Swung-to-Cylinder Projection for Panoramic Image Viewing

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Abstract

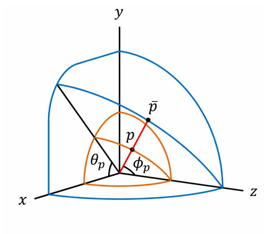
This paper proposes the swung-to-cylinder projection model for mapping a sphere to a plane, which is useful for viewing 360° panoramic images. Our model extends the swung-to-plane model and consists of two steps. In the first step, the sphere is projected onto a swung surface. In the second step, the projected image on the swung surface is mapped onto a cylinder through the perspective projection. The proposed model is simple, efficient and easy to control. Similar to the swung-to-plane model, it makes a better compromise between distortion minimization and line preserving. However, it does not suffer from the distortion problem of the swung-to-plane model when viewing full 360° panoramic images.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Picture/Image Generation—Viewing algorithms

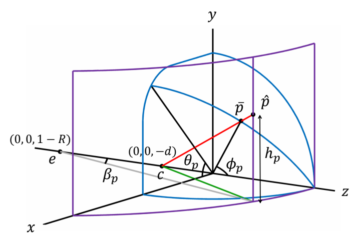
1. Introduction

Capturing a scene with a wide field of view from a single viewpoint records rich visual information of the scene. Recently, many cameras with 360° field of view have emerged into the consumer market, such as Ricoh Theta and V.360. For viewing wide-angle images defined on a viewing sphere, it is often required to map from the viewing sphere to an image plane. However, it is impossible to map from a sphere to a plane without introducing distortions. Thus, projection models have to trade off different types of distortions and none can avoid all distortions. Chang et al. introduced a swung-to-plane projection model [CHCC13] which consists of two steps. The first step projects the viewing sphere onto a swung surface. The second step maps the projection on the swung surface onto the image plane through the perspective projection. It strikes a good compromise between shape distortions and line preserving. The model also unifies several popular models including cylindrical, stereographic and Pannini projections [SPG10].

Although the swung-to-plane projection model has several nice properties, it has a limited horizontal field of view (hFOV) and suffers from serious distortion when viewing with a larger hFOV. Thus, for viewing full 360° panoramic images, it is often necessary to divide the viewing spheres into two hemispheres and apply the projection model separately. Our projection model generalizes the swung-to-plane model [CHCC13] by allowing the second step to map from the swung surface onto a *projection cylinder*. The resultant image is obtained by flattening the projection cylinder. This model is advantageous for viewing panoramas with the 360° hFOV and a large vFOV. We demonstrate that our model gives more pleasant views for panoramic images.



(a) The first step



(b) The second step

Figure 1: *The two projection steps in the swung-to-cylinder model.*

2. The swung-to-cylinder projection model

Our swung-to-cylinder projection model maps from a viewing sphere to the projection cylinder as illustrated in Figure 1. Given a point p on the sphere (the orange surface), the first step projects p onto a point $\bar{p}$ on a swung surface S (the blue surface) through a line emanating from the center of the sphere. This step is exactly the same as the first step in the swung-to-plane projection [CHCC13]. By construction, the 3D Euclidean coordinate $\bar{\mathbf{x}}_p$ of point $\bar{p}$ is $(\bar{x}_p, \bar{y}_p, \bar{z}_p) = (\bar{r}_p \sin\phi_p \cos\theta_p, \bar{r}_p \sin\phi_p \sin\theta_p, \bar{r}_p \cos\phi_p)$.

In the second step, $\bar{p}$ on the swung surface S is projected onto a point $\hat{p}$ on the projection cylinder (the purple surface) through a line emanating from the center of projection c. As shown in Figure 1(b), c is set to lie on the negative z axis with coordinate $\mathbf{x}_c = (0,0,-d)$.

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The projection cylinder has a radius R and is centered at the point e with the coordinate $\mathbf{x}_e = (0,0,1-R)$. The projection cylinder intersects with the surface S at the point $(0,0,1)$. We characterize the projection cylinder by its curvature $\kappa = 1/R$. The point $\hat{p}$ is the intersection between the projection cylinder and a line formed by c and $\bar{p}$ (the red line shown in Figure 1(b)). By expressing $\hat{p}$ as a point on the cylinder and as a point on the line respectively, we have the following equations

$$\hat{\mathbf{x}}_p = \mathbf{x}_c + \alpha_p(\bar{\mathbf{x}}_p - \mathbf{x}_c), \tag{1}$$

$$\hat{\mathbf{x}}_p = \mathbf{x}_e + [R\sin\beta_p, h_p, R\cos\beta_p]^T, \tag{2}$$

where $\hat{\mathbf{x}}_p$ is the Euclidean coordinate of $\hat{p}$; α_p is the parameter on the line; (β_p, h_p) is the coordinate on the projection cylinder. $\hat{\mathbf{x}}_p$ can be derived by solving Equation (1) and (2). The formulae of α_p, β_p and h_p are as follows

