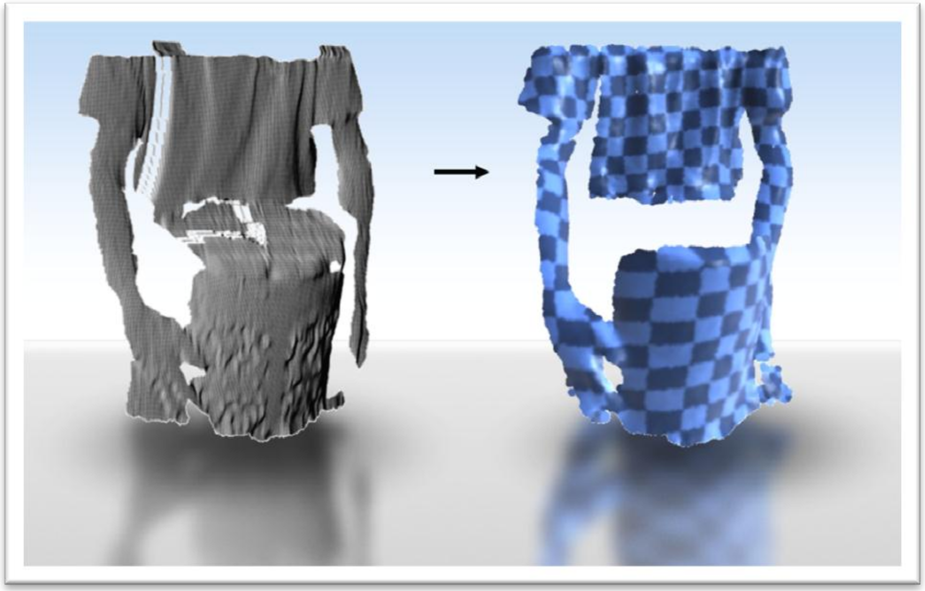
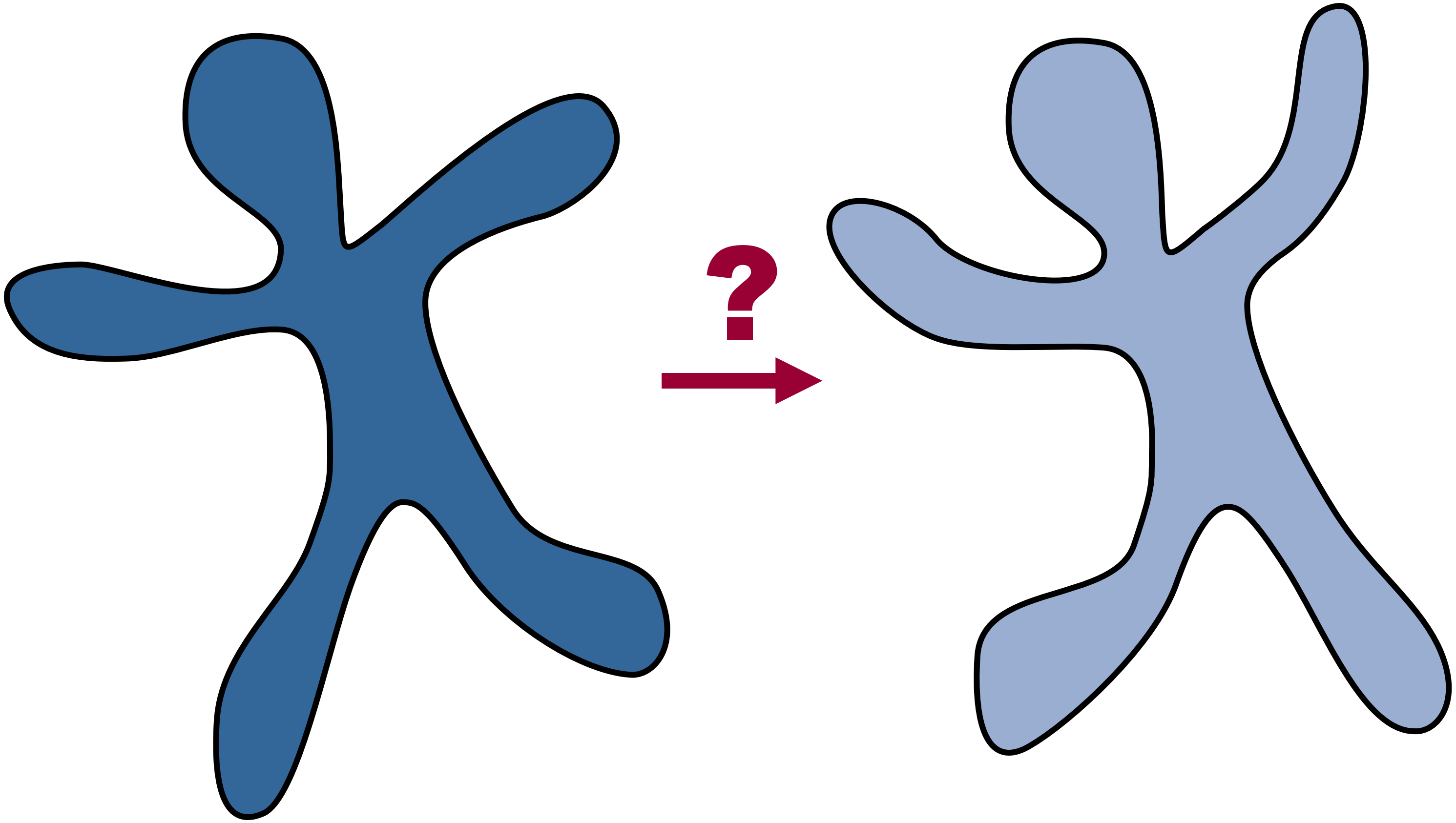


Deformable Sequence Reconstruction

Deformable Shape Matching

Basic Principle

Example



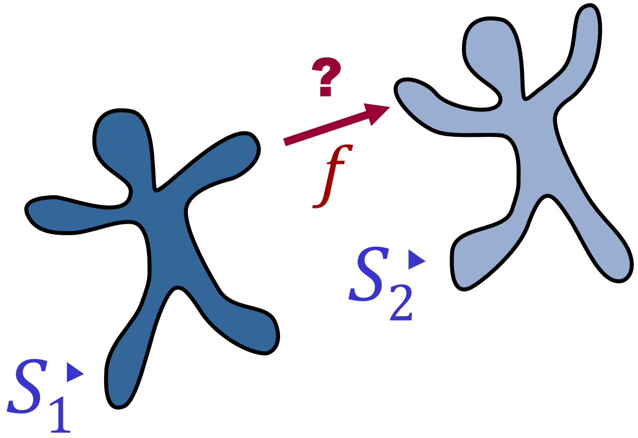
Correspondences?

What are We Looking for?

Problem Statement:

Given:

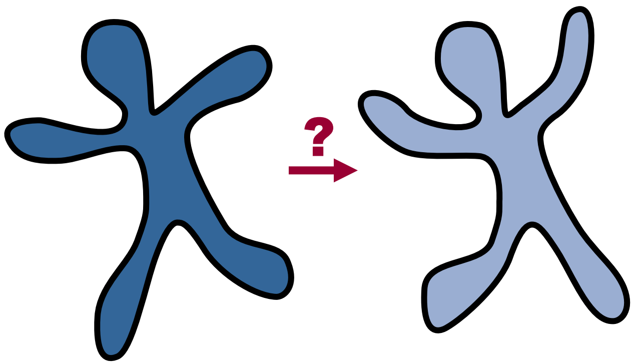
- Two surfaces $S_1, S_2 \subseteq \mathbb{R}^3$



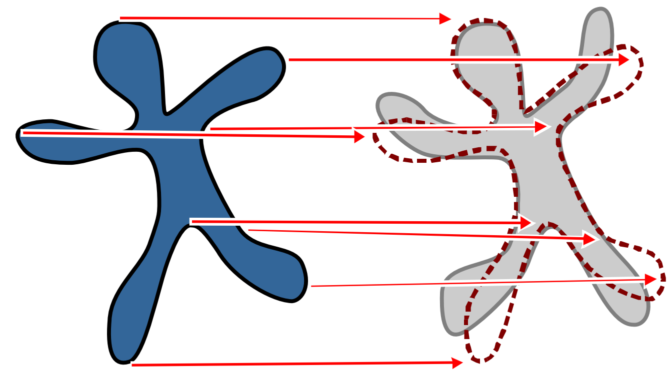
We are looking for:

- A *reasonable* deformation function f: $S_1 \rightarrow \mathbb{R}^3$ that brings S_1 close to S_2

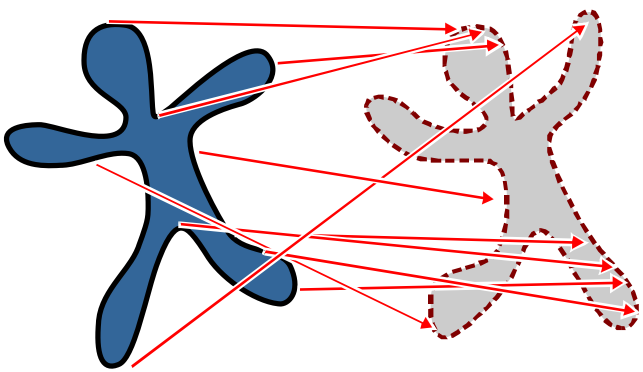
Example



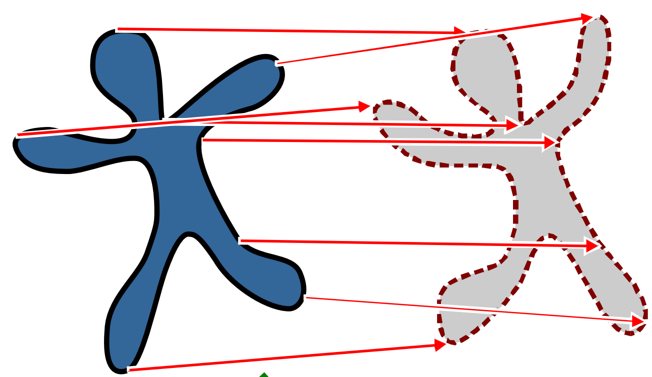
correspondences?



no shape match



too much deformation

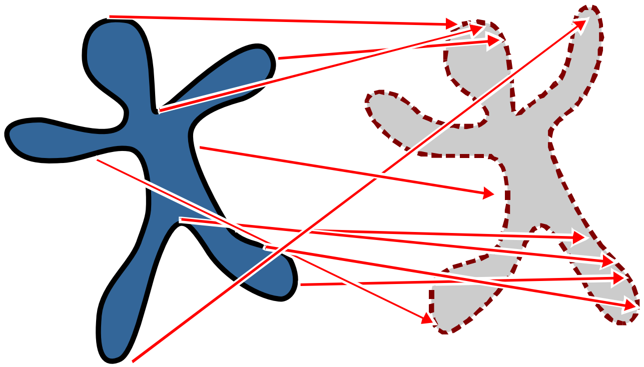


optimum

This is a Trade-Off

Deformable Shape Matching is a Trade-Off:

