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A Technology Cocktail for a 3D Photo-Realistic Model of a I Century Roman Fountain: Range Scanning, RTI and Physically Based Rendering

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Abstract

Archaeology is a science that uses multiple disciplines to ensure the validity of archaeological data and to accurately support theoretical foundations. Archaeology uses computer graphics to create credible virtual representations of archaeological sites and artifacts, which are used for interpretation, research purposes and archiving.
Accurate virtual representations of an archaeological site rely on a balanced mixture of techniques, and some are discussed in this communication aplied to a I century roman fountain. Different imaging techniques were integrated - point cloud from a total station, 3D mid-range laser scanning and reflectance transformation imaging (RTI) - where their best features are extracted and combined to get the most accurate data in different scales. A point cloud enables a fair representation of the entire site (macro-scale), including the surrounding environment features. A 3D mid-range scanning captures the surface detail (medium-scale), but it may not be reliable to capture microstructure elements. An RTI technique (such as polynomial texture mapping, PTM) may overcome this limitation, since it can capture the microstructure elements through the surface reflectance properties. A physically based rendering technique (such as the one used by PBRT) can contribute to produce an improved view of an archaeological site, from a virtual 3D model.
A collection of open-source software tools is under development to efficiently merge these techniques. This integrated set aims to provide a smooth integration of data gathered on the field, while adequately documenting all processing steps for archival purposes. Obtained results so far are promising and suggest some paths for improvements.

Categories and Subject Descriptors (according to ACM CCS): çScatI.3.5Computer GraphicsPhysically based modeling

1. Introduction

The study of an archaeology site requires broad knowledge and competences, including history, physical sciences and engineering techniques. An accurate digital representation of these sites facilitates their remote access to a wider number of scholars and experts, promoting an open and reliable discussion over unbiased views of the available site data.

This communication presents an affordable engineering process pipeline that apply image based techniques to create scientifically robust digital surrogates of small to medium size artifacts with low volumetry, namely façades with bas-reliefs and epigraphs. The granite Idol Fountain was selected to illustrate and discuss some trade-offs on the application and integration of these imaging techniques.

The Idol Fountain, in Braga, Portugal, is a roman religious building from the I century AD which has been discovered in the late XVII century and has been studied since then. The monument has a 6m x 2,20m granite façade with two sculptures and several epigraphs, taking an overall area of about 78 m^2 (13m x 6m). Fig 1 gives an overview of the Idol Fountain in the current museum enclosure, during one of the image capture sessions, while Fig 2 is a snapshot of a simplified 3D model reconstructed with mixed image technologies. The rectangle window drawn on the image selects a detail of the façade that will be used below to discuss the processing pipeline and illustrate some results.

Figure 1: *The Idol Fountain during a capture session.*

Figure 2: *A simplified reconstructed 3D model of the Idol Fountain.*

The various interpretations of the fountian's epigraphs have not been always consensual [EMM08]. To study the epigraphs scholars physically went to the fountain's natural location, to measure the size of the characters and to apply and document multiple (biased) views, some requiring hard to place raking lights. Most of these views were not fully logged and documented, and in some case images were adjusted and/or changed with no record on the applied modifications to the original view. Since current imaging software lets a user modify image parameters - and very often these are not registered - most scientific discussions are based on weak evidence from biased and tampered empirical provenance data.

Scientifically robust digital surrogates, based on empirical provenance such as digital photography or laser scanning, can contribute to improve the discussion and the outcome quality in the interpretation of archaeology sites, providing a common framework for different scholars to study a site or artifact and compare their results using common premises.

This communication gives a concise view on digital surrogates in Section 2. Section 3 addresses the geometry and texture data acquisition components to build a digital model of a site and Section 4 completes the pipeline process integrating the textured data into the geometry model to get to the final site scene description. Section 5 evaluates and discusses the cocktail of techniques that produced sets of different images, each displaying advantage features and limitations over the competitive alternatives. The concluding section presents a critical view of the overall process and the trade-offs between costs and quality of the results.

2. Digital Surrogates

Scientifically robust digital surrogates must be built in a credible and accessible way and must provide a set of useful features from the end user perspective.

