Mesh Simplification With Curvature Error Metric

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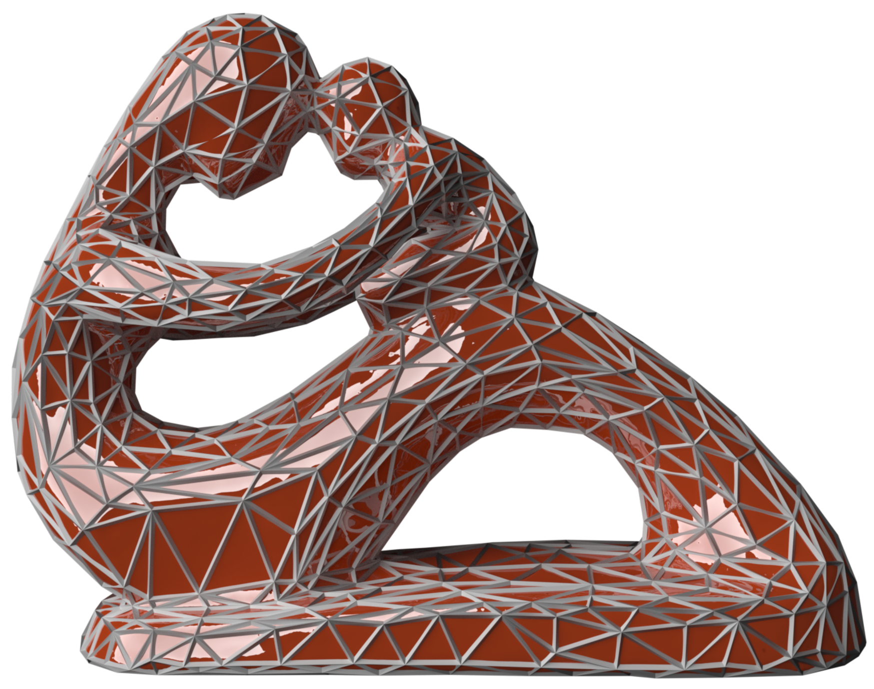
Overview

We use a reconstructed surface based on algebraic spheres to control the mesh decimation



Mesh simplification from the reconstructed surface (red) based on the interpolation of algebraic spheres

Context

The use of proxies as a high level control of the surface has been studied for mesh simplification with planes [3] and spheres [4]. We propose as a proxy the reconstructed surface of the input mesh based on algebraic spheres, which handles curvatures as well as sharp features.

Proposition

Mesh simplification algorithm

Input: High resolution mesh
Output: Coarser mesh

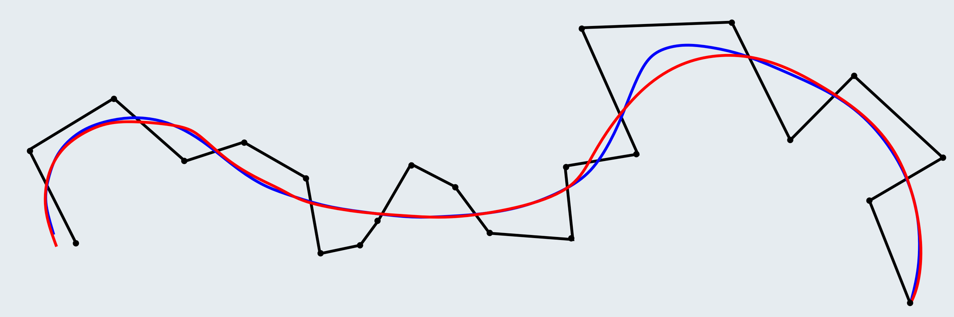
begin
  for *each vertex* x_i **do**
    p_{x_i} = **curvature estimation** of x_i;
  end
  for *each pair* (x_i, x_j) **do**
    $(c_{(x_i,x_j)}, x_{\alpha_i})$ = **curvature error metric** at (x_i, x_j);
    push (x_i, x_j, x_{α_i}) in a heap keyed on cost $c_{(x_i,x_j)}$;
  end
  while *heap not empty* **do**
    collapse (x_i, x_j) on x_{α_i};
    $p_{x_{\alpha_i}} = p_{x_i} + \alpha_i(p_{x_j} - p_{x_i})$;
    update cost of x_{α_i} neighbors in the heap ;
  end
end

Curvature estimation

We compute the algebraic sphere S_i for each vertex $\mathbf{x}_i$ of the mesh.

