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Analyzing Fracture Patterns in Theran Wall Paintings

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Abstract

In this paper, we analyze the fracture patterns observed in wall paintings excavated from Akrotiri, a Bronze Age Aegean city destroyed by earthquakes preceding a volcanic eruption on Thera (modern Santorini) around 1630 BC. We use interactive programs to trace detailed fragment boundaries in images of manually reconstructed wall paintings. Then, we use geometric analysis algorithms to study the shapes and contacts of those fragment boundaries, producing statistical distributions of lengths, angles, areas, and adjacencies found in assembled paintings. The result is a statistical model that suggests a hierarchical fracture pattern, where fragments break into two pieces recursively along cracks nearly orthogonal to previous ones. This model could be useful for predicting fracture patterns of other wall paintings and/or for guiding future computer-assisted reconstruction algorithms.

1. Introduction

Reconstruction of fractured ancient artifacts such as frescoes, pots, statues, and tablets is important because it helps archeologists make inferences about past civilizations and cultures. Unfortunately, reconstruction is usually a painstakingly labor-intensive job which may take several months or even years to complete by hand if the number of fragments is very large.

To overcome this problem, several computer scientists have worked on automated reconstruction systems that acquire photographs and/or laser scans of fragments and then use computer algorithms to assemble them [WC08]. Example projects of this type include Stitch [CWAB01] and Forma Urbis Romae [KL06]. They typically use combinatorial algorithms to search for the arrangement of fragments that optimizes a scoring function, usually designed heuristically based on the compatibility of properties in adjacent fragments. While they have demonstrated some success, they still are not able to assemble complete artifacts from many fragments automatically [WC08].

In this paper, we utilize analysis of previously reconstructed wall paintings to learn statistics of correct fragment arrangements. Our goal is to gather data that can be used to characterize the arrangements of fragments typically found in reconstructions so that more principled scoring functions can be developed and generative models of crack formation can be evaluated. We use interactive programs to trace detailed fragment boundaries in images of manually reconstructed wall paintings. Then, we use geometric analysis algorithms to study the shapes and contacts of those fragment boundaries, producing statistical distributions of lengths, angles, areas, and adjacencies found in assembled arrangements of fragments.

We believe that these statistics reveal valuable information that could guide future scoring functions and/or generative models. Loosely speaking, we find that: 1) fragments tend to be nearly convex polygons with 3-8 sides, 2) the distribution of fragment areas roughly follows an exponential distribution, 3) "edges" between two adjacent fragments tend to be nearly straight, and 4) "junctions" most often appear with three fragments coming together in a T-junction.

We believe that these observations support the hypothesis that the cracks formed as the result of a hierarchical process, where fragments were broken recursively into two subfragments along nearly straight cracks nearly orthogonal to previous cracks. Investigating this hypothesis with statistical analysis of continuous-valued properties of reconstructed wall paintings is the most novel contribution of our work.

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2. Related Work

There has been a long history of work on computer-aided reconstruction of fractured objects in archeology [KDS09, WC08]. Most previous work has focused on finding pairwise matches between adjacent fragments by aligning patterns in their surface colors [PPR*08], polygonal boundaries [PPE02, dGLS02], normal maps [TFBW*10], and/or fractured edges [BTFN*08, HFG*06, PK03]. These methods have been successful in cases where the fragments have highly distinctive features [HFG*06], the reconstructed objects are surfaces of revolution [KS04, KS08, WC06]), and/or when domain-specific features can be used to identify potential matches [KL06]. However, they have not been able to automatically reconstruct archoelogical artifacts with a multitude of flat, partially eroded fragments [WC08].