Credibility can be achieved through an adequate pipeline of open, proven, reliable, correct and fully documented processes; these issues are the guidelines for our current projects in the cultural heritage area, but these are not the aim of this communication and they will not be fully covered here.

To mass build scientifically robust and long lasting digital surrogates of an archaeology site requires tools that are open (non-proprietary), affordable (in instrumentation and human resources) and easy to use. The literature describe some of these modelling pipelines for archaeological visualization (e.g., [AFT*04]), but most do not satisfy all these requirements. We present and discuss yet another process pipeline, where new code was developed to merge with open software products and to also display photo-realistic views of a site scene. Soon the complete pipeline will be fully open and complying with international standards, such as OASIS [OAS] or CIDOC-CRM [ICM], for long term digital preservation of cultural heritage surrogates.

Usefulness from the point of view of an archaeologist require some or all these features:

- the overall context of the site should be modelled and visualized;
- the model geometry and texture, and its visualization should follow rules that are physically correct;
- the end user should be able to apply to the digital model different light sources and/or light source positions, and it should be correctly seen from different viewpoints;
- users should be able to accurately analyse a photo-realistic digital model at several levels of detail.

3. Data Acquisition

To fully model the overall site context, precise geometry can be obtained through 3D scanning instrumentation, which may range from moderate cost optical instruments with reduced mesh density, such as a total station, to higher resolution laser scanning devices. Both allow the reconstruction of a 3D geometric model from a point cloud and adequate software tools. A short overview of these tecnhiques and how they can be mixed, simplified and integrated opens the section below on 3D Geometry Acquisition.

To reconstruct the textured appearance of an artifact surface with photo realistic properties, two main features are required: (i) the reflectance properties of the surface should react both with a change on the light source(s) positioning and with the viewpoint, and (ii) the rendering process to get the final scene should be computed following physically correct light propagation models (global illumination). The Reflectance Transformation Imaging (RTI) technique is a photo-based approach to capture surface reflectance properties, which lets the user visualize how the model of an artifact behaves under changing light conditions. A concise introduction to the RTI technique and how it can currently be captured into a Polynomial Texture Map format (PTM) is further detailed in the section below on Texture Acquisition.

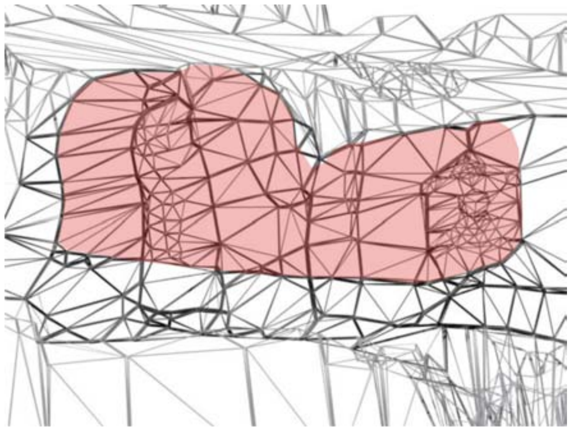

Figure 3: *A partial wire frame view of the reconstructed fountain with a total station.*

3.1. 3D Geometric Acquisition

To build a 3D model of a site, geometric data can be acquired using different techniques. In the current archaeological survey of the Idol Fountain, a total station was used to get an geo-referenced, but less detailed, 3D mesh of the granite monolith that surrounds the fountain; to obtain an accurate and very detailed 3D model of the sculptured façade of the monument a 3D laser scanner was selected.

The total station is an optical electronic device, used mainly for surveying purposes, that combines the ability to measure a position horizontally and vertically at the same time. It was developed mainly for civil engineering as an alternative to the traditional theodolite. Soon archaeologists realised their utility for archaeological fieldwork, where the total station is mainly used to impose a grid on the site to locate every artefact/feature in 3D space and to make topographic maps of the site [AFT*04].

In the Idol Fountain the total station was used to incorporate the surveyed data in the modelling pipeline: the sparse 3D point cloud registered 1640 point coordinates over 25 different layers, and it took approximately 2 working days. The point coordinates were stored in different layers to enable a more precise volumetric reconstruction of the fountain site.