Computing the curvature at any point consists in interpolating the spheres along the edges and faces, e.g. for the edge $(\mathbf{x}_1, \mathbf{x}_2)$, we have:

$$S_\alpha = S_1 + \alpha(S_2 - S_1) \quad (1)$$

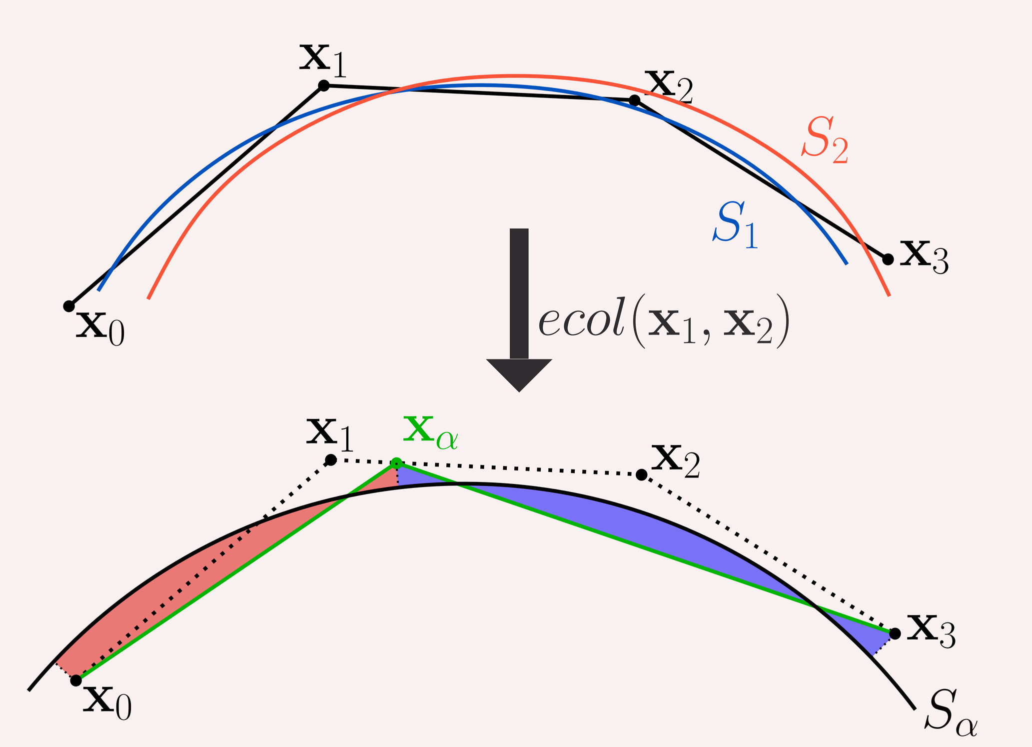


Reconstructed APSS curve (blue) from the polyline (black). Interpolated spheres (red).

Curvature error metric

The contraction cost $c_{(x_1,x_2)}$ is defined as the volume between the sphere representing the curvature and the mesh.

Thanks to the algebraic formulation, the distance from a point $\mathbf{x}$ to the sphere is obtained by computing the field value $S(\mathbf{x})$.