There are two significant problems. First, it is intractible to exhaustively search the space of potential fragment alignments ($\Theta(N!)$ for N fragments) [DD07]. Second, it is difficult to devise a "scoring function" that effectively discriminates correct alignments from incorrect ones. As a result, most prior reconstruction algorithms have employed heuristics to prune the search space based on expected relationships between adjacent fragments. For example, in the domain of jigsaw puzzle solving [FG64], the boundary of each piece can be partitioned robustly into discrete features representing "tabs," "indents," "corners," and "border edges," and only certain types of arrangements are possible when joining those features (e.g., corners abut with corners, tabs align with indents, border edges continue across several pieces, etc.). Thus, it is possible to prune the space of potential matches significantly, and puzzles with up to 200 pieces can be solved [GMB04]. Our goal is to generalize these methods to the domain of wall painting reconstruction, where relationships between adjacent fragments are not so clear-cut and thus must be described statistically.

Statistical analysis of fracture patterns has a long history in mechanics [Gri21], geology [Clo55], forensics [MMR06], paleontology [BRY09], and several other fields. In 1962, Lachenbruch classified fracture patterns as either orthogonal (e.g., hierarchical) or non-orthogonal (e.g., hexagonal) [Lac62]. He observed that junctions are often at right angles in orthogonal structures. In 1968, Rats proposed a "rule of identical areas" suggesting that stone blocks break into nearly equally-sized pieces until a minimum block size is reached and observed that orthogonal fracture patterns have predominantly four-sided fragments [Rat68]. Mulheran observed that crack networks in thin films are statistically self-similar, suggesting that fragment areas have an exponential distribuiton. Other specialists have studied hierarchical fracture patterns in ceramics [KMM98], chalk [Caw77], clay [TS08], polymer coatings [Han02], mud [BPC05], and other materials.

Within archeology, the most closely related work is by McBride et al. [MK03], who observed that the vast majority of junctions appear where three fragments join in a "T" junction (70-89%) and that discernable corners in fragment contours usually align in pairwise fragment matches (77-78% have at least one corner aligned). They utilized these observations in a reconstruction algorithm that considered only pairwise matches that align corners, demonstrating results for test data sets with 13-25 fragments. While this work takes a significant first step in the direction of our paper, it provides only very coarse statistics (i.e., counts of the number of junctions falling into certain pre-defined cases). The authors do not trace the boundaries of fragments and/or gather continuous-valued descriptions of how fragments are arranged (e.g., areas, lengths, angles, etc.), and they do not advocate a specific model of the fracture process, and thus their statistical analysis and range of applications are not as general as ours.

3. Methods

The main contribution of our work is a statistical model for the crack pattern of a fractured wall painting. Starting from a high-resolution image of a manually reconstructed fresco, we use an interactive program to trace contours around every fragment in the image and then gather statistical distributions of spatial properties that characterize the observed arrangement of fragments. These statistics form a data-driven probabilistic model for the fracture pattern.

Our test case is a wall painting called "Crocus Gatherer and Potnia" (top left of Figure 1), which was recovered from the Xeste 3 building at the archeological site of Akrotiri, Thera. This was a Late Bronze Age city destroyed circa 1630 B.C. by earthquakes preceding a volcanic eruption. The city, well preserved by volcanic ash, has been the site of an excavation since 1967, and dozens of wall paintings have been recovered and are being reassembled [Dou92].

The wall paintings were constructed on an interior wall that was covered first by mud and straw, then by a layer of lime plaster roughly 1 cm thick. The designs, which include both *buon fresco* and *fresco secco* areas, were applied to a final thin layer of fine plaster [Dou92]. The wall paintings are excavated in thousands of small fragments, cleaned and conserved, then manually reassembled by skilled curators over the course of many years. The "Potnia" measures 3.2 m in width and 2.3 m in height, and we work with a high-resolution image produced by stiching together a large number of digital photographs [Pap09].

Our goal is to develop a processing pipeline to analyze the fracture patterns in this and other wall paintings. The following two subsections describe the main steps of this processing pipeline: contour tracing and contour analysis. Potential applications are discussed in Section 4, and Section 5 provides a summary of our findings and topics for future work.