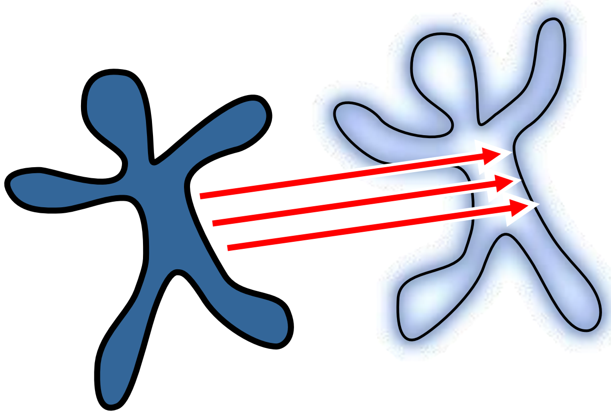
- We can match any two shapes using a weird deformation field



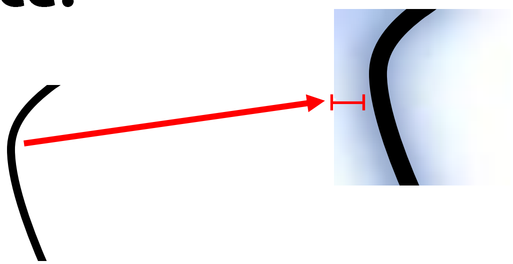
- We need to trade-off:
  - Shape matching (close to data)
  - Regularity of the deformation field (reasonable match)

Variational Model

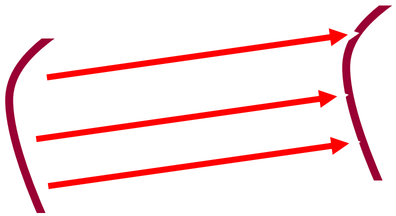
Components:



Matching Distance:



Deformation / rigidity:

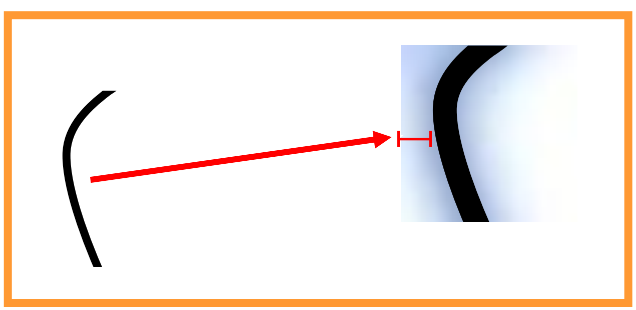


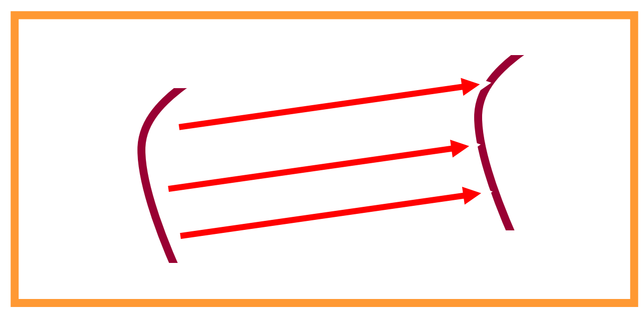
Variational Model

Variational Problem:

- Formulate as an energy minimization problem:

$$E(f) = E^{(match)}(f) + E^{(regularizer)}(f)$$



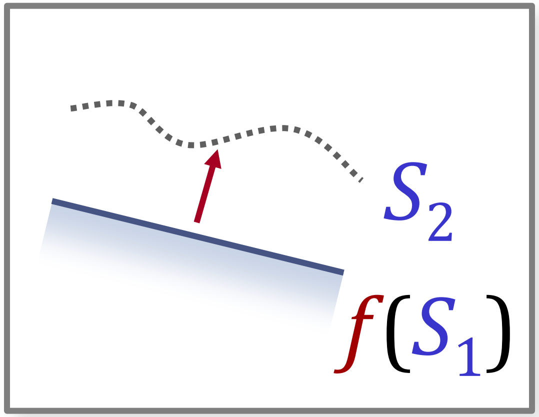


Part 1: Shape Matching

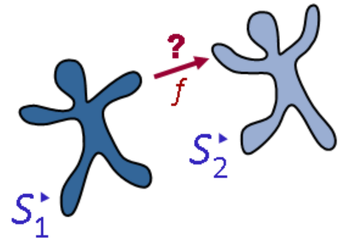
Data Term:

- Objective Function:

$$E^{(match)}(f) = dist\left(f(S_1, S_2)\right)$$



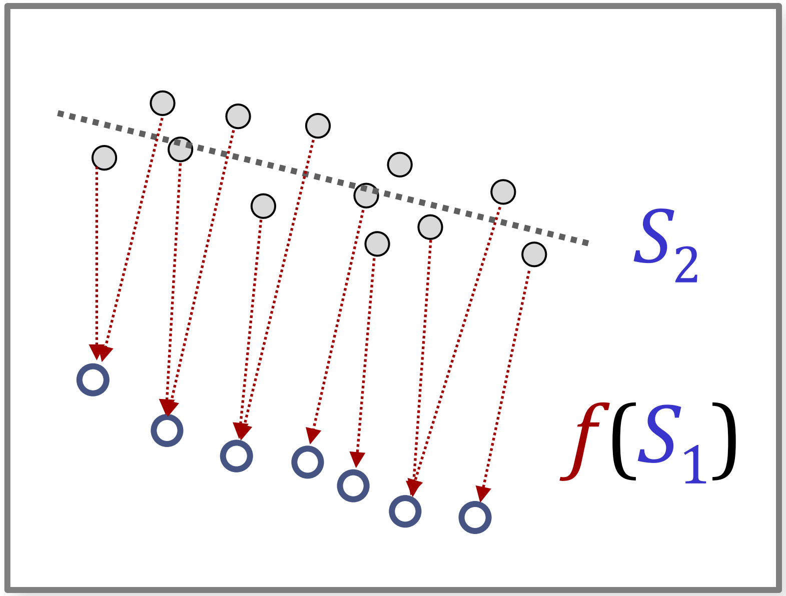
- Distance measures:
  - Least-squares (L_2)
  - Reweighted (robustness)
  - Hausdorf distance
  - L_p-distances, etc.

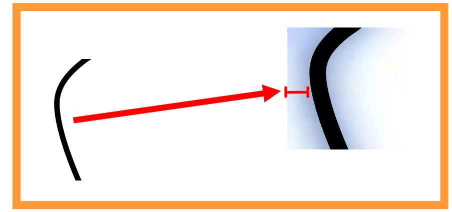


- L_2 measure is frequently used (models Gaussian noise)
  - Reweighting/truncation for robustness

Surface Approximation

$$E^{(match)}(f)$$

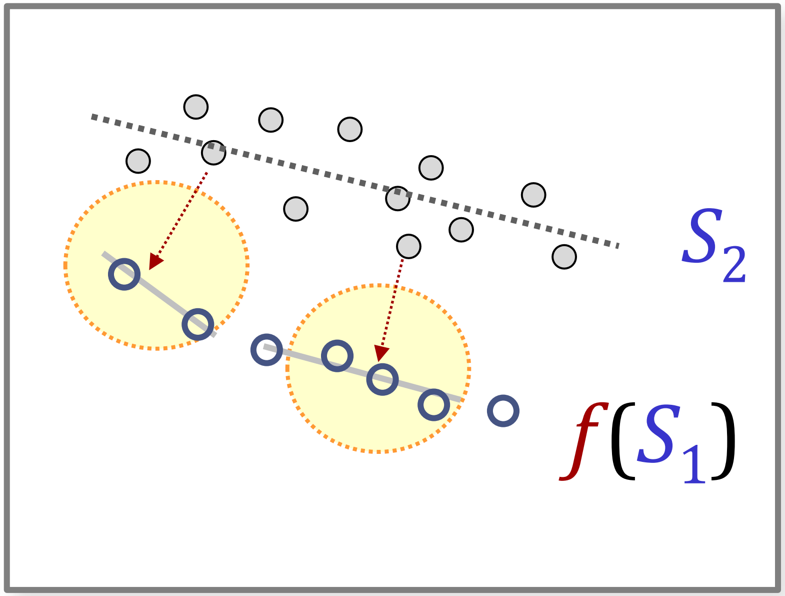


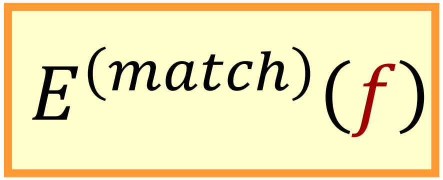


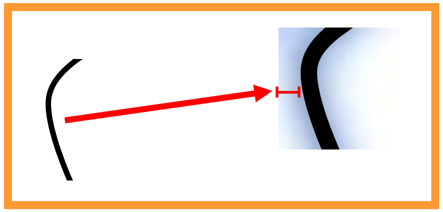
Basic: Closest point matching

- "Point-to-point" energy
- Usually iterated: "Iterated Closest Points (ICP)"
  - Establish nearest-neighbor correspondences
  - Minimize energy (with regularizer)