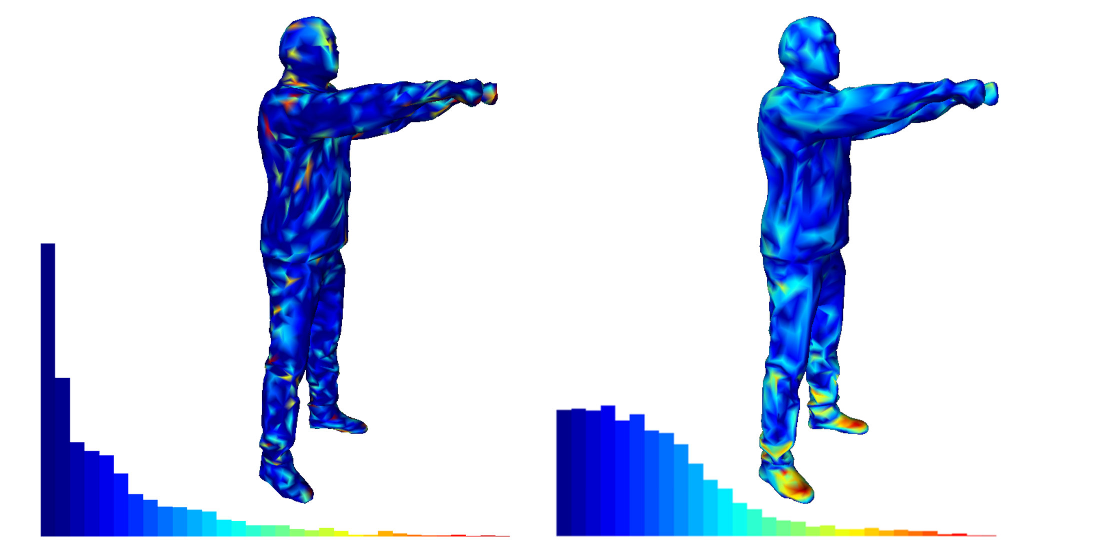
Edge contraction of $(\mathbf{x}_1, \mathbf{x}_2)$ on $\mathbf{x}_\alpha$. The energy to minimize is equal to the filled area.

The resulting position $\mathbf{x}_\alpha$ is obtained by minimizing the following energy:

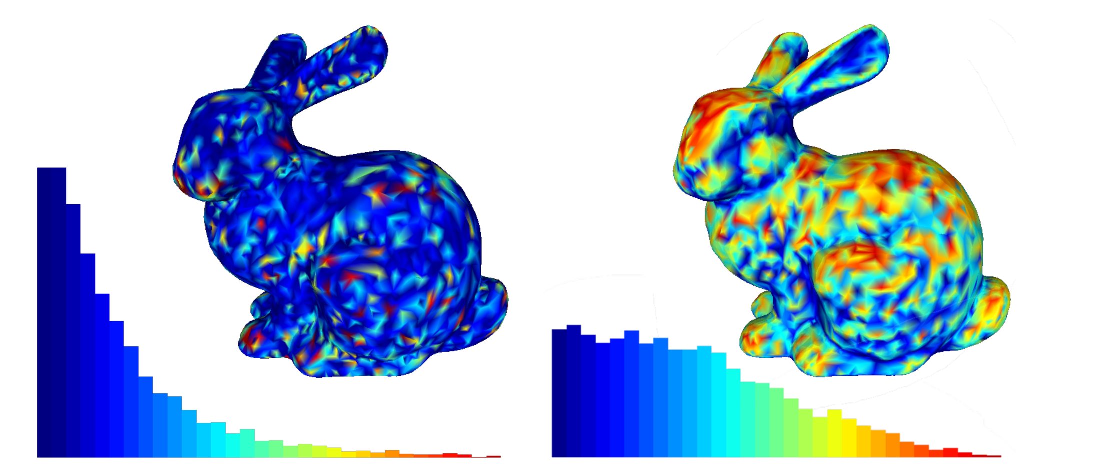
$$\underset{\alpha}{\operatorname{argmin}}\Big(\int_0^1 S_\alpha(\mathbf{x}_0 + \alpha(\mathbf{x}_\alpha - x_0))\mathrm{d}\alpha \|\mathbf{x}_\alpha - \mathbf{x}_0\| + \int_0^1 S_\alpha(\mathbf{x}_\alpha + \alpha(\mathbf{x}_3 - \mathbf{x}_\alpha))\mathrm{d}\alpha \|\mathbf{x}_3 - \mathbf{x}_\alpha\|\Big)$$

Comparison

Our simplification shows more vertices with a low error.



Simplification and error distribution of Human (8k faces) with our method (left), QEM method (right)



Simplification and error distribution of Bunny (12k faces) with our method (left), QEM method (right)

Conclusion

We present a new error metric for mesh simplification which preserves local curvature. Thanks to the properties of interpolated algebraic sphere, the curvature is easily computed.

Future work

- Finding the 3D optimal position by minimizing the distance face-sphere
- Investigate adaptive kernel size when computing the algebraic spheres w.r.t. the surface features

References

[1] G. Guennebaud and M. Gross. Algebraic point set surfaces. In *ACM SIGGRAPH 2007 Papers*, 2007.

