From spectra to perceptual color: Visualization tools for the dimensional reduction achieved by the human color sense

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Abstract

Physical colors, defined as unique combinations of photon populations whose wavelengths lie in the visible range, occupy an infinite-dimensional real Hilbert space. Any number of photon populations from the continuous spectrum of monochromatic wavelengths may be present to any positive amount. For normal vision, this space collapses to three dimensions at the retina, with any physical color eliciting just three sensor values corresponding to the excitations of the three classes of cone photoreceptor cells. While there are many mappings and visualizations of three-dimensional perceptual color space, attempts to visualize the relationship between infinite-dimensional physical color space and perceptual space are lacking. We present a visualization framework to illustrate this mathematical relation, using animation and transparency to map multiple physical colors to locations in perceptual space, revealing how the perceptual color solid can be understood as intersecting surfaces and volumes. This framework provides a clear and intuitive illustration of color metamerism.

1. Introduction

Physical colors occupy an infinite-dimensional real Hilbert space, H_{color}, and may be represented by countable infinite column vectors $\boldsymbol{s}$. At the retina, this space collapses to three dimensions of sensor values, that is, the photon catches and resultant excitations of the three classes of photosensitive cones [Koe10]. The cone spectral responses, L, M and S, of cones with response function row vectors $\boldsymbol{r}$ are then given by:

$$\begin{bmatrix} L \\ M \\ S \end{bmatrix} = \begin{bmatrix} \boldsymbol{r_L} \\ \boldsymbol{r_M} \\ \boldsymbol{r_S} \end{bmatrix} \boldsymbol{s}$$

While the three-dimensionality of perceptual color space is a familiar and intuitive concept, the same cannot be said for H_{color}. We are accustomed to conceptualizing spaces of three dimensions or fewer, living in a spatially three-dimensional world. Perceptual color space is, by nature, relatable to the perceptual experience of color, which may be used to navigate the space such as when selecting colors with a hue-saturation-value (HSV) or red-green-blue (RGB) color-picker. Any projection of physical color space must violate our intuitions of either space or color: namely that objects should not occupy the same space, or that colors should be separated according to their perceptual dissimilarity. The visualization framework proposed here maintains the distance relations of color dissimilarity, using transparency and animation to illustrate the intersecting surfaces and volumes of physical colors projected throughout the perceptual color solid.

2. Two-wavelength color set surfaces

We begin by considering physical color sets comprising just two wavelength populations, w_1 and w_2. Each color set, C, is defined by the intensities of each population, $C\{w_1, w_2\}$, giving rise to a surface extended between the monochromatic axes $C\{w_1\}$ and $C\{w_2\}$. Figure 1 shows two color sets, $C\{610, 540\}$ and $C\{460, 540\}$.

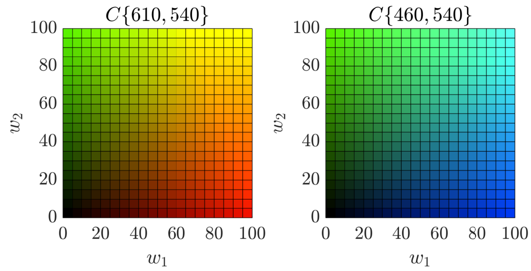

Figure 1: *Projection of the $C\{610, 540\}$ and $C\{460, 540\}$ two-wavelength physical color sets onto the $[w_1, w_2]$ plane.*

Projection to a fully three-dimensional color space will result in continuous $C\{w_1, w_2\}$ surfaces. These spaces may be geared towards intuition (HSV), practical considerations (RGB), or representing perceptual space (La*b*). Figure 2 shows several $C\{w_1, w_2\}$ color sets projected to a HSV space, using the spherical coordinate system (r,θ,ϕ) such that r=value, θ=hue, and ϕ=saturation. The summation of primaries rather than proportional mixing makes surfaces and primaries more readily identifiable.