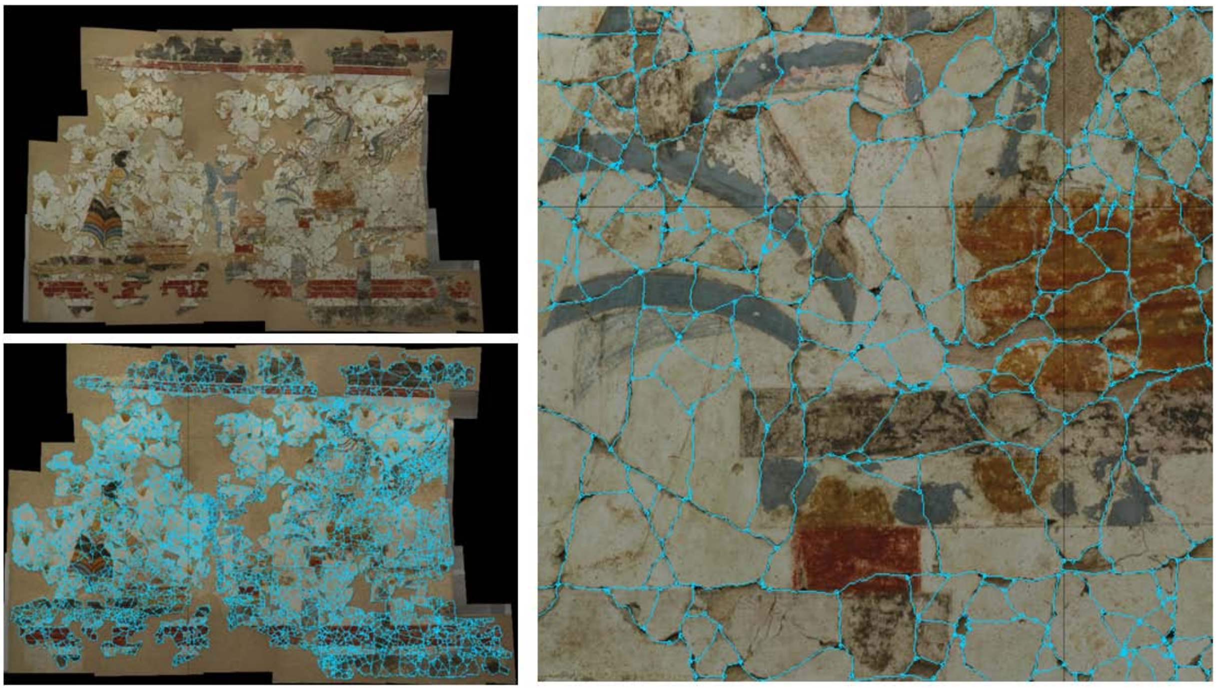


Figure 1: High-resolution color image of a reconstructed Theran wall painting called "Crocus Gatherer and Potnia" (republished with permission from Figs 122-128 of "The Wall Paintings of Thera" [Dou92]). The painting (3.22 x 2.30 meters) was photographed at 80 pixels per centimeter by George Papandreou [Pap09] (top left), and then every fragment was outlined with a contour tracing program to yield a polygonal mesh representation (bottom left) with 4,147 fragments, including 156 gaps, 10,994 edges (blue lines), and 4,921 junctions. A zoomed view of the polygonal mesh is shown on the right.

3.1. Contour Tracing

Beginning with a high-resolution color image of a manually reconstructed fresco (top left of Figure 1), our first goal is to trace the contour outlining the perimeter of every fragment (bottom left of Figure 1). This goal is challenging because a single fresco may have thousands of fragments and the cracks between those fragments may form a complex network of contours, requiring millions of points to capture the paths of all cracks accurately (Figure 1).

Ideally, we could write a computer program that would extract fragment contours from a color image automatically. However, cracks are difficult to detect robustly, especially in painted regions where crack patterns are inter-mixed with color patterns [BH03]. As a result, it is difficult for a computer to discover the topology and gross placement of contours completely automatically, as there are many situations in which the global structure of the fracture pattern must be understood in order to determine the correct placement and connectivity of junctions (a task that people are very good at). Alternatively, we could ask a person to trace every contour with an interactive tracing program. However, it would be tedious and error-prone for a person to trace the path of every crack accurately (at pixel precision), since cracks typically have many "wiggles" that would be difficult to trace interactively (a task that computers are good at). So, we take a hybrid approach.