$$\alpha_p = \frac{-b_p t + \sqrt{\bar{x}_p^2(R^2 - t^2) + b_p^2 R^2}}{\bar{x}_p^2 + b_p^2}, \tag{3}$$

$$\beta_p = \tan^{-1}\left(\frac{\alpha_p \bar{x}_p}{\alpha_p b_p + t}\right), \tag{4}$$

$$h_p = \alpha_p \bar{y}_p, \tag{5}$$

where $b_p = \bar{z}_p + d$ and $t = R - d - 1$. The formula of $\hat{\mathbf{x}}_p$ can be obtained by substituting Equation (3) into Equation (1). After projections, the projection cylinder is flattened as the image plane. Thus, the 3D coordinate of $\hat{p}$ is mapped to a 2D coordinate (u_p, v_p) on the image plane. Based on the formula in Equation (2), the 2D coordinate of $\hat{p}$ can be written as

$$(u_p, v_p) = (R\beta_p, h_p). \tag{6}$$

To sum up, with Equation (3), (4), (5) and (6), one can relate the 3D spherical coordinate $(1, \theta_p, \phi_p)$ of a point p on the viewing sphere with the 2D coordinate (u_p, v_p) of its projection on the image plane. There are four parameters in our swung-to-cylinder projection model, d (projection center's position), κ (projection cylinder's curvature), h and l (for the trajectory curve of the swung surface). To reduce visual distortions, the parameters can be determined by optimizing an energy function formulated for simultaneously minimizing the distortions on shapes, areas and lines.

3. Results

Figure 2 compares the swung-to-cylinder projection model with central cylindrical projection and stereoscopic Pannini projection using the Tissot's indicatrix and the grid pattern. In Tissot's indicatrix, the grey lines are contours of either constant θ or constant ϕ. The blue ellipses are projections of circles on a sphere. In an ideal projection, they should remain circle and has the same area. The grid patterns shows the projections of three sets of orthogonal scene lines. The proposed model has better properties on preserving shapes, areas and lines. Figure 3 compares the cylindrical projection, the Pannini projection [SPG10], the swung-to-plane projection [CHCC13] and our swung-to-cylinder projection when viewing full spherical panoramas. The cylindrical projection (Figure 3(a)) maintains areas better while our swung-to-cylinder model (Figure 3(d)) reduces the bending of lines and thus provides better perspective perception. The Pannini projection (Figure 3(b)) is not suitable for viewing with large vFOVs because it would require infinite space to show the whole view. Much content is lost. Although having a better compromise among distortions, the swung-to-plane model has a limited hFOV. It either has to divide hFOVs (Figure 3(c)) or suffers from serious distortions when viewing panoramas with a larger hFOVs. By projecting to a cylinder, our swung-to-cylinder model is capable of viewing full panoramas with better perspective perception.

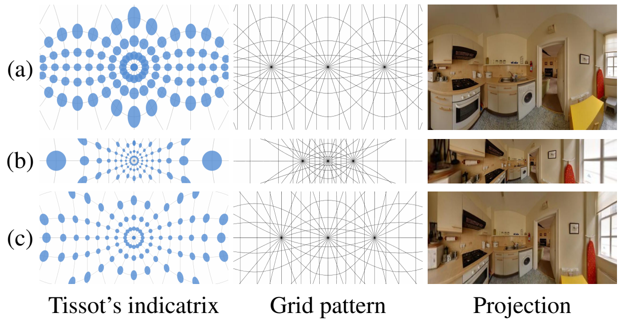

Figure 2: *Tissot's indicatrix and grid patterns. (a) Central cylindrical projection. (b) Stereographic Pannini projection [SPG10].(c) Our swung-to-cylinder projection.*

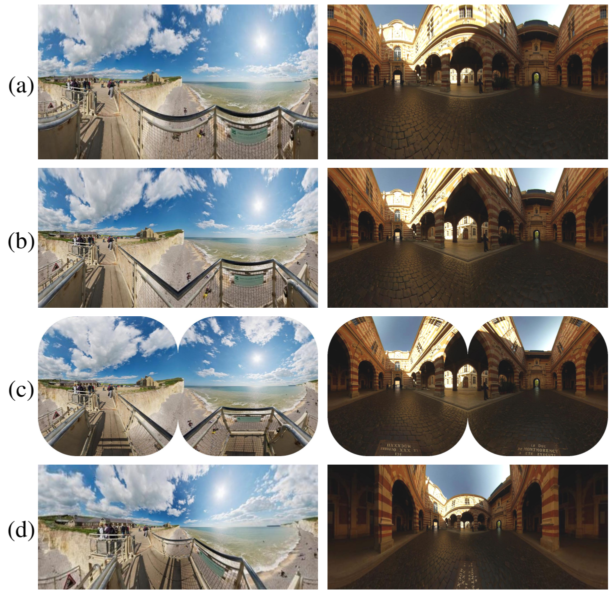

Figure 3: *Full spherical panorama visualizations. (a) The cylindrical projection. (b) The Pannini projection. (c) The swung-to-plane projection. (d) Our swung-to-cylinder projection.*

References

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