The equipment used in the survey was a *Nikon Total Station DTM 310*; the logged 3D points were imported into *progeCAD 2008 Smart* for pre-processing and later into *Blender 2.46* to define the 3D triangle mesh; the former is a free software version and the latter is open source. Fig 3 is a partial wire frame view of the reconstructed fountain, based on this point cloud.

The laser scanner technique analyses a real-world object or environment to collect data on its shape, relative location and, in some cases, colour. The surveyed data can then be used to create extremely accurate digital 3D models, for a wide range of application fields, such as medicine, reverse engineering or cultural heritage. Laser scanning instruments project a laser dot or line beam onto the part surfaces, while cameras continuously triangulate the changing distance and profile of the laser beam. The image of the beam is then translated into 3D coordinates.

Different scanners serve different purposes. For the Idol Fountain a hand-held portable 3D laser scanner was used, which allowed to obtain more accurate 3D coordinates from partially occluded surfaces, and in a faster and practical way. Less then 3 hours were required to manually capture all fountain façade, a very dense point cloud with 1,137,721 3D points and 2,263,090 faces.

To perform the data acquisition of the monuments' sculptured granite façade the team used the *ZScanner 700* portable laser scanner from *Z Corporation*, with a 0.1 mm z-axis resolution and an accuracy up to 0.05 mm. The *ZScanner 700* uses high-speed CCD camera units and a laser projection unit to triangulate its position from a series of targets randomly arranged on the objects.

The point cloud from a 3D scanner in medium to large size objects is seldom suitable for modelling purposes, due to the huge point cloud density. Several mesh simplification algorithms have been surveyed in the literature [Lue01]. To simplify the triangle mesh of the Idol Fountain's façade the team adopted the Normal-based Simplification Algorithm (NSA) [Sil07]. The NSA is a very fast edge collapsing-based simplification algorithm for polygonal models that achieves a good compromise between time performance and mesh quality. In some cases, the visual quality of simplified CAD models with planar zones using the NSA-algorithm is better than the one created by other algorithms. Indeed, for archaeological data virtual representation purposes the highly simplified models have an extremely satisfactory visual quality.

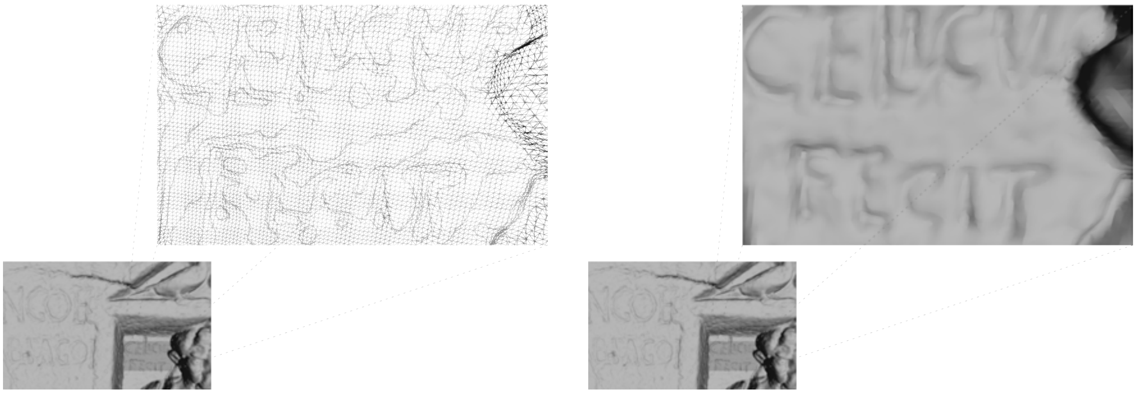

Figure 4: *Detail of a 3D scanner reconstruction without mesh simplification: the wire frame view and the surface model*