We have implemented a semi-automatic program to trace fragment boundary contours inspired by the "intelligent scissoring" approach of Mortensen and Barrett [MB95]. The user begins by clicking on a junction or other point of interest. Then, as she moves the mouse away from the clicked "anchor" point, the computer interactively displays the optimal computed path between the anchor and the current cursor position. The user may click to place another anchor point, freezing the current curve, or click on a previously-placed curve or anchor, joining the curve to a newly-created junction.

The optimal path minimizes an energy function designed to snap to cracks in the image. Because the cracks appear darker than the surrounding plaster, we define the energy function $\mathscr{E}$ at each pixel to be smaller when the pixel's intensity is lower than its neighbors':

$$\mathscr{E} = \left[1 + I(p) - \max_{q \in \mathscr{N}(p)} I(q)\right]^n. \tag{1}$$

The exponent n is used to determine the strength of snapping: lower values favor short paths, while higher ones allow the path to deviate more from a straight line in order to follow cracks. (We found that $n = 8$ provides a reasonable tradeoff.) Dijkstra's algorithm is used to efficiently find shortest paths at run time.

The bottom-left and rightmost images in Figure 1 show screenshots of the program after the contour tracing process is complete (i.e., after approximately 20 hours of user input). The result is a polygonal mesh covering the image, where each polygon represents a fragment or gap (marked by the user), each edge represents a sequence of points along the boundary between two fragments (shown in blue), and each vertex represents a junction at a position where multiple edges/fragments meet. Since junctions and edges are shared between fragments, topological (adjacencies) and geometric properties (e.g., angles, lengths, and areas) are easily computed from the polygonal mesh.

3.2. Contour Analysis

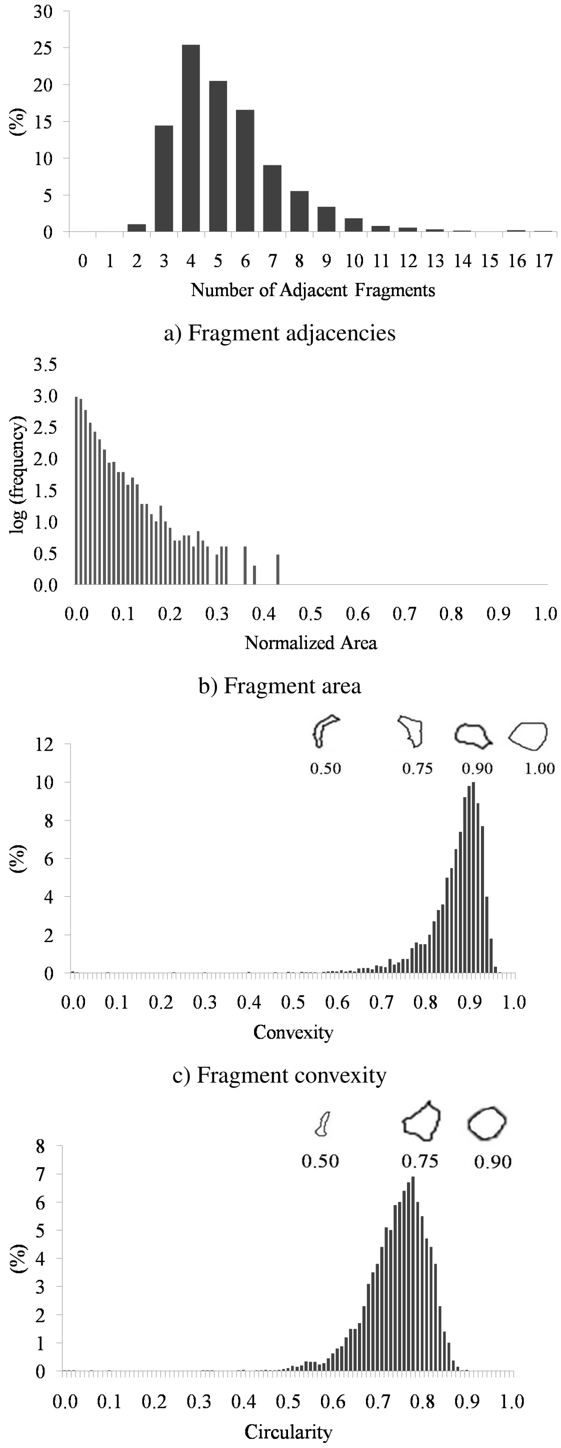
Once we have a polygonal mesh output by the contour tracing program, we analyze its geometric and topological properties with the goal of building a statistical model of its crack pattern. Specifically, we investigate properties of fragments (adjacencies, area, convexity, and circularity), edges (lengths, angles, straightness, and corner types), and junctions (adjacencies, angles, and corner types). We present statistics gathered during our analysis in this section and discuss possible applications in the next.

