The 8th International Symposium on Virtual Reality, Archaeology and Cultural Heritage
VAST (2007)
D. Arnold, F. Niccolucci, A. Chalmers (Editors)

Shape Capture Assisted by Traditional Tools

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Abstract
We present new techniques for capturing the shape of physical objects using simple tools. From a set of caliper distance measurements between object points, we reconstruct a three dimensional structure. We show that we can refine the model using planar contours obtained with a gage that are placed in three dimensions using the caliper measurements. We demonstrate that the model we construct can be used to assist optical approaches for model capture.

Categories and Subject Descriptors (according to ACM CCS): I.3.8 [Computer Graphics]: Applications

1. Introduction

Shape capture of objects in the field for study in cultural heritage applications can still be difficult. With the wide variety of possible shapes, materials, and capture conditions, there is no “one size fits all" solution for shape capture. A suite of methods documented with their strengths, weaknesses, and range of application is needed by the cultural heritage community. Suitable methods and equipment for acquiring shape in the field (rather than in the laboratory or museum) need to be robust, accurate, easily portable and inexpensive. Three dimensional laser scanners, either triangulation or time-of-flight, can produce detailed, accurate models [Bla04], but are not always practical or inexpensive enough for field studies. Image-based methods using hand held still or video cameras [PVGV*04] have been shown to be a portable, inexpensive alternative to produce excellent results. In this paper we introduce methods using simple traditional, non-electronic, tools that complement inexpensive image-based methods to increase their robustness, document their accuracy, and to fill in some data holes when camera views may be inaccessible.

The traditional tools used in the methods we propose are calipers, contour gages, and sketches on paper. These have long been used in archaeology, architectural documentation, biological morphometrics and related fields. In this work we show how such measurements can be organized to create simple 3D models, in addition to providing linear measurements for validation.

Our goal in this work is not to offer a substitute to image-based methods, or to suggest that researchers should move backward to pre-digital camera techniques. Instead, our goal is to complement new digital techniques by taking advantage of some of the strengths of traditional methods. Specifically, three ways that these methods complement image-based techniques are:

- *Verification of Accuracy and Confidence in Results:* For cases where image-based methods are successful the question remains whether there has been any systematic or human error in applying the method. The methods describe here are also subject to human error, but represent a completely independent measurement path. The construction of a second model with a separate technique increases the confidence in the results.
- *Robustness:* Methods using digital cameras rely on either pre-calibration, calibration targets used on site, or on the presence of identifiable landmarks for self calibration. In the event a camera changes to an uncalibrated state, a target is damaged, or there are not enough distinct landmarks visible, it may not be possible to reconstruct a model. The simple 3D models built by the methods proposed here do not rely on calibrations or imaging landmarks. Results can be obtained even if there is a complete failure of electronics in the field or corruption of electronic media.
- *Modeling In Difficult Imaging Conditions:* In some cases immovable features such as architectural ornamentation are the shapes to be captured. For such features it is not always possible to place a camera in the views necessary to reconstruct the shape. In some cases however it may be possible to obtain caliper or contour gage data that can be used to fill in the data holes in the image-based model.

The caliper, gage and paper methods we propose can be applied without any electronics in the field. All electronic processing of data can be done in the office after returning from the site. A major disadvantage of the methods is that they require physical contact with the object, which may not be desirable in all cases.

Caliper measurements form the basis for our methods. We use multiple sets of linear measurements to compute the three dimensional locations of points on a surface. Dense sets of points along a planar curve on a three dimensional object can be recorded with a contour gage. Multiple curves from a contour gage can be positioned correctly relative to one another in three dimensions using points from the caliper measurements. Annotations on paper can be used to form a triangle mesh from the measured points, and to record the location of surface details on the measured shape.

We begin with a discussion of building 3D models from caliper distance measurements. Next we show how these models can be enhanced using contour gage data and notes or tracings on paper. We compare results obtained with varying amounts of redundant input data. We also compare our results with a model obtained using inexpensive photogrammetry. We show how data from simple tools can be combined with optically acquired data. We demonstrate the use of our methods combined with an inexpensive optical technique, passive stereo vision, to record cultural heritage artifacts.

