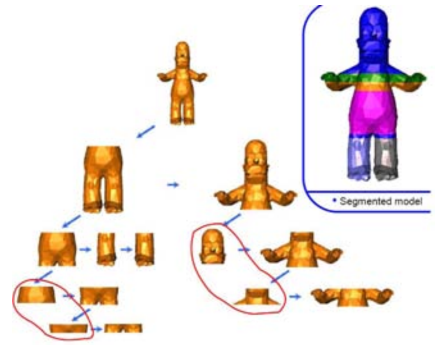

Figure 1: *Iterative segmentation and postprocessing merging. The model is taken from AIM@SHAPE Shape repository [Aim].*

experiments showed that the height function gives better results for CAD models, where under better results we mean larger simple and smaller complex areas. However such segmentation is not rotation invariant. Integral geodesic distance function requires more computational time, but it is invariant to the position in space and quite robust to different postures of a model.

2.2. Extended Reeb Graph

After segmentation we construct ERG as proposed in [ABS03] and [Bia04]. The nodes of the graph represent the volumetrical segments of the model. The segmentation described in the previous section guarantees that all topological characteristics of a shape are revealed in the ERG, unless they are concentrated in relatively small area with respect to the overall model size (less then the threshold).

If we analyze geometric properties of the shape of each segment and store this information together with the topological structure, then the composition of all nodes and corresponding shape characteristics give the overall shape description.

2.3. Geometric Shape Characteristics

In this section we provide the methodology for the analysis of the shape of each component represented by a node in the graph.

Each component of a model obtained after segmentation can be characterized by the following values:

- the number of boundary components N_b;
- the nondimensional measure of segment's weight in the whole model $SW = \frac{V_C}{V_M}$ defined by the relation of the volume of the the component V_C and the volume of the whole model V_M. This value can be used in graph matching to estimate the influence of a node on the whole graph structure, i.e. bigger segments have greater impact;
- convexity measure $Convexity = \frac{A_C}{A_{CH}}$ which is defined as

the ration of the surface area of the component A_C to the surface area of its convex hull A_{CH} [CRC*02];

- hull packing $H_p = 1 - \frac{V_C}{V_{CH}}$ defined as the convex hull volume V_{VH} not occupied by the volume of the component V_C [CRC*02];
- compactness measure $Comp = \frac{{A_C}^3}{{V_C}^2}$ defined as the ratio of the surface area cubed over the volume of the component squared.

For cone- and cylinder-like components we propose to perform more detailed shape analysis. The main idea of the proposed methodology is that the components with one and two boundaries have two reference points which define the main axis of the component. For the components with one boundary, the first reference point is the tip of the component, which is the point with the maximum/minimum value of the measuring function. The second reference point is the barycenter of the boundary component. For the components with two boundaries, both reference points are the barycenters of the boundaries. The two reference points define the line which we propose to use as the main axis for further shape analysis. We align the component so that its main axis coincides with the axis X. Further, intersecting the component with equally spaced planes perpendicular to the main axis we obtain the set of contours. The analysis of the evolution of these contours gives us advanced information about the shape of the component. We provide three tables for shape analysis for components both with one and two boundaries. The tables contain the name of a component and the image of its standard shape with corresponding characteristics.

Table 1 gives the description of component's taper/enlargement through tracing the changes of the area of the cross sections. If the area of the cross sections is constant to the extent of a predefined threshold then the component belongs to the first type of the proposed classification. If the cross area changes (increases or decreases) we analyze the average speed of the cross area alteration. The value of the difference in the area of two neighbor cross sections $\frac{A_{i+1}}{A_i}$ depends on the distance at which they are placed. Suppose we locate $N-1$ cross sections at equal distances along the main axis of a model component M. The inserted contours define N equal intervals

$$(M_{min}, M_1) \bigcup (M_1, M_2) \bigcup \ldots \bigcup (M_{N-1}, M_{max}).$$

We calculate the average cross area alteration and we define that the alteration is smooth if the apex angle of the induced cone or pyramid is less than 90°, which corresponds to the change in the area of two neighbor cross sections less than $\frac{(N-1)^2}{N^2}$. We define the average cross area alteration to be sharp in the opposite case. If the alteration of cross areas changes its behavior, for example increasing and then decreasing, then we define the component as having multiple area alterations.

Table 2 gives the description of component bending, specifying if it is straight or curved. In order to classify a component according to its bending we keep track of the position of the main axis with respect to each cross section. If the axis passes through all sections then the component is straight. If the axis is located outside several consequent sections then the component has single bending. In the case when the axis is outside two or more sections while passing through the sections located in between, the component has multiple bending.

Table 3 classifies components with respect to the average curvature of each cross section. We propose to distinguish

Table 1: *Analysis of a component's taper/enlargement*

	Constant cross area	Cross sections have approximately equal areas
	Smooth increasing\ decreasing cross areas	The area of cross sections changes smoothly increasing or decreasing
	Sharp Increasing\ Decreasing cross area	The area of cross sections changes sharply increasing or decreasing
	Multiple cross area alterations	The area of cross sections changes increasing and decreasing, characterized by the number of alterations

Table 2: *Analysis of a component bending*

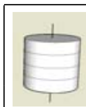	No bending	The main axis of the component passes through all cross sections
	Single bending	The medial axis passes through the cross sections which are close to the reference points
	Multiple bending	The main axis passes through the cross sections occasionally. Characterized by the number when the main axis is outside the cross section

Table 3: *Analysis of a curvature alteration*

	Constant curvature	The average curvature is preserved throughout all sections. The single curvature value characterizes the whole component
	Curvature alteration	The average curvature changes along the component. The component is characterized by the values of curvature in the vicinity of reference points

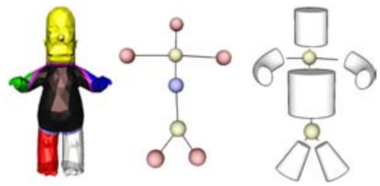

Figure 2: *Rough approximation of a model as the result of shape classification.*

only tree kinds of cross sections, namely sections having the average curvature $60^\circ, 90^\circ$ and more degrees. If the curvature of the cross sections is preserved along the main axis, then the component has the constant curvature, otherwise it has curvature alteration.

Shape classification given in Tables 1, 2 and 3 is not exhaustive and can be extended in future. The intersection of the three defined categories produces 24 kinds of shapes which can be used for rough approximation of each segment of a model. Figure 2 demonstrates the example of such approximation.

The final topological graph augmented with the geometrical characteristics can be stored in the following structure $(V,E) = (\{(N_{bi}, SW_i, H_{cp_i}, H_{p_i}, H_{ci}, Class1_i, Class2_i, Class3_i) | v_i \in V\}, \{(v_i, v_j) | (v_i, v_j) \in E\})$.

3. Conclusions and Future Work

In this paper we gave the brief description of the shape segmentation process which minimizes branching areas of a 3D model by maximizing cone- and cylinder-like parts. We proposed deeper shape analysis for the latter two types of segments. The results of the shape analysis can be stored as weights of the nodes of the Extended Reeb Graph. In this way we augment the topological graph with the geometrical shape characteristics.

In future we will examine the proposed methodology for shape analysis and elaborate a graph matching algorithm for the augmented topological graphs. We will also experiment the proposed methodology for shape segmentation using the Morse function proposed in [DBG*06], [NGH04].

4. Acknowledgements

This work has been supported by the CFP6 IST NoE 506766 AIM@SHAPE. We would like to thank the reviewers for the valuable comments and suggestions.

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