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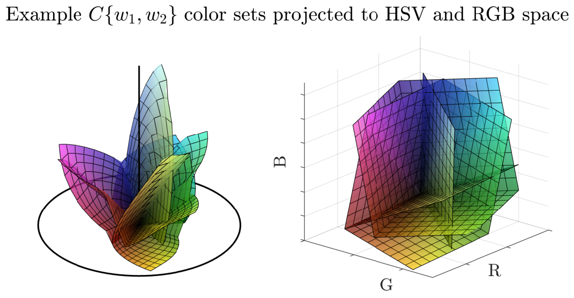

Figure 2: *Left: $C\{w_1, w_2\}$ color sets projected to HSV space. Black circle = plane of saturated hues; central vertical line = gray axis. Right: the color sets projected to RGB space. Sets were selected to span the spaces. In both projections, metamers are clearly identified by the intersections of two or more surfaces.*

3. Polychromatic primaries

As well as the simple case where both w_1 and w_2 are monochromatic, color sets may be generated by combining polychromatic spectra. Figure 3 illustrates the color set generated by combining the D65 standard daylight spectrum (**D65**) with a monochromatic 610 nm light.

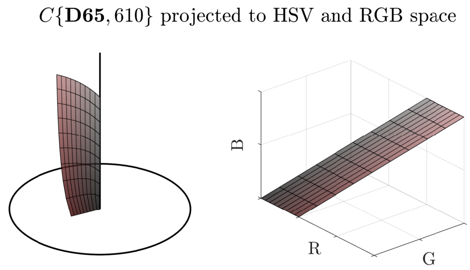

Figure 3: *The surface generated by the color set $C\{\boldsymbol{D65}, 610\}$.*

4. Visualizing anomalous trichromacy

Anomalous trichromacy is a type of color vision deficiency affecting 6.3% of the male population; color perception in such individuals is still three-dimensional, but with an altered spectral sensitivity of the L or M cones, giving a reduced wavelength separation between the L and M sensitivities. While this results in impaired discrimination overall, some physical colors that are metameric to normal observers may be discriminable to the anomalous trichromat. Color vision deficiencies have been simulated [BVM97], but visualization tools for the relationship between physical color and an individual's sensor space are lacking. Figure 4 shows the projection of two $C\{w_1, w_2\}$ color sets to normal- and anomalous trichromacy-defined RGB spaces. Matrices show inter-surface inter-point distances (darker = shorter distance), with distances below a threshold colored red, corresponding to metamers (i.e. difference signals overwhelmed by neural noise). While there are more metamers in the anomalous space (2.09%, vs 1.50% for the normal space), some colors that are metameric in the normal space are now discriminable. This is further illustrated by a difference matrix.

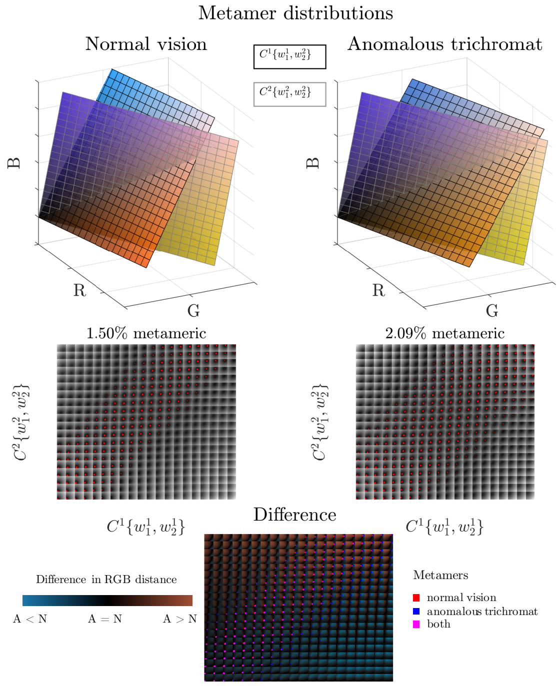

Figure 4: *Two $C\{w_1, w_2\}$ color sets projected to RGB spaces defined by normal vision and anomalous trichromacy. While there is a greater number of metamers in the anomalous space (colored red in the matrices), some colors that are metameric in the normal space have inter-point distances above threshold.*

5. Three-wavelength color set volumes

Just as we may project $C\{w_1, w_2\}$ physical colors to surfaces, so too can we project three-primary $C\{w_1, w_2, w_3\}$ colors to volumes. The resultant volume would be an opaque solid, but instead may be regularly sampled for combinations of primaries, as in Figure 5. The addition of a fourth primary may be visualized with animation.

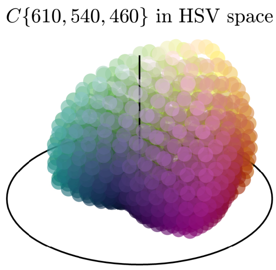

Figure 5: *The three-primary physical color set $C\{610, 540, 460\}$.*

References

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