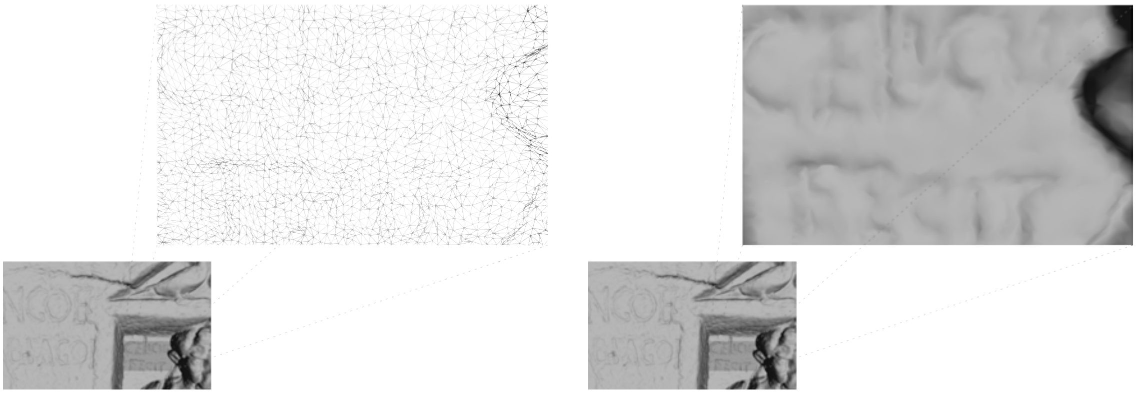

Figure 5: *Detail of a 3D scanner reconstruction with NSA mesh simplification: the wire frame view and the surface model*

In the fountain's façade the mesh complexity was reduced to 22% of its original size, while maintaining an acceptable visual quality. Fig 4 shows a reconstructed detail of the façade, both the wire frame and the surface model view, before the simplification, while Fig 5 displays the same view after applying the NSA simplified algorithm.

After simplifying the façade model, the 3D model of the fountain site is obtained by merging the two sets of point clouds, that were previously captured with common GPS reference points, to help the alignment procedure. The process pipeline to get an untextured 3D model of the Idol Fountain is represented in Fig 6.

3.2. Texture Capture

Archaeology sites are not only rich in 3D geometric data but also in texture (including colour). Current data acquisition process pipelines can capture some of the needed texture data together with the range data, although the most common approach is simply to add a synthetic colour as texture (as done to reconstruct the overall view of the Idol Fountain in Fig 2) or, when a more realistic view is required, a sample photograph (a micro-texture) is replicated throughout the whole scene. However, these textures generally fail to capture mesostructure data from the surface.

One way to overcome these limitations is to capture reflectance information from the surface using methods provided by techniques such as RTI. The RTI goal is to empirically recover the BRDF (Bidirectional Reflectance Distribution Function) of the target surface through the analysis of a set of object images taken under varying light directions.

The Idol Fountain is a granite monolith that can not be moved or rearranged, creating difficulties to apply most available RTI techniques. However, this can be overcome with a view-dependent approach, where the acquisition only requires images from a single fixed camera position under varying light source positions. To compute the light source direction from each image without further user interaction, glossy sphere(s) can be placed next to the target object and image processing code can estimate these directions from the highlights on the ball(s). This technique, known as HRTI (Highlight based RTI), has been succesfully applied in several capture sessions [MMSL06, BSP07]. Current HRTI out-

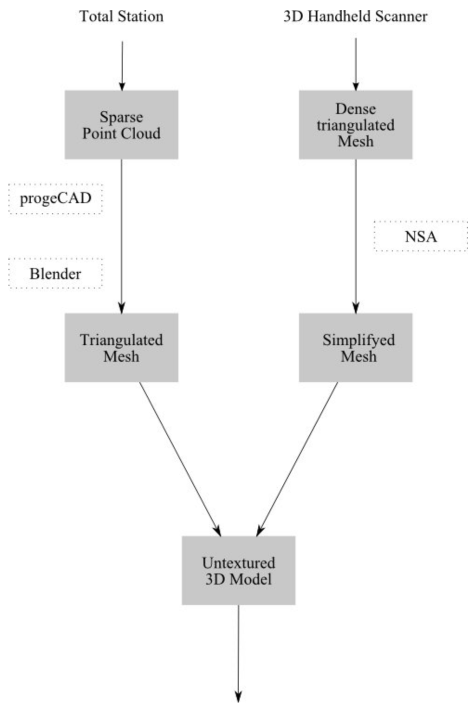

Figure 6: *A flow chart to get an untextured 3D model.*

put is a single PTM file [MGW01] which stores for each pixel the coefficients of a bi-variable polynomial approximation of the BRDF for a specific point of view.