2. Traditional Tools Alone

In this section we introduce capture with calipers, gages and paper. Calipers and contour gages are conventional manual measurement tools that are durable and do not necessarily require electric power. Calipers are used to measure the straight line distance between two points. Contour gages record offsets along a curve that lies in a plane. Both tools come in a variety of styles and a range of prices. Errors on the order of a millimeter or less can be obtained from tools obtained for under ten euros.

2.1. Calipers

Two styles of calipers are shown in the top row of Fig. 1. Calipers can readily be used to measure the diameter of a sphere, or the length of the edge of a cube. In this section we demonstrate that sets of pairwise caliper measurements between N points can also be used to determine the 3D locations of the points that can be connected to form an approximation of an arbitrary shape.

When distance measurements between all pairs of N points are given, multi-dimensional scaling MDS (e.g. see [PFG00]) can find the 3D positions of these points. This approach has been used in measurements for biological morphometrics [CSIM96]. Unfortunately, when only some subset of these measurements are given, the problem is, in general, NP-complete [Sax79]. For appropriately chosen measurement subsets however, the correct solution can be obtained by a greedy "trilaterization" algorithm [Ere03]. Since we have control over the measurements, we pursue this approach and use a simple scheme for measuring an adequate number of pairwise distances.

We know that a rigid configuration of three points, a triangle, is formed given the three unique pairwise distances between them. Given the distance to these three points from a fourth point, and one bit of data indicating which side of the plane of the triangle the fourth point lies on, a unique tetrahedron is specified. Additional points can be positioned relative to this tetrahedron by measuring distances to each of four previously measured non-coplanar points.

As in any measurement, there is some error in each measured distance d relative to the true value. The error is a combination of the resolution and accuracy of the calipers we use, and the accuracy with which we can locate points on the object. This error can be spread over the estimates $\mathbf{x}^*$ of all of the positions by solving simultaneously for the positions $\mathbf{x}$, rather than sequentially. Furthermore, the errors are random, so measuring more than the minimum number of distances can improve the estimate. We estimate the three dimensional structure by minimizing the following objective function f of the unknown coordinates x_i, y_i, z_i given the measured distances d_{ij} :

$$f() = \Sigma_{ij}(\sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2} - d_{ij})^2 \quad (1)$$

The result of computing this minimum is a set of computed three dimensional coordinates $\mathbf{x_i^*}$. From the coordinates we can compute distances d_{ij}^* that give a quantitative indication of the quality of the locations computed from the measured input d_{ij}.

2.2. Contour Gages

Interesting curved surfaces could in theory be captured by tediously locating large numbers of points with the caliper measurements just described. However, we can capture points along planar curves more conveniently using a contour gage. Contour gages are routinely used for rotationally symmetric objects or extruded shapes. The gage is pressed against the object to record the shape, the shape is traced onto a page and digitized for import into a modeling system, e.g. as shown in the middle row of Fig. 1.

We extend the use of contour gage data using the caliper method just described. In using the gage, we trace the contour onto paper and measure the two dimensional coordinates $\mathbf{s}$ along the curve. We can position a contour gage curve in three dimensions then by marking three points along the curve $\mathbf{s_1}, \mathbf{s_2}$, and $\mathbf{s_3}$ and their corresponding positions $\mathbf{x_1}, \mathbf{x_2}$ and $\mathbf{x_3}$ on the object. We compute estimated values $\mathbf{x_1^*}, \mathbf{x_2^*}, \mathbf{x_3^*}$ for these positions using the network of caliper points. Provided that the three points are not colinear, we then use the