Fragment Adjacency: Figure 2a shows a histogram of the number of fragments adjacent to each "interior" fragment (ones that do not share an edge with a gap). We observe that most fragments are adjacent to 3-8 other fragments, and the mode is four. However, there are a few small fragments surrounded by two (concave) fragments and a few large fragments adjacent to more than ten.

Fragment Area: Figure 2b shows a histogram of fragment areas (normalized to lie between 0 and 1 by dividing by the area of the largest fragment) plotted on a log scale. The observed distribution seems to roughly follow an exponential distribution with many small fragments and few large ones.

Fragment Convexity: Figure 2c shows a histogram of fragment "convexity" (computed as the area of the fragment divided by the area of its convex hull), with representative examples for four different convexity values inset along the top of the histogram. This distribution indicates that the vast majority of fragments are almost, but not perfectly, convex. We observe that deviations from perfect convexity are mainly due to small concavities that form along edges that are not completely straight (rather than a few deep concavities amongst perfectly straight edges), as shown in the inset examples.

Fragment Circularity: Figure 2d shows a histogram of fragment "circularity" (computed as



a) Fragment adjacencies

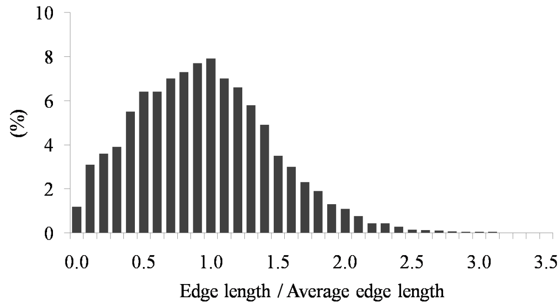
b) Fragment area

c) Fragment convexity

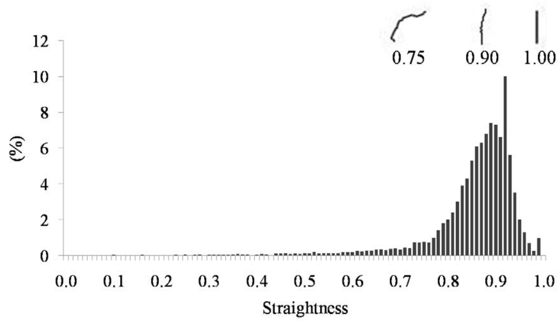
d) Fragment circularity

Figure 2: Histograms of fragment properties.