The HRTI data from the fountain façade was captured with a *Nikon D40* reflex camera (6 Mpx), with a 60 mm macro *Nikon* lens and neutral density filters. The light source was a standard 500 W incandescent light projector, to overcome the strength of neutral density filters required to remove the ambient light. Since HRTI computes the light source directions from the highlights on glossy spheres placed next to object, two black snooker balls were placed next to the relevant targets. The second ball is used for triangulation purposes, to compute not only the light source direction but also its spatial location (for upgraded versions of the current PTM file format).

With this setup several scene details were captured to illustrate the critical areas of the façade, each being a collection of 45 images fed into the *PTMbuilder*. The *PTMbuilder* is a software package that automatically produces the final PTM file from a set of captured images [BSP07]. The image on the left in Fig. 7 shows a snapshot from the PTMviewer with a detailed view of the façade region shown in Fig. 1, where the light source direction was specified in the PTM viewer and graphically represented in this figure.

4. Visualization and Rendering Textured 3D Surface Models

A PTM file is a single viewpoint representation of a textured surface, with no geometry underneath. Although it accurately shows how the surface pixels react to changing lights, different viewpoints of the same artifact may present a flat and distorted representation of the object. When rendering a site scene, if a PTM file is used as a texture applied to a flat surface in the rendering process, this distortion is cleared displayed, and this will also occur if the PTM is merged into the 3D surface model (with added alignment or registration complexities). This section gives an overview of the technology pipeline required to get to a rendered textured 3D surface representation of an object. It also presents an alternative approach to replace the conventional image-based sample in texture mapping by one that reacts to the light position while minimizing the PTM distortion when navigating through a site model: the use of PTM micro-textures.

To produce a photo-realistic 3D virtual representation of the whole archaeology site two additional steps are still required: to map the textures in the resulting geometry meshes, and to render the model through a photo-realistic engine.

The use of photo-based micro-textures fails to accurately represent the texture mesostructure, as mentioned before. On the other hand, current view-dependent RTI representations of a larger area (such as the PTM representation of a fountain façade detail) present a distorted and incorrect view of the object when the viewpoint is changed. The visual quality of micro-texture mapping can be considerably improved if the applied micro-texture also reacts to a dynamic variation of the light source: if the sample photograph used in texture mapping is replaced by a PTM-based sample of the surface material (granite, in the fountain façade), the overall scene behaviour in changing light conditions can be enhanced.

Micro-texture mapping does not require the registration process and can be applied through simple projective unwrapping of the resulting 3D meshes, using any available texture mapping tool. Although PTM micro-textures and photo micro-textures share the same constraints due to the viewpoint dependency, the PTM approach does not require additional bump mapping to simulate the mesostructure on the surface, as demonstrated in [MGW01, HoG03].

No current software tool is ready to accept the PTM format as an input texture. However, since PTM files keep the same size as the original photographic images, the texture is mapped using one of the original images and is later replaced by the corresponding PTM file. The open product *3D Blender 2.46* was used to apply a texture to the 3D model, since it offered an user friendly interface for face selections and UV unwrapping. The final result model is stored into a single *Wavefront* OBJ file containing both the geometry 3D data with the corresponding UV mapping and material.