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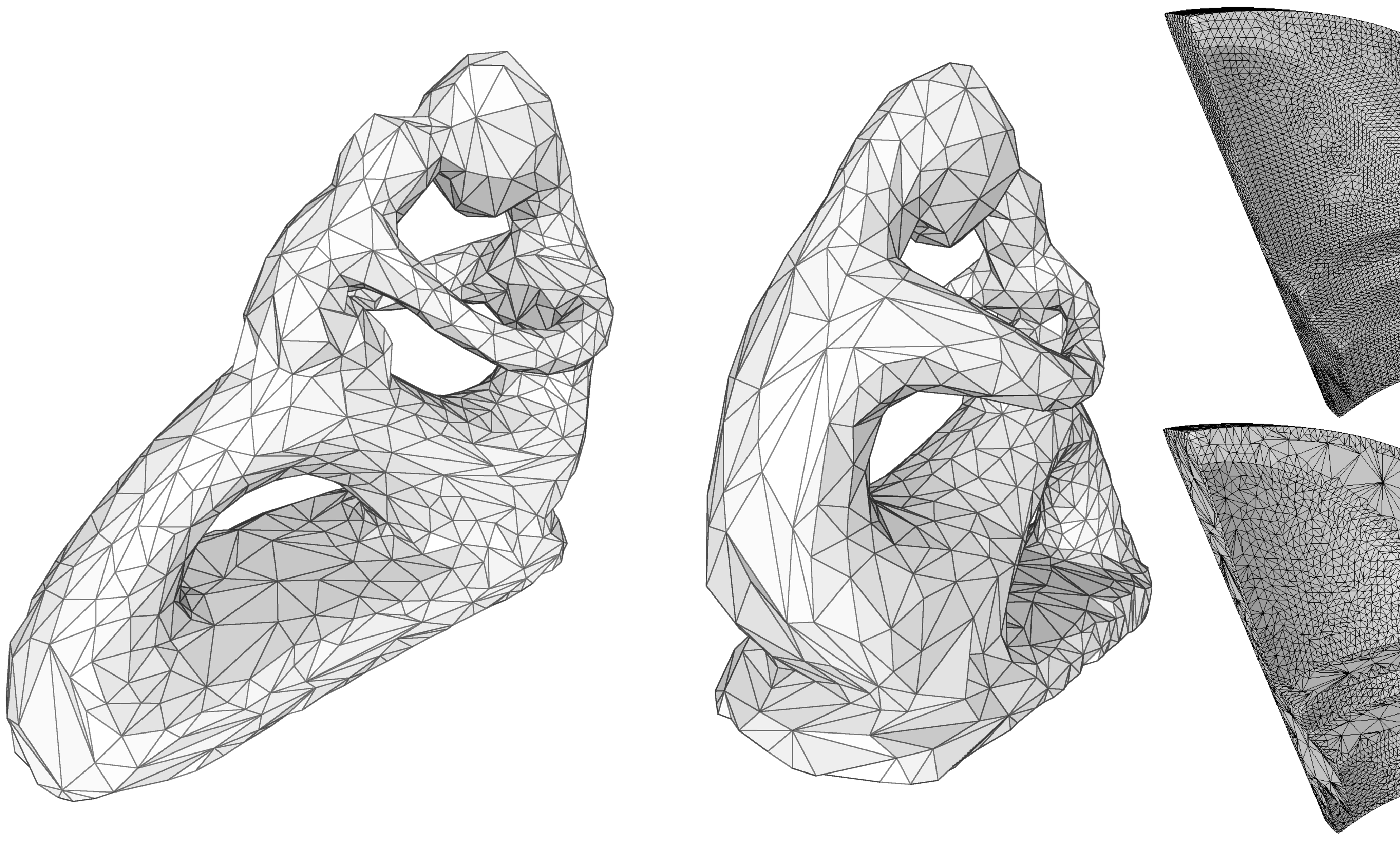
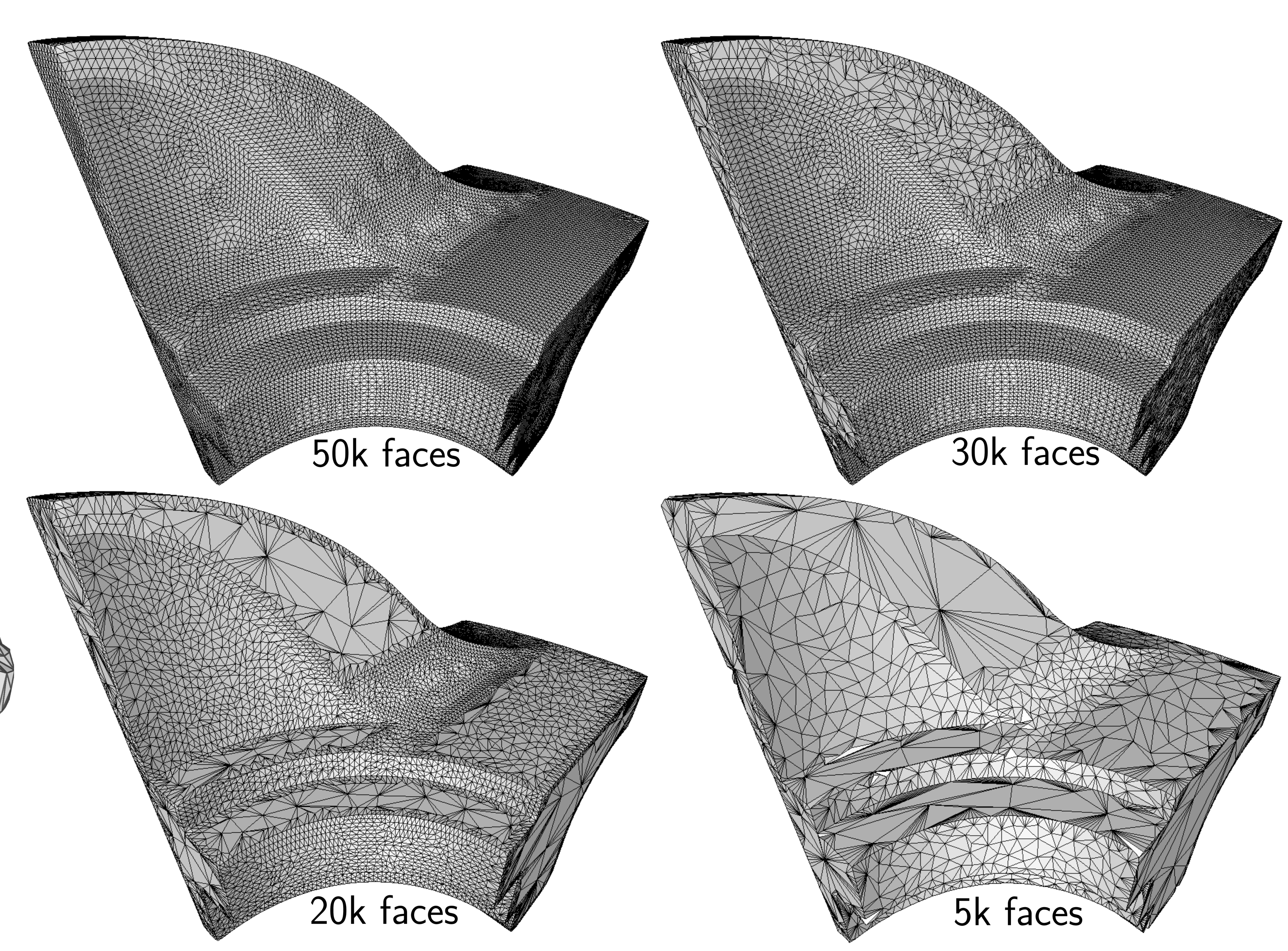
[3] D. Salinas, F. Lafarge, and P. Alliez. Structure-Aware Mesh Decimation. *Computer Graphics Forum*, Jan. 2015.

[4] J.-M. Thiery, E. Guy, and T. Boubekeur. Sphere-meshes: Shape approximation using spherical quadric error metrics. *ACM Trans. Graph.*, 32(6), Nov. 2013.



Two views of the Fertility surface simplified to 2k faces



Original Fandisk (top left) and its simplifications