Surface Approximation



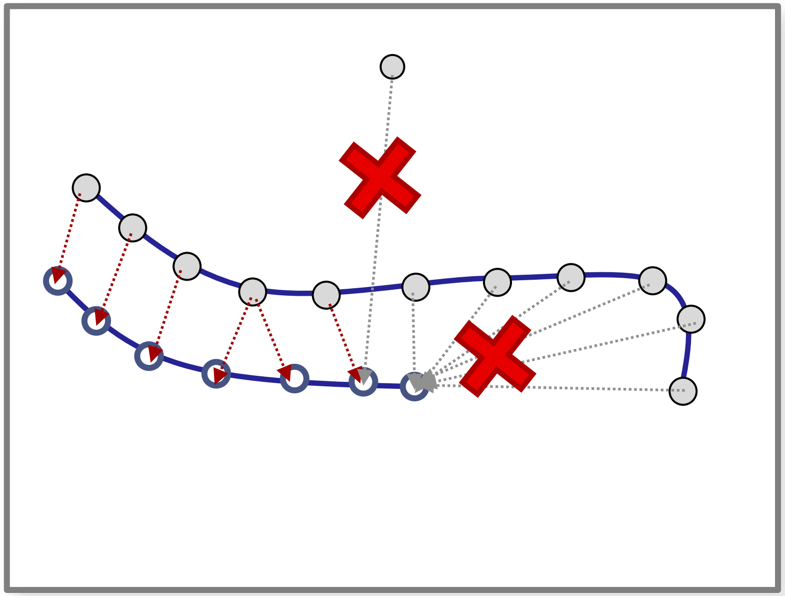




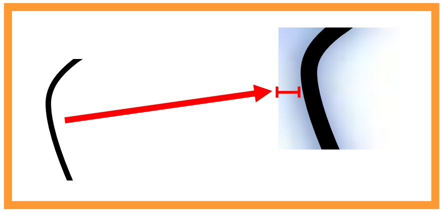
Improvement: Linear approximation

- "Point-to-plane" energy
- Fit plane to *k*-nearest neighbors

Robust Least-Squares



$$E^{(match)}(f)$$



Robustness: Reweighting

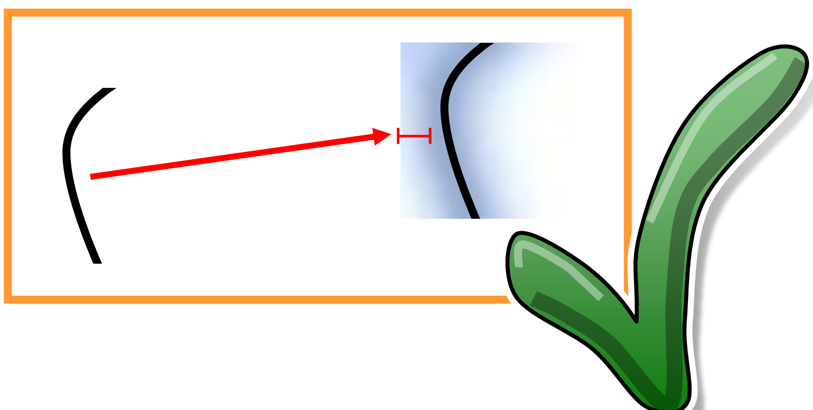
- Ignore Outliers
  - Large distance
  - Connection to normal at large angle
  - Many matches to one point

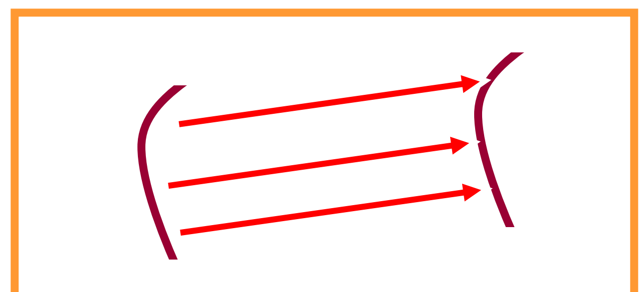
Variational Model

Variational Problem:

- Formulate as an energy minimization problem:

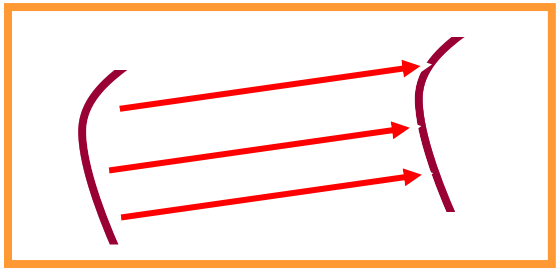
$$E(f) = E^{(match)}(f) + E^{(regularizer)}(f)$$





Deformation Model

$$E^{(regularizer)}(f)$$

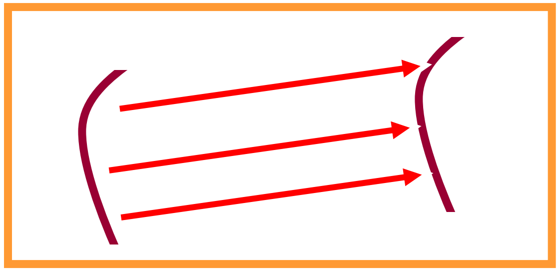


What is a "nice" deformation field?

- Elastic deformation
  - Volumetric elasticity
  - Thin shell model (more complex)
- Intrinsic
  - Isometric matching
- Smooth deformations
  - "Thin-plate-splines" (TPS)
  - Allowing strong deformations, but keep shape

Deformation Model

$$E^{(regularizer)}(f)$$



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  - Isometric matching
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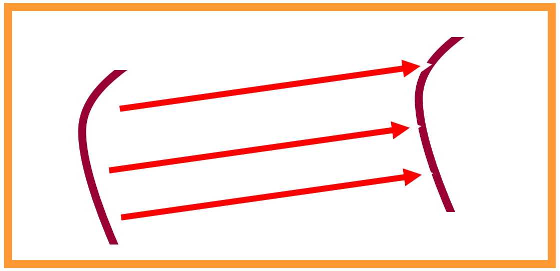
  - Allowing strong deformations, but keep shape

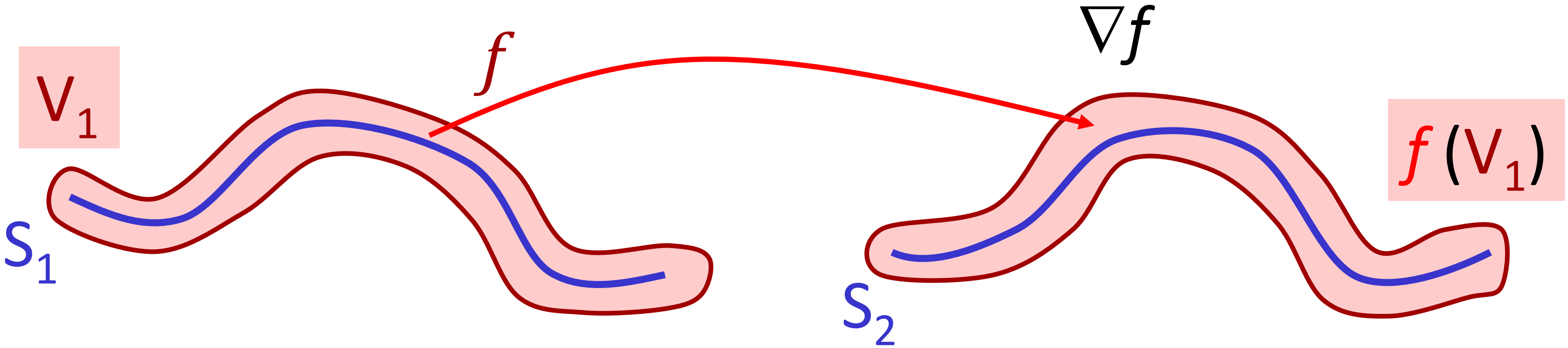
How to Detect Deformations?

$$E^{(regularizer)}(f)$$



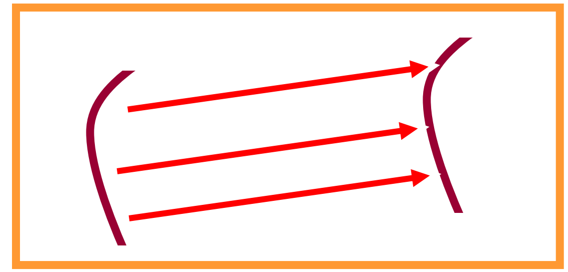
Model

- Map volume to volume
- Function $f: V \to \mathbb{R}^3$



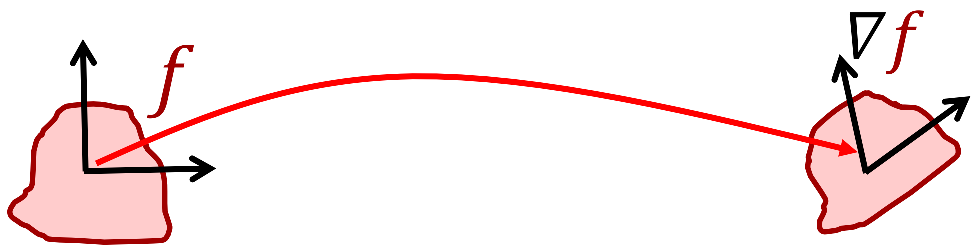
How to Detect Deformations?

$$E^{(regularizer)}(f)$$



Detect deformation

- Look at "deformation gradients"
- Jacobian matrix ∇f
- Function $\nabla f : V \rightarrow \mathbb{R}^3$



Criterion

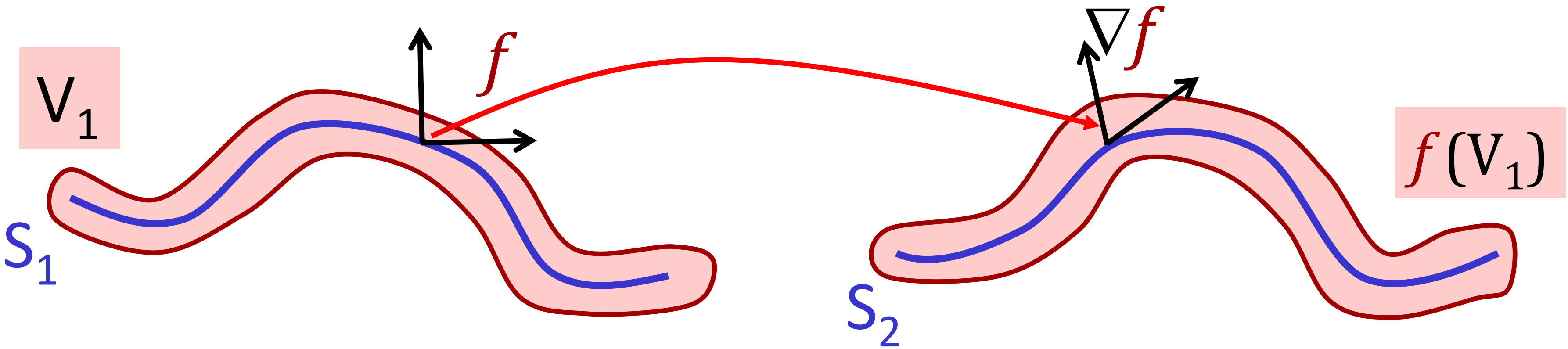
- *No deformation:* ∇f orthogonal
- *Deformation:* ∇f non-orthogonal

Elastic Volume Model

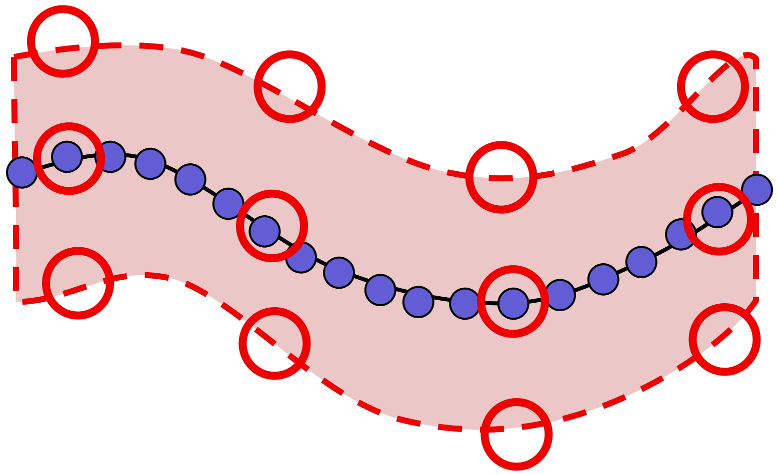
Extrinsic Volumetric "As-Rigid-As Possible"