A software tool was developed (the *PTMmeshViewer*) to

Figure 7: *Three snapshots of a rich textured surface (left to right): a) a PTM view, b) a photo micro-textured 3D model, c) a PTM micro-textured 3D model*

Figure 8: *Three snapshots of a façade detail, from a bottom-right viewpoint (left to right): a) a PTM view, b) and c) PBRT rendered views (photo- and PTM-based textured)*

visualize in real-time the final textured model. This tool reads an OBJ file from the previous processing step, replaces the 2D image texture by the corresponding PTM texture, renders the scene in real-time with local illumination, and provides an interactive mode to drive-through the model. The other two images in Fig. 7 show the same façade region rendered with this tool, using these two micro-texture mapping approaches (from left to right): photo-based textured and PTM-based textured.

The site scene, rendered and visualized in real-time by the *PTMmeshViewer*, can be fed to a physically based ray tracer, using a file format such as the PBRT format. *PBRT* (Physically Based Ray Tracer) is an open source ray tracer that generates images with global illumination, producing realistic scenes that are physically correct in terms of light propagation across the scene. A plug-in was developed to let *PBRT* use PTM files as input textures. Fig. 8 show 3 views of the same façade detail, from a viewpoint quite distinct from the original images captured for the RTI representation: the distorted left image is the PTM view, while the other two represent rendered views with global illumination of the two micro-textured approaches, the photo-based and the PTM-based (from left to right).

The processing pipeline to get a digital surrogate of a site and rendered photo-realistic views of the whole or part of the site is now fully described. Fig. 9 below complements the pipeline flow chart of Fig. 6 above.

5. Qualitative Evaluation of Competitive Techniques

Digital surrogates in archaeology must be credible, accessible to build and useful, as earlier stated. The process pipeline here proposed is based on standard well known and proved equipments (and their output formats) and can be built from fully open software products (exception to the PTM fitter, that soon will be replaced). Cost of equipments is affordable for most archaeology teams, the expertise to take full advantage of them can be easily obtained and the required human and computing resources to acquire and process most data is low. The digital representation of an

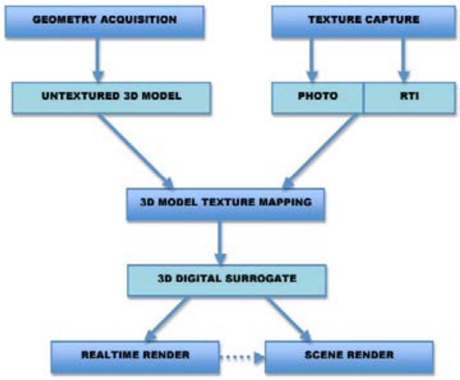

Figure 9: *The full process pipeline flow chart to get and visualize a textured model.*

archaeologic site such as the Idol Fountain is usefull not only for archiving, but also to contribute to its (remote) study - particularly the controversial epigraphs - and to provide its access to wider audiences.

An overall unbiased view of the fountain site can be obtained from the 3D model built out of blended data from range scanning, micro-textured with light direction sensitive photo-based samples. A rendering engine that fully considers the light propagation (e.g., supports global illumination) is able to provide rich set of views based on physically correct parameters, to be displayed as photo-realistic views of the site. The level of detail of such views ranges from the overall façade view (with some simplifications, due to current computing limitations) to fine details of the façade (few square mm). Finer details get blurred due to the 3D geometric mesh simplification, and alternative approaches for visualization can be recommended.

Fig. 7 above illustrates the application of the 3 main competitive techniques to display closer details of the façade. These 3 views have a similar viewpoint to the one used to capture the original photos and the light source was placed during the visualization process on a similar position for all 3 images (coming from the bottom-right). A qualitative comparative evaluation among these rendered views (without global illumination), clearly shows the richer display of details of the RTI approach, when compared to the other 2 with simplified 3D geometric data. This suggests that when the current RTI technique implementation (PTM-based) is upgraded to a multi-viewpoint approach, this technique will become a very attractive complementary approach to study and analyse very fine details of full 3D artifacts. Looking now to the 2 textured 3D geometric meshes (Fig. 7 b and c), the light direction sensitive micro-texture gives a clearer and more realistic view of the carved details on the granite surface, when compared to a conventional photo-based sample texture.