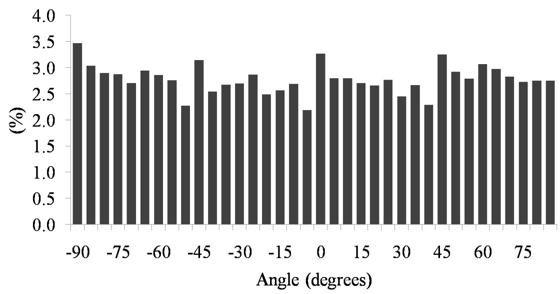
$\sqrt{4 * \pi * area} / perimeter)$, with representative examples for three different circularity values. The circularity measure provides a value between zero (line segment) and one (perfect circle) indicating how "round" the fragment is. From the histogram, we observe that fragments rarely have extremely elongated or almost perfectly circular shapes (like the examples shown on the left and right), but rather tend to have circularities like a nearly regular 4-6 sided polygon (like the example shown in the middle).



a) Edge length



b) Edge straightness



c) Edge orientation

Figure 3: Histograms of edge properties.

Edge Length: Figure 3a shows a histogram of edge lengths (normalized by the average edge length of its fragment). This distribution shows a peak near the value one, which shows that most edges of the same fragment have approximately the same lengths (note that all edges of a perfectly regular polygon would provide a value of one).

Edge Straightness: Figure 3b shows a histogram of edge "straightness" (computed by dividing the length of the straight line segment between the two adjacent junctions by the length of the path following the sequence of points along the edge). This straightness measure has values between zero (loop) and one (line segment) indicating how much the edge path deviates from a straight line. We see that the majority of edges are almost straight (65% have straightness above 0.9). However, some edges are highly curved (e.g., straightness below 0.5), probably due to inhomogeneities in the wall materials.

Edge Orientation: Figure 3c shows a histogram of edge "orientation" (the counter-clockwise angle in degrees between an edge's vector $\vec{v}$ and the positive X axis, where the edge's vector $\vec{v}$ spans the positions of the two adjacent junctions). Note that this distribution shows no strong preference for any particular crack direction (small peaks at multiples of 45 degrees are probably due to discretization of measurements on the pixel image).

Edge Type: Figure 4 shows the distribution of edge types defined by McBride et al. [MK03]. In 14.9% of the edges (type 1), the contours of both adjacent fragments have a corner (bend by more than 45 degrees from perfectly straight) at both of its adjacent junctions. In 44.3% of edges (type 2), both adjacent fragments have a corner in only one of its adjacent junctions. The remaining 40.8% (type 3) have no corners matching at its junctions. These results are roughly similar to ones reported by McBride et al. [MK03].

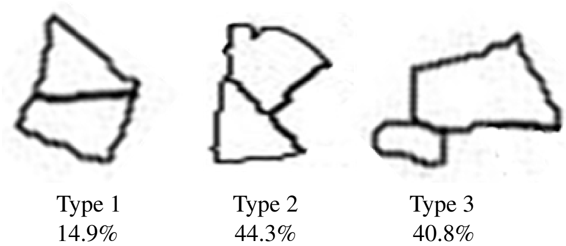
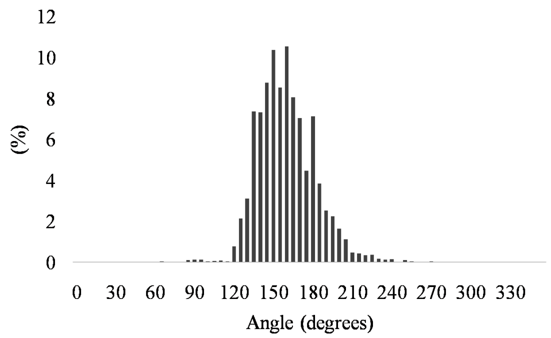
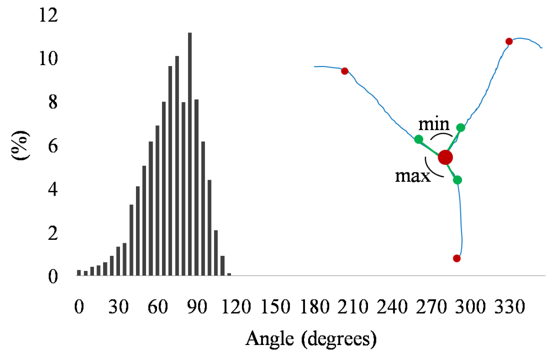


Figure 4: Frequencies of edge types (as defined in [MK03]).