- Measure orthogonality
- Integrate over deviation from orthogonality

$$E(f) = \int_{V_1} \left\| [\nabla f(\mathbf{x})][\nabla f(\mathbf{x})]^{\mathrm{T}} - \mathbf{I} \right\|_F^2 d\mathbf{x}$$



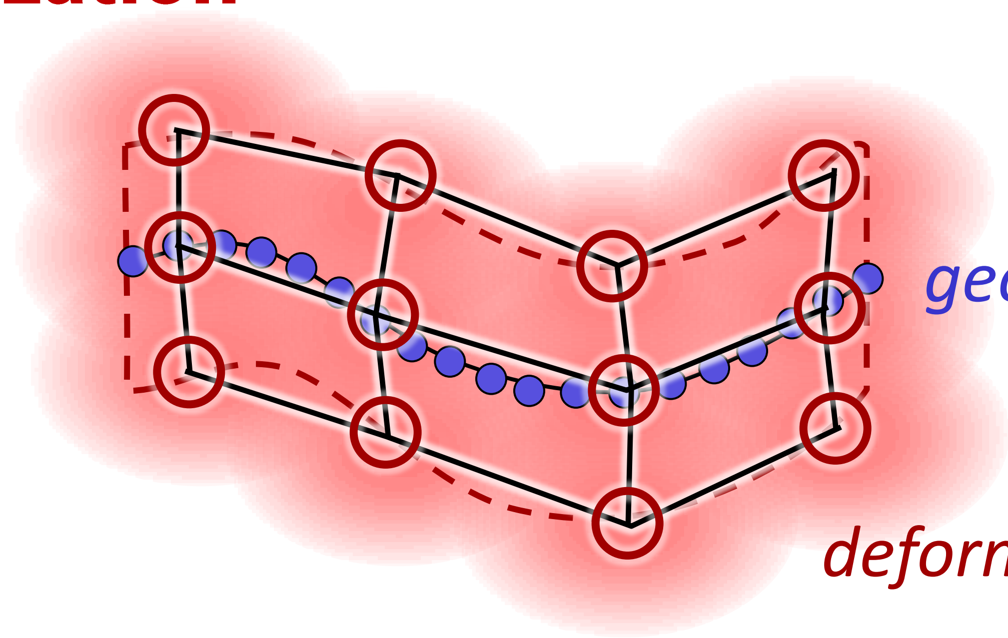
Discretization



Example Approach:

- Full resolution *geometry*
- Subsample *deformation*

Discretization



"Subspace" Approach:

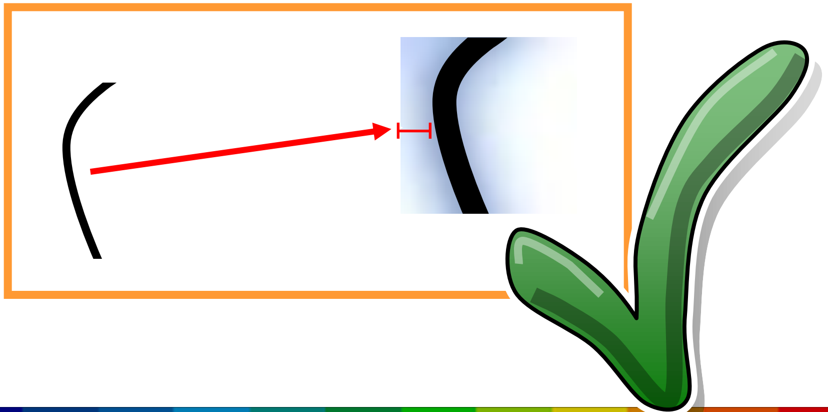
- Sample volume
- Place basis functions
- Decouple from resolution of geometry

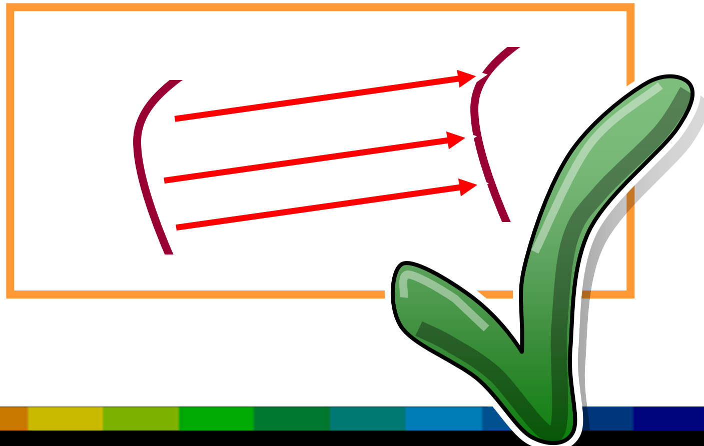
Deformable ICP

How to build a deformable ICP algorithm

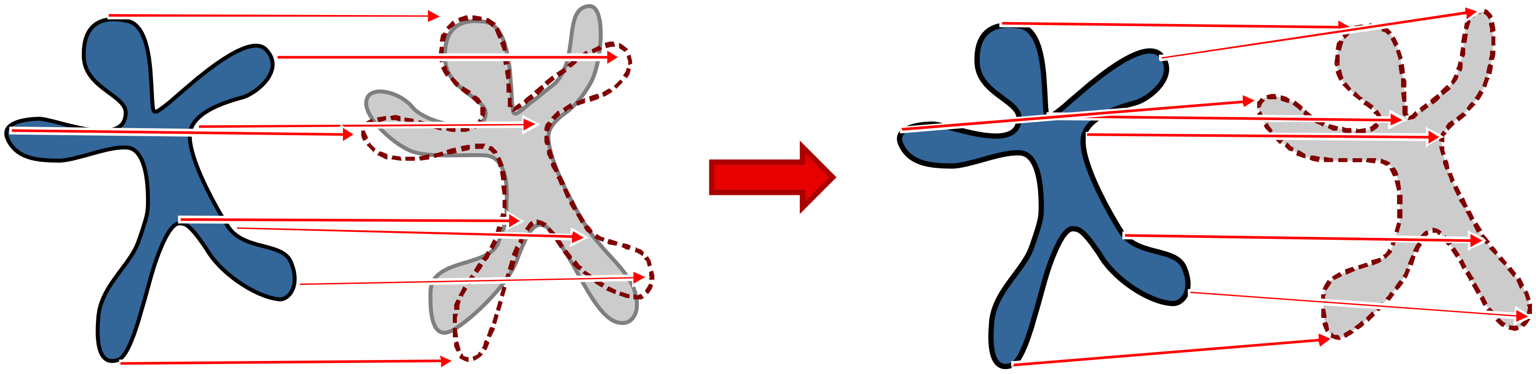
- Pick a surface distance measure
- Pick an deformation model / regularizer

$$E(f) = E^{(match)}(f) + E^{(regularizer)}(f)$$





Deformable ICP



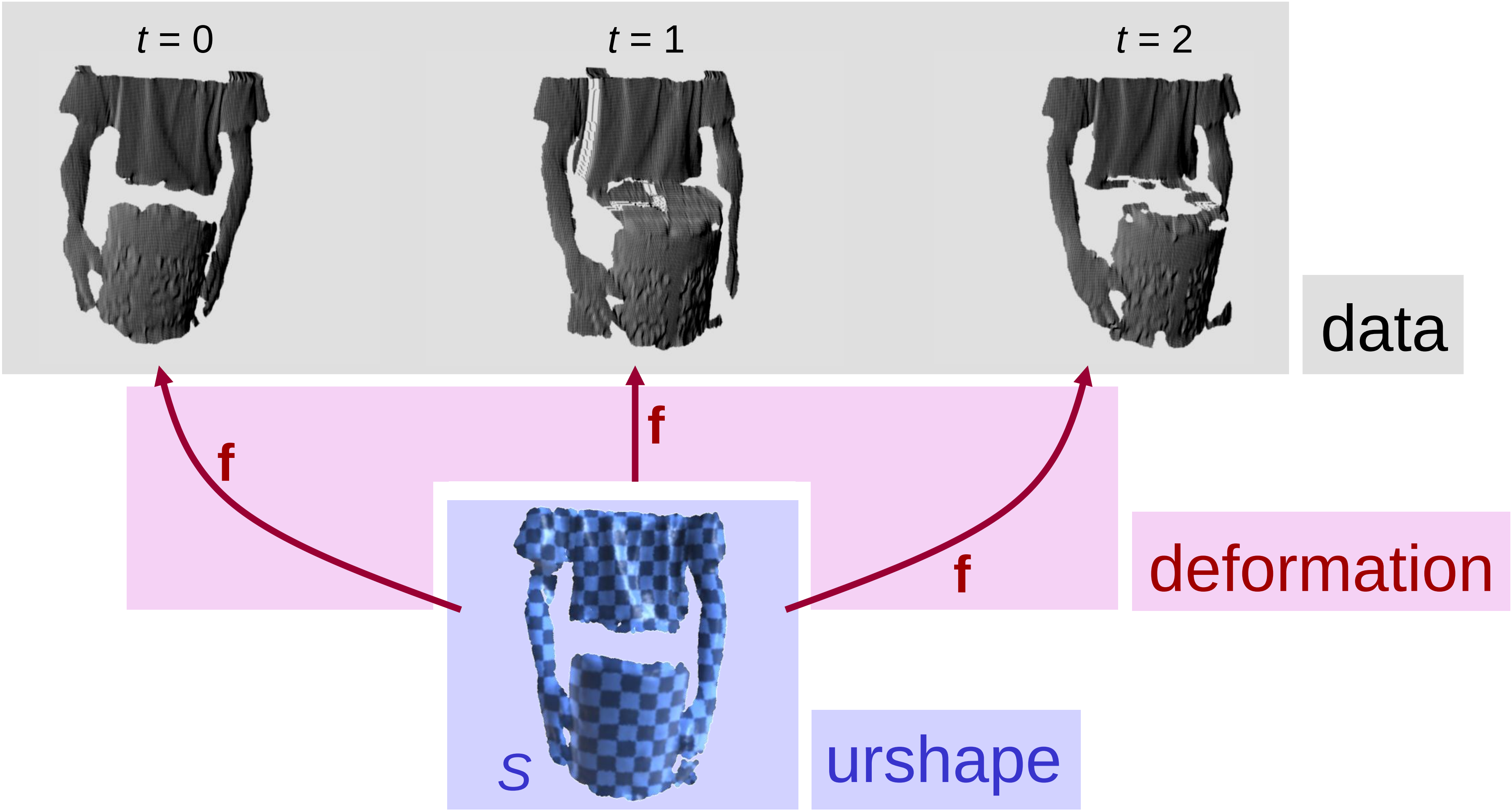
Deformable ICP Algorithm

- Select model: $E^{(match)}$, $E^{(regularizer)}$
- Initialize $f(S_1)$ with S_1 (i.e., $f = \mathrm{id}$)
- (Non-linear) optimization:
  - Newton, Gauss Newton
  - LBGFS (quick & effective)