Façade views from multiple viewpoints that are considerably dissimilar to the ones used to capture the images for the RTI capture are clearly disadvantageous for the RTI view: this view is clearly distorted, as expected and as displayed on the left image in Fig. 8 above, which represents the 3 views from the same competing techniques, but from a bottom-right viewpoint. The 2 images on the right were rendered with global illumination, but since no additional light sources were added, the differences are very subtil. The PTM-based micro-texture (the far right image) still displays a clearer and more realistic view of the façade detail, as expected (notice the beter readability of the top left epigraph).

6. Concluding Remarks

The requirement for scientifically robust digital surrogates of archaelogy sites is growing for three main reasons: (i) to provide a common framework for scholars and experts to reliably promote credible scientific discussions and to give remote access of a site data to a wider audience, (ii) to build repositories for long term digital preservation of the human cultural legacy, and (iii) the physical detachment enables the portability of the site allowing a wider audience access through the publication of the digital surrogate. These three goals impose a set of constraints that must be complied to enforce the credibility, accessibility and usefulness of these digital surrogates. This communication mentioned the credibility issues as the team guidelines in cultural heritage projects, and presented an open pipelined software tool to build an adequate digital surrogate of an archaeology site, based on imaging technologies and addressing the accessibility and usefulness issues.

To build the 3D digital model of the archaeology site Idol Fountain, 3 complementary physical instruments and techniques were required:

- a total station with GPS referencing for the overall site context, with open software to convert a point cloud data into a triangle mesh; fast learning, easy to use, but data capture is time consuming;
- a laser scanner for range data of the details, also with GPS referencing (a hand laser scanner is simpler to use and to get data from occluded objects, although it is still an expensive tool), with open software to convert the high density point cloud into the 3D geometry mesh, and adequate mesh simplification to support photo-realistic representations (simplification algorithms should be carefully chosen and biased to the surface texture); in the Idol Fountain, the granite texture may require a more efficient algorithm then the adopted NSA approach;
- a digital camera for texture capture: high quality RTI representations require large sets of images (usually over 40

for each viewpoint), and the lights placement may take some time (with practice this varies from minutes to 1 or 2 hours).

Current open software tools are efficient to generate only one type of RTI file, the proprietary single viewpoint PTM file format; this limits the visualization of PTM files to a single viewpoint (otherwise distortion is perceptible and annoying), and PTM file stitching is not available yet; soon, new products and file formats will be available to overcome these limitations; in the meantime, for multiple viewpoints the PTM format is only used to simulate micro-textures that react to the light source position, with better photo-realism than conventional photo-based micro-textures.

To merge and integrate all geometry and texture data, this process follows separate steps: geometry data, and these with textured data. Registration of the geometry data is simplified when there are control points that were accurately measured using GPS. However, alignment of texture data from a digital camera with geometry data requires additional complex procedures (including camera calibration) and texture misalignments produce virtual artifacts during visualization that removes credibility to the displayed scene. No simple procedures were yet devised to overcome this difficulty, and since there are yet no view-independent RTI files for texture mapping, this path is not worth pursuing at this moment. The approach based on a single PTM-based micro-texture that is replicated through the whole scene is still the most attractive in scenes with the same texture; if different textures are required, then the rules presented in [RCMS99] can be applied. Future replacement of these PTM-based micro-textures by richer RTI-based ones may present an acceptable compromise between cost of acquisition and overall visual quality.

The final stage in this process pipeline is the generation of rendered scenes that allow the user to navigate in the 3D model under mixed and varying light conditions; the user may wish to dynamically modify the scene lighting to better observe relevant details (e.g., raking lights on epigraphs). These wish features require real-time physically based ray tracers, which are not available yet. Current state of the art only allows visualization of separate views (even in HPC clusters), and the user interface needs to be improved to support these interactive operations. A full scene with the simplified mesh of the Idol Fountain façade is still too computationally intensive for the open product PBRT on a typical destop/notebook system. There is room for several improvements in this final pipeline stage.

As an overall comment on this work, the results obtained so far exceed the expectations of the archaeologists that followed the project. Other expert teams will test and validate the pipeline process applied to other archaeology sites, and their feedback will provide useful tips for later versions.

7. Acknowledgements

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