Junction Angles: Figures 5a-b show distributions of "interior angles" formed by adjacent edges at junctions. For each junction adjacent to K edges (and no gaps), there are K interior angles formed by pairs of adjacent edges (i.e., corner angles of the adjacent fragments). We compute the measure for each of these angles by forming vectors from the junction position to a point 1/3 of the way along the two adjacent edges and then measuring the angle between those vectors (see inset in Figure 5b). We find the minimum and maximum interior angle at each junction and include them into the histograms shown in Figures 5a and 5b, respectively. Note that the distribution of minimum angles is centered around 80-90 degrees, while the maximum angles are centered around 140-160 degrees, with a secondary peak at 180 degrees.



a) Maximum junction angle



b) Minimum junction angle

Figure 5: Histograms of junction angle properties.

Junction Type: Figure 6 shows the distribution of observed junction types. A junction is labeled 'K-way' if it has K adjacent fragments/edges. For 3-way junctions, we call it a '3-way T' if its maximum interior angle is between 135 and 225 degrees (two of its three edges almost form a straight line) and a '3-way Y' otherwise. It is interesting to note that we observe only 3-way, 4-way, and 5-way junctions. Amongst those, the vast majority are 3-way junctions (94%), and most of them are '3-way T's (76%). These results are roughly consistent with those of McBride et al. [MK03].

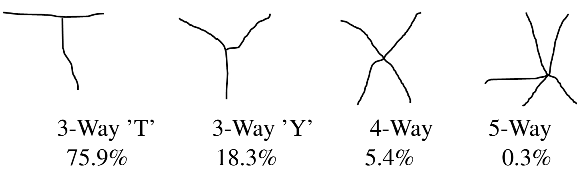


Figure 6: Frequencies of junction types.

4. Applications

We believe that the statistics presented in the previous section are useful for multiple applications, including prediction of fracture processes and reconstruction of wall paintings from fragments. While space limitations prevent us from investigating these applications in depth, we discuss how they might be approached using the statistics produced by our system.

4.1. Failure Analysis

One potential application is trying to understand the process by which wall paintings fractured by examination of their fragments. This application is interesting not only from an academic standpoint, but also for forensics (to describe how a particular wall fractured) and archeology (the fracture process may reveal information about what materials supported the wall, whether the wall fell in an earthquake or was destroyed by a sledgehammer, etc.).

Based on the statistics gathered from the "Crocus Gatherer and Potnia," we conjecture that it was fractured by a sequential, hierarchical process, where brittle fragments broke recursively into two nearly equal size pieces along cracks nearly orthogonal to previous ones. This conjecture is supported by the following data:

- The predominance of 3-way 'T' junctions suggest a sequential fracture process where subsequent cracks appear orthogonal to previous ones (the direction that optimally relieves load is orthogonal to the fragment boundary) [Lac62].
- The exponential distribution of fragment areas (Figure 2b) suggests a hierarchical fracture process where fragments are broken recursively into statistically self-similar patterns. Similar distributions of fragment areas has been observed in hierarchical fracture processes for mud [TS08], ceramics [KMM98], thin films [Mul93], and polymer coatings [Han02].
- The distributions of fragment adjacency (Figure 2a), fragment convexity (Figure 2c), and edge straightness (Figure 3b) suggest that most fragment boundaries can be well-approximated by a convex polygon with a small number of sides (3-8). The predominance of four-sided fragments has been observed in hierarchical fracture processes for stone [Rat68, TS08, Tsy00] and desicated gels [BDC05].
- The distributions of edge lengths (Figure 3a) and fragment area, convexity, and circularity (Figure 2b-d) suggest that hierarchical cracks tend to split fragments into subfragments with nearly equal areas. To produce a hierarchical sequence of nearly convex and regular polygons, cracks tend to form across the shortest dimension of each fragment, splitting it into two with almost equal lengh sides and high circularity. This observation is supported by the "rule of identical areas," which was previously proposed for the fracture of stone [Rat68].