Animation Reconstruction

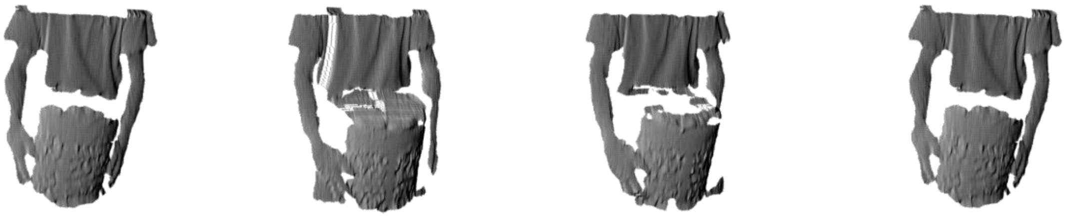
Reconstructing Sequences of Deformable Shapes

“Factorization” Approach



Hierarchical Merging

data

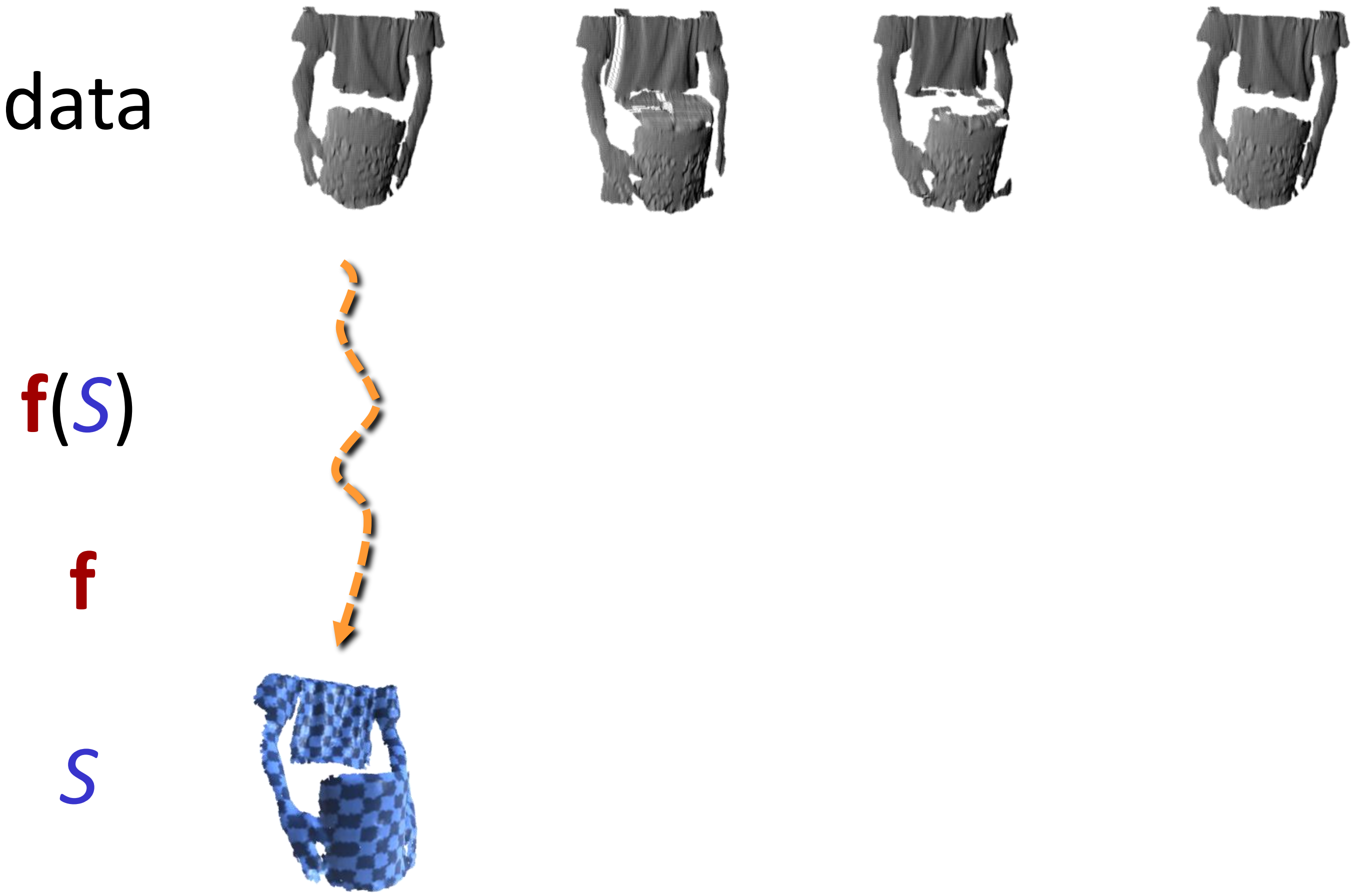


$\mathbf{f}(S)$

$\mathbf{f}$

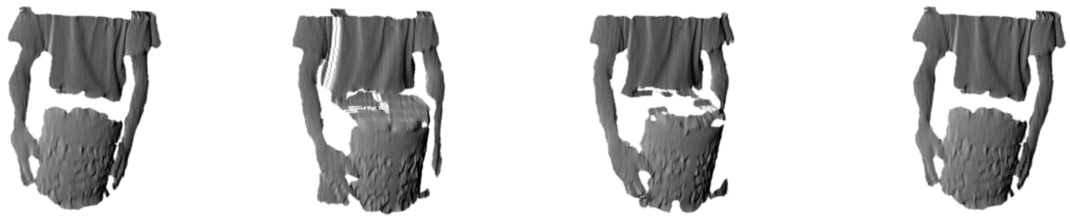
S

Hierarchical Merging



Initial Urshapes

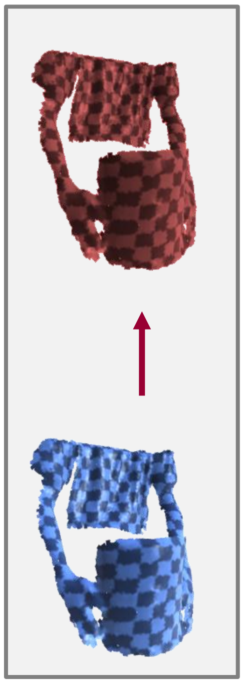
data



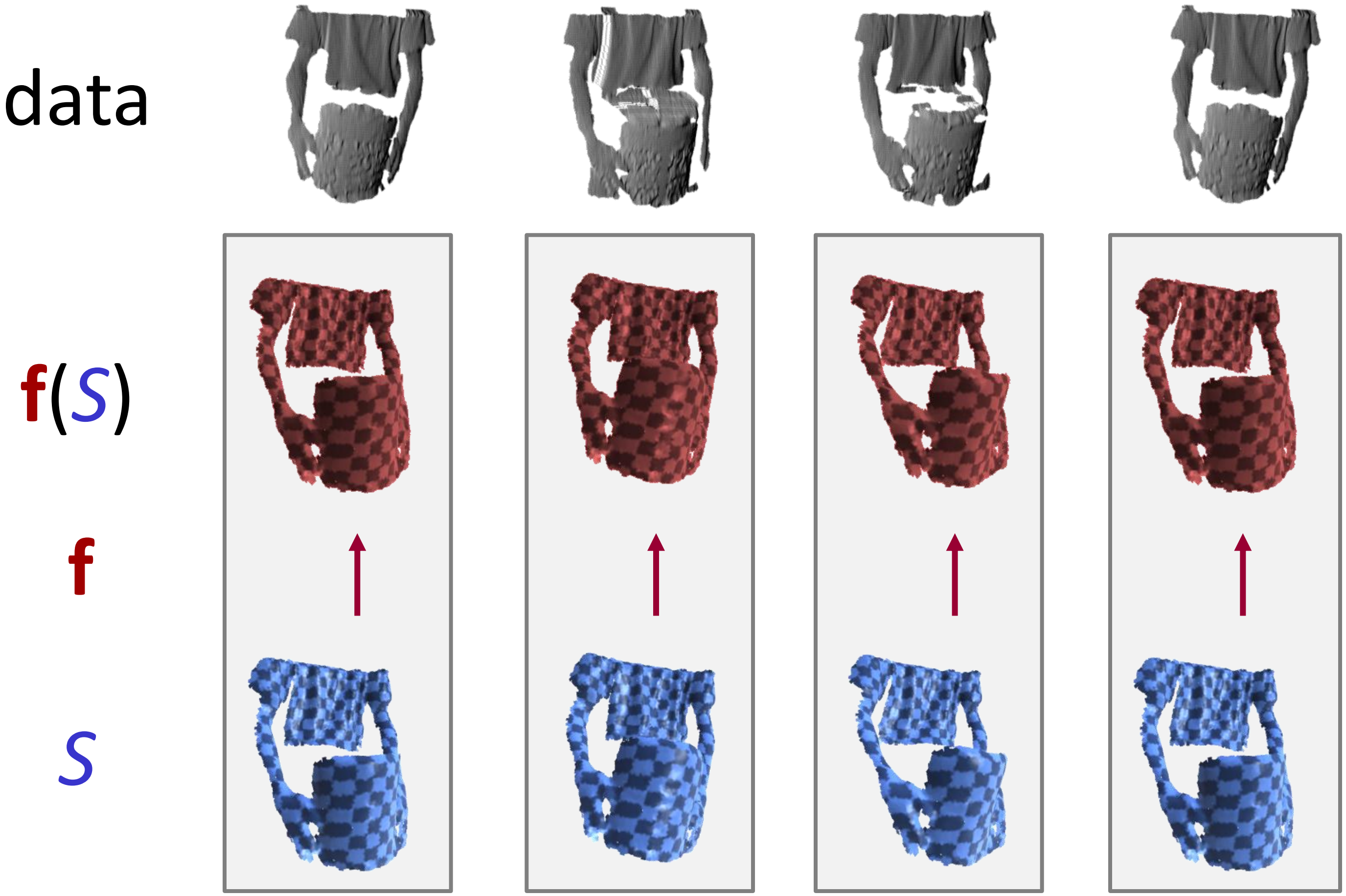
f(*S*)

f

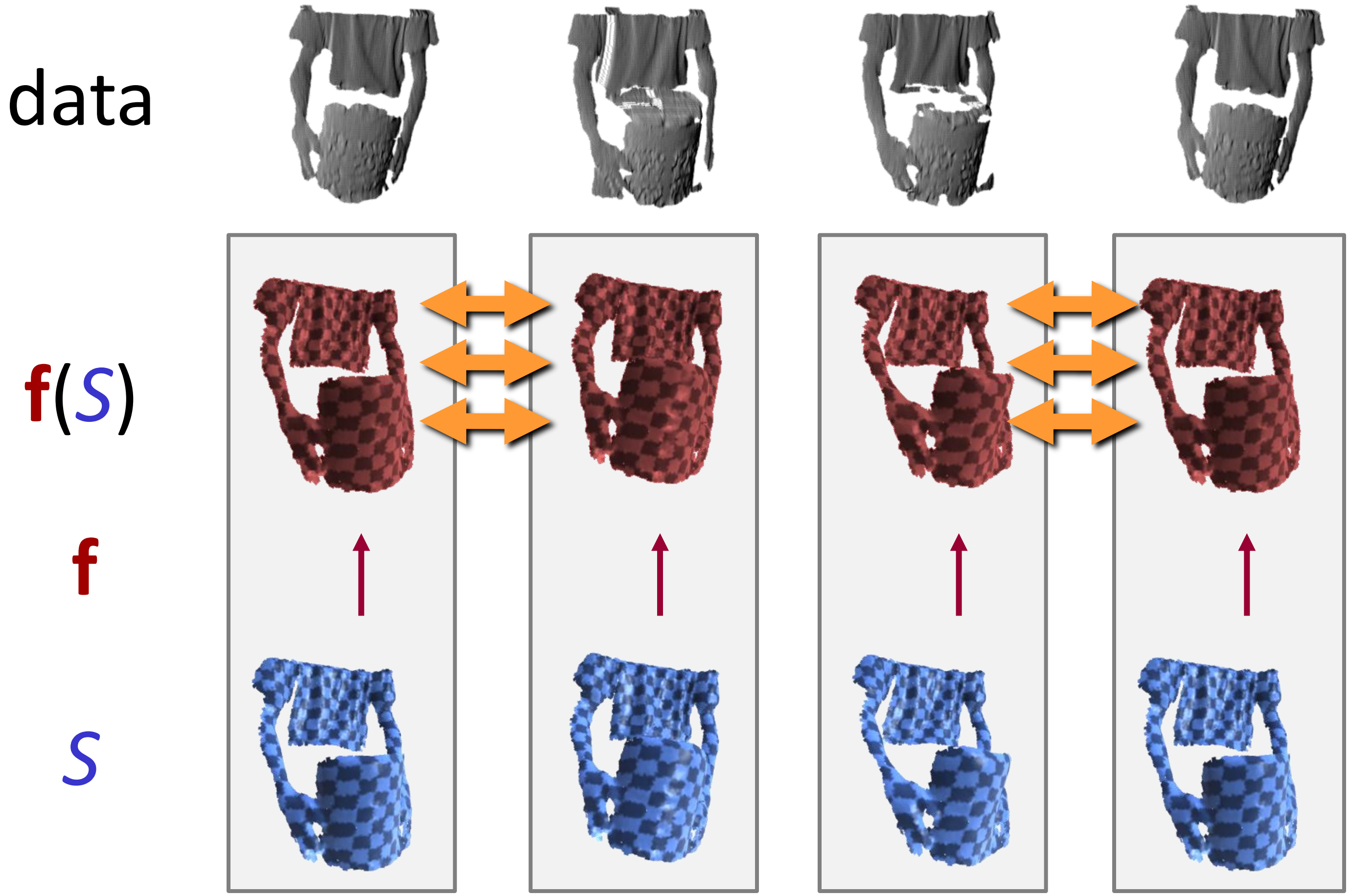
S



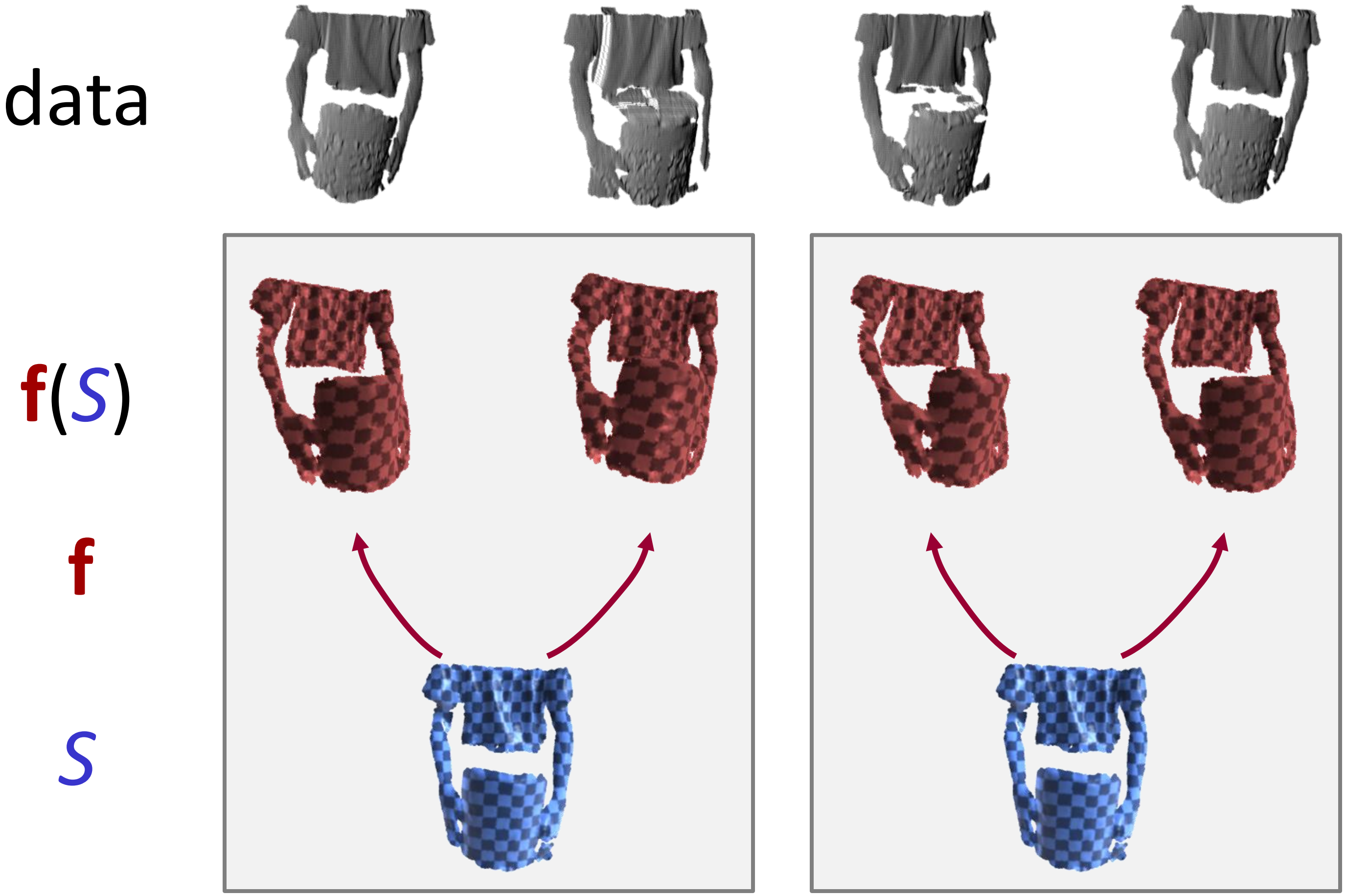
Initial Urshapes



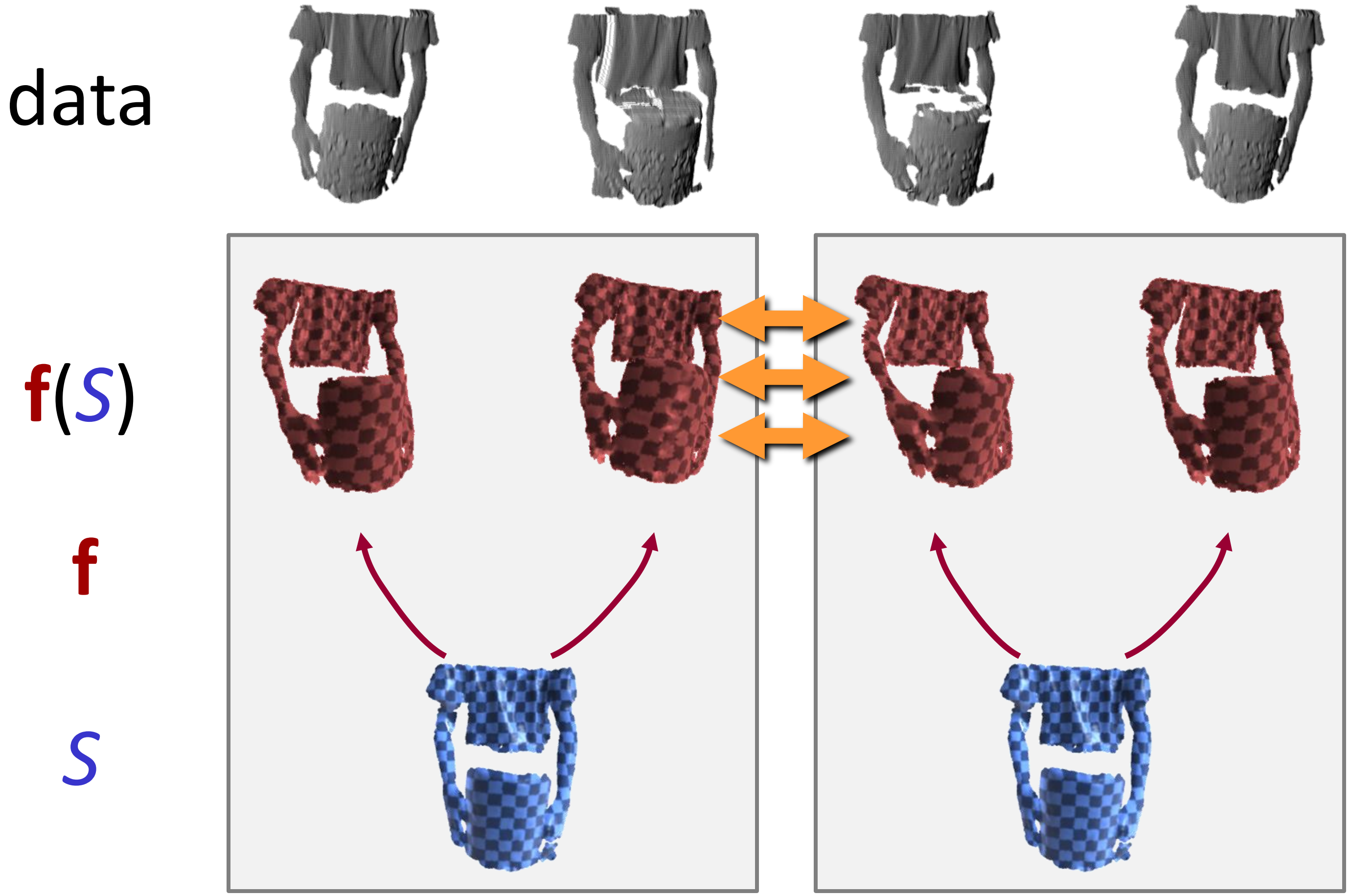
Alignment



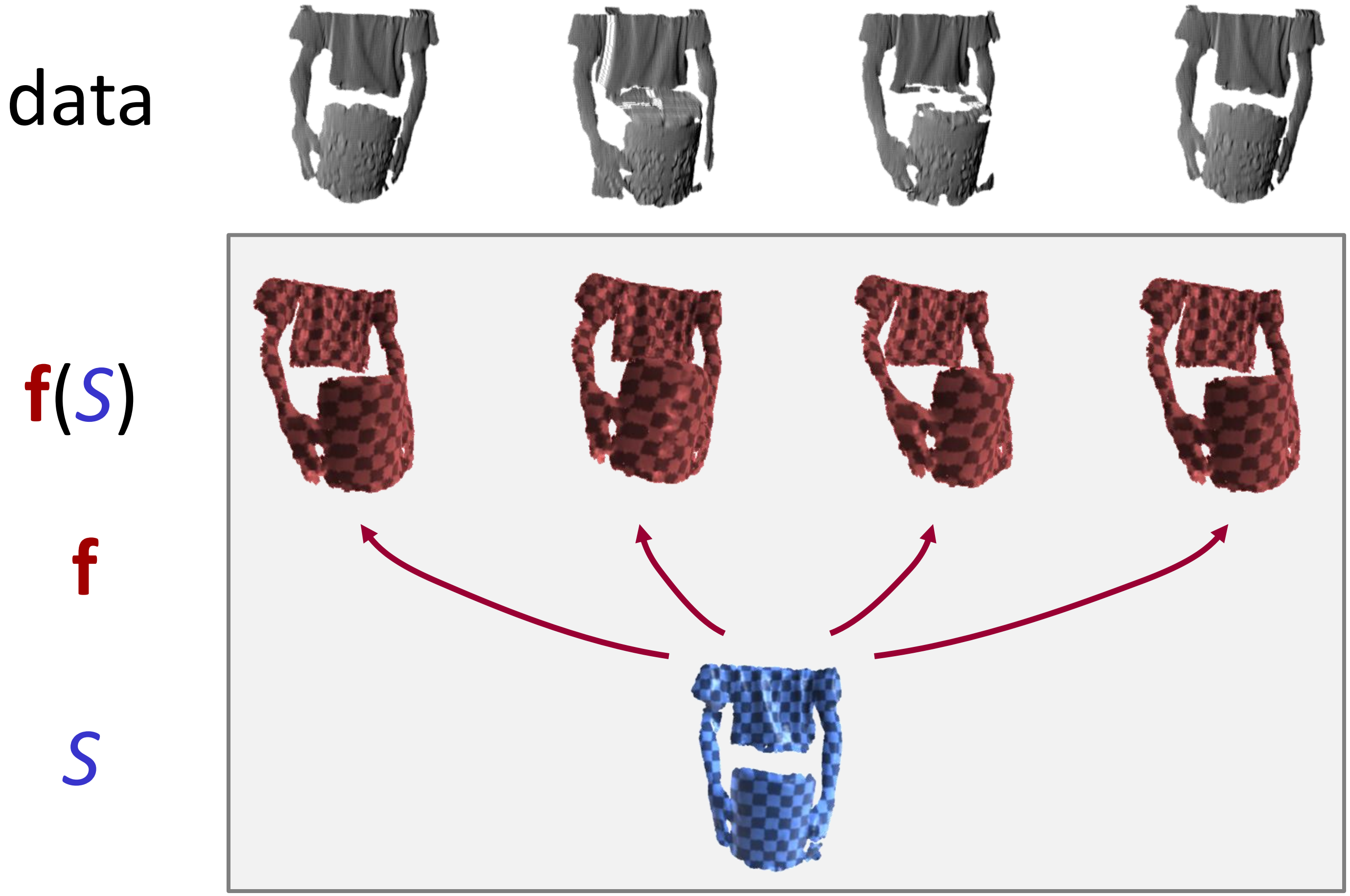
Align & Optimize



Hierarchical Alignment



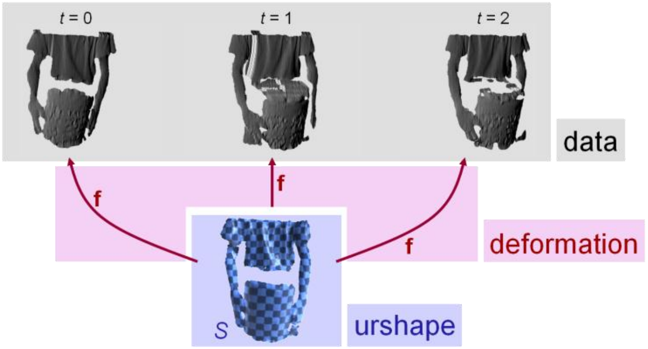
Hierarchical Alignment



Global Optimization

Energy Function

$E(\mathbf{f}, S) = E_{data} + E_{deform} + E_{smooth}$



Components

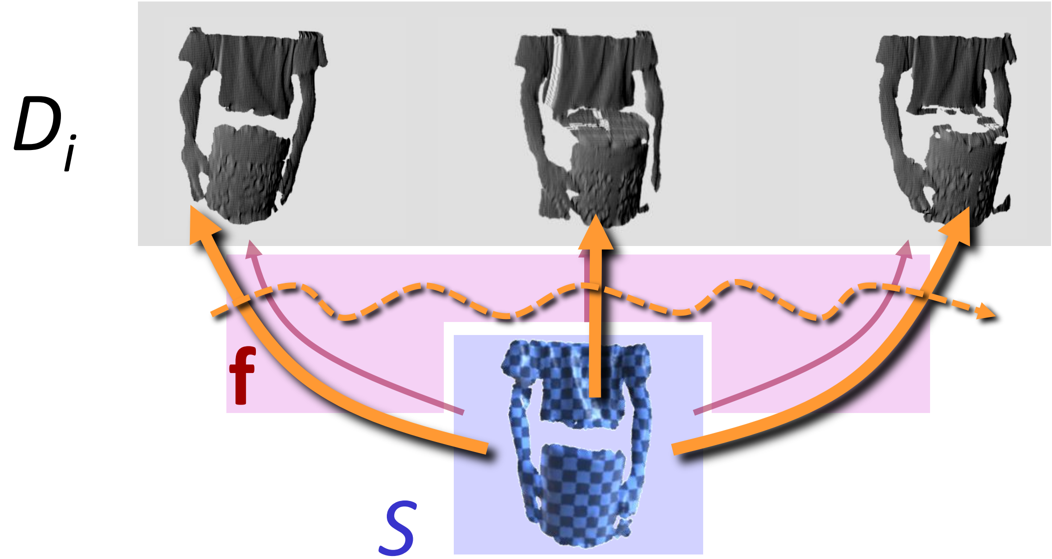
- $E_{data}(\mathbf{f}, S)$ – data fitting
- $E_{deform}(\mathbf{f})$ – elastic deformation, smooth trajectory