The combination of these observations is consistent with a orthogonal fracture process, as described in [Lac62]. This fracture model has been observed with time-lapsed photography in other domains (e.g., desication of clay [TS08]). Our contribution is mainly to argue for it statistically as a possible model for fracture of the Theran wall paintings.

4.2. Reconstruction

A second application is computer-assisted reconstruction of objects from fragments. Much research has been devoted to this problem over the last few decades [WC08], but to our knowledge none has explicitly utilized a statistical model of a fracture process to guide reconstruction.

Most systems use combinatorial algorithms to search for arrangements of fragments that optimize a scoring function measuring the compatibility of properties in adjacent fragments. For example, Brown et al. [BTFN*08] built a system for reconstruction of Theran wall paintings that used laser scanners to acquire 3D surface geometry for every fragment, exhaustive search algorithms to test pairwise alignments of fragments at regularly spaced contact points and a function based on distances between aligned surface points to score potential matches. The system does not explicitly consider properties of junctions or clusters of more than two fragments.

We conjecture that it is possible to improve both search algorithms and scoring functions by considering the process by which artifacts were fractured. In particular, if the Theran wall paintings were indeed broken by the hierarchical process proposed in the previous subsection, then that process could be reversed by a bottom-up reconstruction algorithm. Since hierarchical fracture processes produce self-similar patterns at every scale, correctly arranged clusters of fragments should have the same statistical properties as those observed for the original fragments. So, an algorithm could utlize the statistical distributions reported in Section 3.2 to design a scoring function that guides a bottom-up search algorithm.

For example, rather than exhaustively searching all pairwise alignments of fragments [BTFN*08], it would be possible to consider only matches where corners align (as in [MK03]) and then score them probabilistically based on the statistical properties of the edge, junctions, and merged fragments. For the Theran wall painting studied in this paper, highest probability matches produce edges that are almost straight, junction angles that sum to approximately 180 degrees, and merged fragments that are almost convex, highly circular, and have 3-8 approximately straight edges (of these criteria, only edge straightness has been considered as a matching criterion by previous reconstruction systems [KK01, MK03]). We believe that these criteria could be incorporated into a scoring function for reconstruction of other Theran wall paintings and possibly other archeological artifacts to improve efficiency and accuracy. A study currently being performed to validate this hypothesis has shown promising initial results.

5. Conclusion and Future Work

In this paper, we have described methods to analyze the fracture patterns observed in a manually reconstructed wall paintings. We describe an interactive program to trace detailed fragment boundaries in images and geometric analysis algorithms to produce statistical distributions of lengths, angles, areas, and adjacencies found in the traced fragment arrangements. The result is a statistical model that suggests a hierarchical fracture pattern, where fragments break recursively into two nearly equal size pieces along cracks nearly orthogonal to previous ones. We believe that this model could be useful for understanding fracture mechanisms and/or for guiding future computer-assisted reconstruction algorithms.

While this study takes a small step, our approach has several limitations and there are many avenues for future research. As a next step, future work should perform analysis of many different wall paintings and ideally many different artifact types. Although our data set has over 4,000 fragments, they are all from the same wall painting, and so it may not be representative of other wall paintings or other artifacts made from different materials. We plan to gather more data and to make our tools publicly available so that analysis and comparison of different archeological artifacts can be performed in the future.

A second interesting topic for future work is to develop simulations and mathematical models of hierarchical fracture processes and then compare the statistics predicted by those simulations/models with the ones observed in this paper. This approach is taken in previous work in other application domains [DGA05, IO09, Mou05], but it would be interesting to investigate (parameters for) a model of hierarchical fracture of brittle materials to match the patterns observed in archeology. Moreover, the proposed hierarchical fracture model could be confirmed with high speed video, as was done for fracture under biaxial loading in [Ols07].

Finally, further research is required to understand exactly how to utilize the observed statistics and proposed fracture model presented in this paper to guide a reconstruction algorithm. Several ideas of how to do this are discussed in Section 4.2, but exploring them fully is still a topic for future work.

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