- $E_{smooth}(S)$ – smooth surface

Final Optimization

- Minimize over all frames

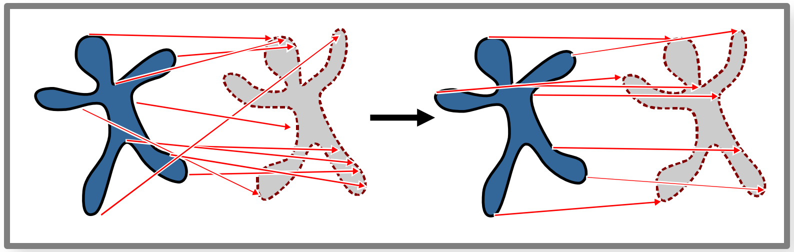
Elastic Deformation Energy

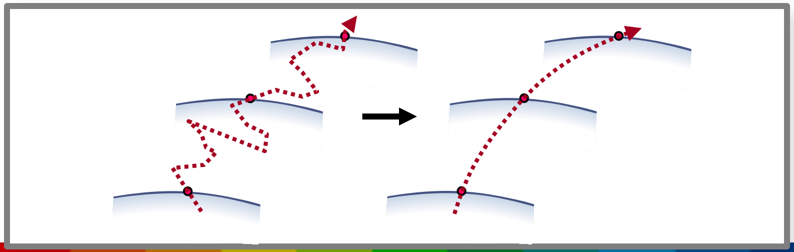
$E_{deform}(\mathbf{f})$



Regularization

- Elastic energy
- Smooth trajectories

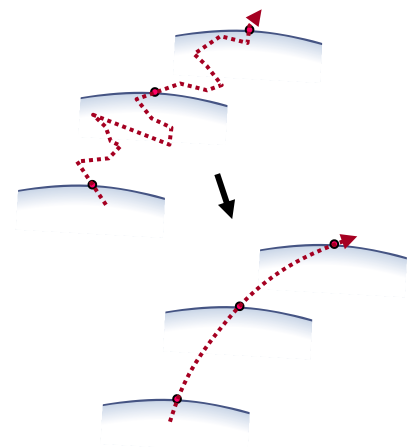




Additional Terms

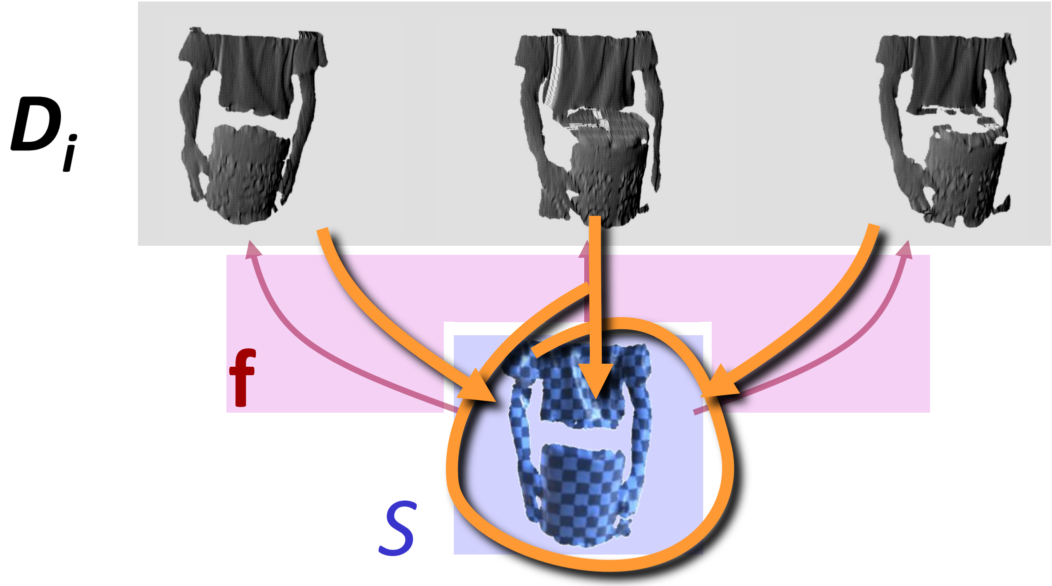
More Regularization

- Acceleration: $E_{acc} = \int_T \int_V |\partial_t^2 \ \mathbf{f}|^2$
  - Smooth trajectories
- Velocity (weak): $E_{vel} = \int_T \int_V |\partial_t \ \mathbf{f}|^2$
  - Damping



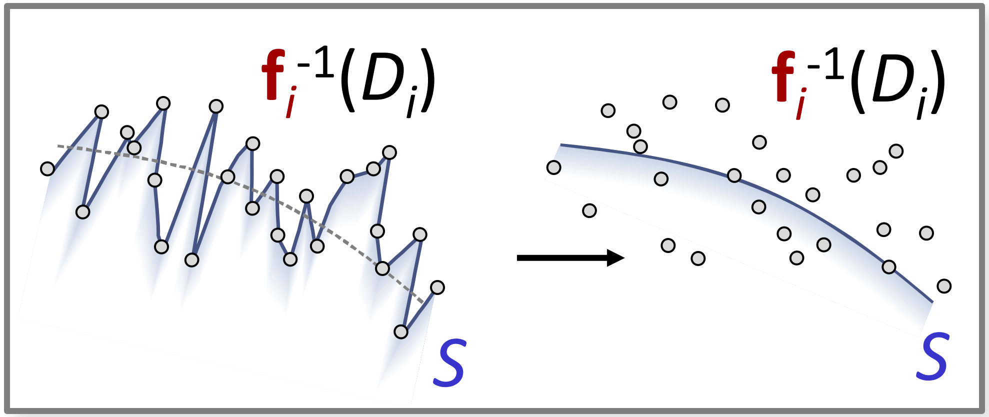
Surface Reconstruction

$E_{smooth}(S)$

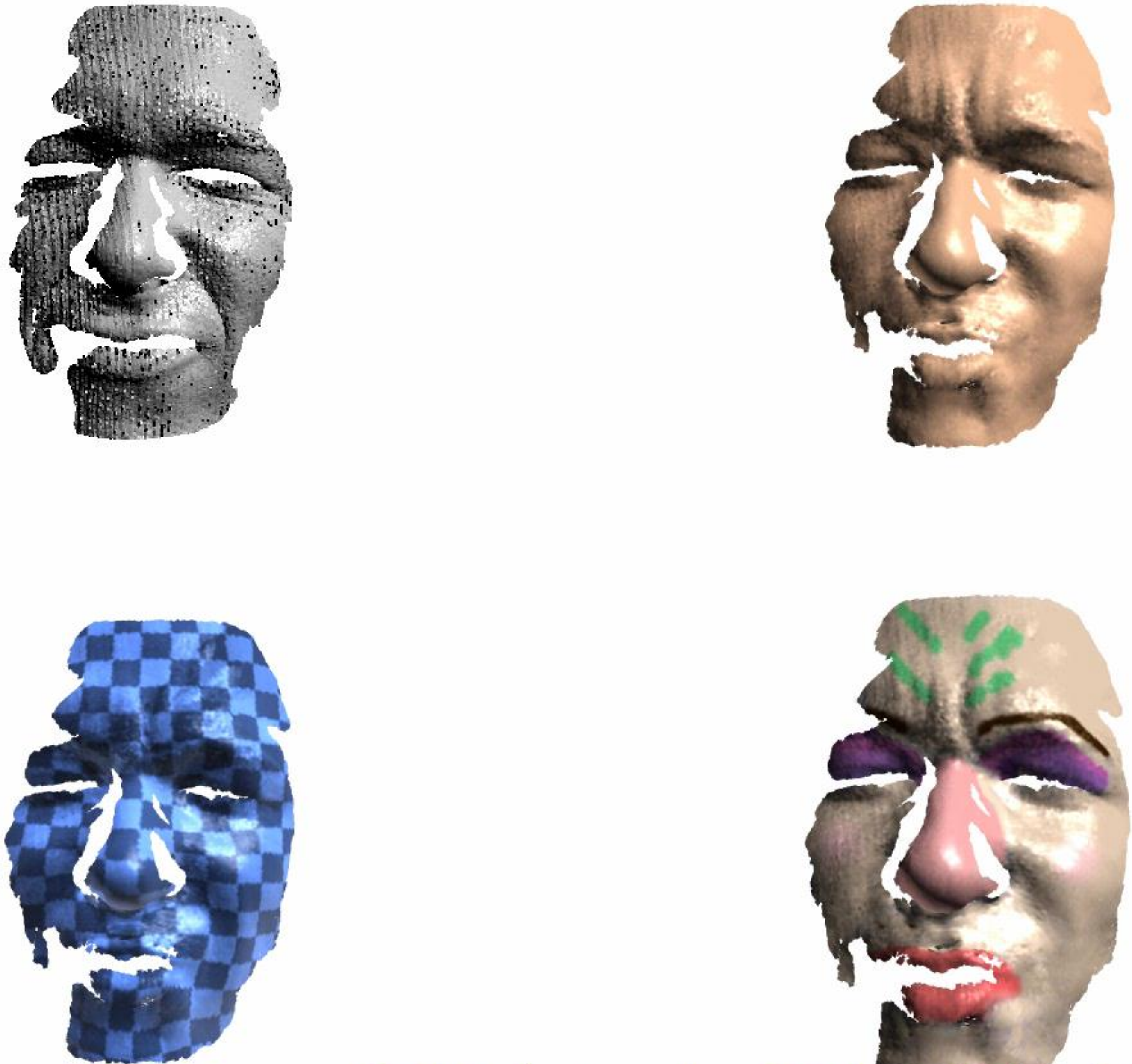


Data fitting

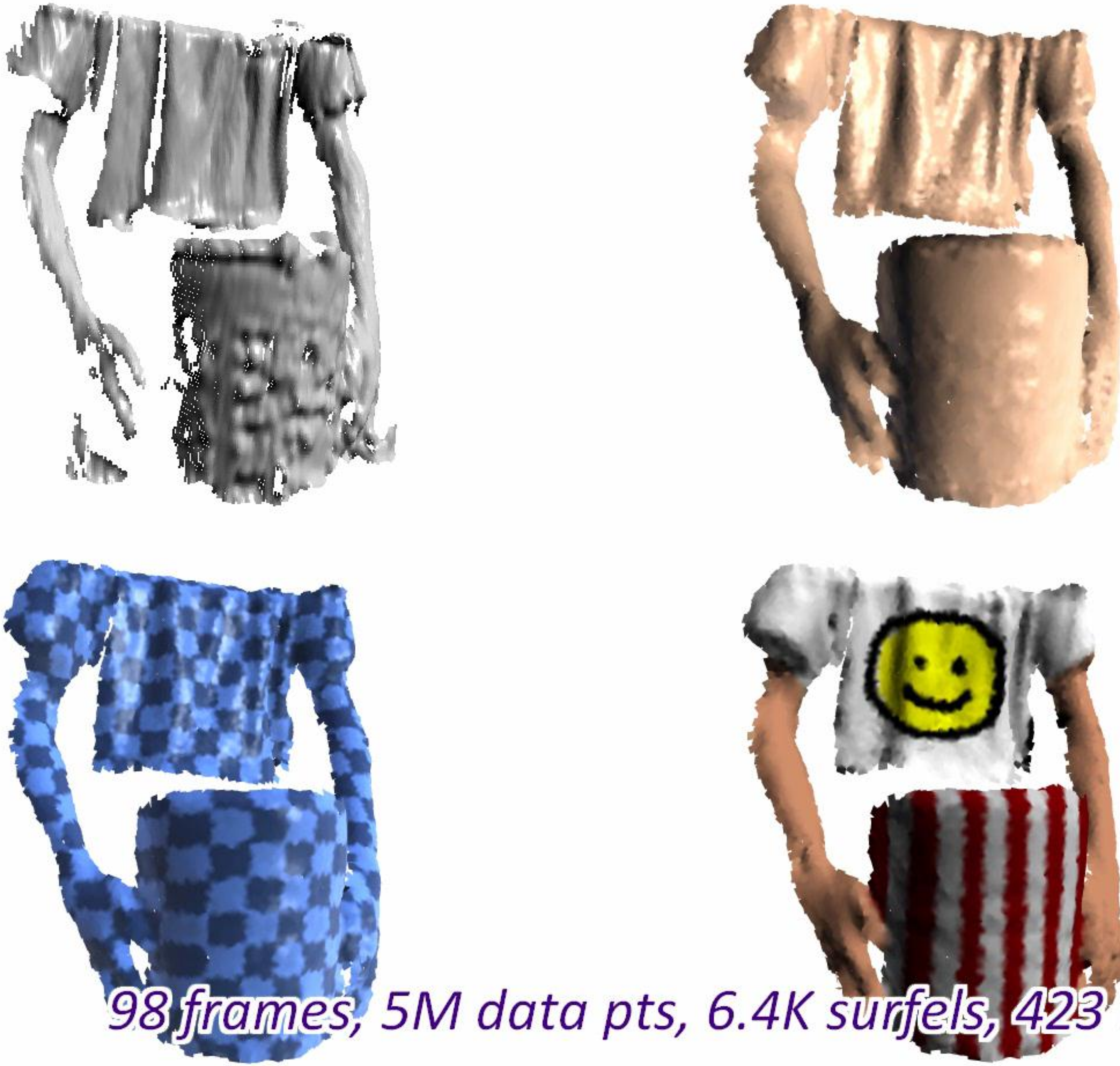
- Smooth surface
- Fitting to noisy data



Results

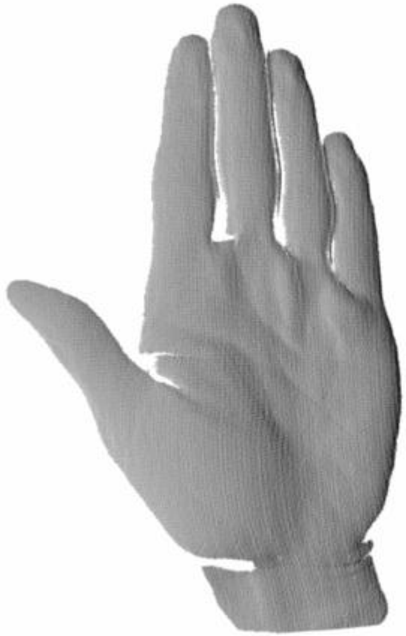


79 frames, 24M data pts, 21K surfels, 315 nodes



98 frames, 5M data pts, 6.4K surfels, 423 nodes

120 frames,
30M data pts,
17K surfels,
1,939 nodes



34 frames,
4M data pts,
23K surfels,
414 nodes