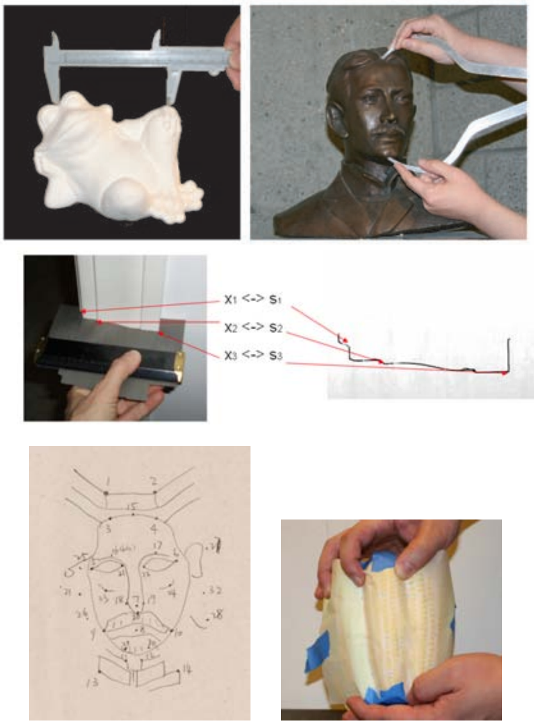

Figure 1: *Simple Tools – (Top) Two styles of calipers used for measuring point distance. (Middle) A standard use of a contour gage is to record the cross section of moldings. (Bottom) Paper annotations may consist of a crude sketch (left) or markings made on paper wrapped on an object (right).*

points that have been positioned by the caliper technique and transform the points along the planar contour into the object coordinate system by finding the rigid transformation from the points $(s_i, t_i, 0)$ to (x_i^*, y_i^*, z_i^*). To account for the impact of measurement errors, in practice we find four or more corresponding points on the contour and model and find a least squares solution for the rigid transformation.

2.3. Paper Annotation

Another simple non-electronic method for recording information is to make approximate sketches on paper or to mark on paper that is wrapped over an object. The lower row of Fig. 1 illustrates these two alternatives.

An approximate sketch such as shown on the left can be used to compute the connectivity between the measured points to form a triangle mesh. Back in the office after data is taken the sketch can be digitized using a 2D office paper scanner. Connectivity can be computed in two dimensions using a method such as Delaunay triangulation. The connectivity that is computed can then be applied to the three dimensional points that are computed by using the point indices to relate the 2D coordinates from scanned paper to the 3D coordinates from the caliper model. Multiple overlapping sketches can be used to triangulate models that are not height fields. While the triangulation found in overlap regions can not be guaranteed to be the same, the sections can be easily adjusted into a single mesh manually.

Wrapping the object in paper is shown on the lower right. By using semi-transparent paper, the positions of the measured points can be marked on the paper. Other notes can be written about specific features, or pencil or wax rubbings recording surface relief can be made on the paper. Clearly any direct contact between the marking instruments and material surfaces must be avoided. Because the measured points are located on the paper these markings can be transferred to the measured model as a texture map, essentially allowing for annotation in 3D.

3. Practical Implementation and Sample Results

Our modeling process begins with entering the pairwise distance measurements we obtain with calipers. A variety of software options are available for finding the $\mathbf{x}_\mathbf{i}^*$ that minimize the objective function f(). We have experimented both with the existing multidimensional scaling (MDS) *mdscale* and unconstrained miminization *fminunc* routines in MATLAB and with the Generalized Reduced Gradient (GRG2) nonlinear optimization code used in the Solver in Microsoft Excel.

Our MATLAB implementation follows a conservative approach. To reliably obtain an accurate base model, we exhaustively measure the pairwise distance between 10 points on the object, rather than the minimum 4 required. Using *mdscale* in MATLAB we solve for the point locations for these 10 points. For the additional points, positions are added by using unconstrained minimization, *fminunc*, from four length measurements from the new point to four of the original 10 points. In the script for applying *fminunc* for each additional point, a check is made that the four points used as reference are not coplanar. If reference points are coplanar these are flagged as sources of potential ambiguities. The value of the objective function returned by *fminunc* is checked, since large values indicate the true minimum has not been found. For these cases, point locations are reestimated with a new starting condition. In practice we found few measurements that did not converge in the initial calculation, and found that one small perturbation in the starting conditions resolved the problem in each instance.

Our Excel implementation requires exhaustive measurements for only 4 initial points. Excel provides a natural user interface to enter the distance measurements. We use macros to reference the coordinates that are used to compute the distance between each pair of points. A macro is also used to conveniently write out the results of the calculations in *obj* format. All measurements can be entered at once, or the data for each point may be added incrementally, and intermediate results inspected.

The MATLAB and Excel implementations give comparable results (i.e. they converge within the range of measurement uncertainty) for the same input data. The advantage of the MATLAB implementation is that it is easier to implement checks on data and automate perturbations in initial conditions. The advantage of Excel is that it is more widely available.

We use a normal office paper scanner to digitize the contours found with the contour gage and and the points on sketches or markings from paper wrapped on the objects. We used a custom user interface to compute the 2D coordinates of the marked points. The 2D coordinates from curves traced from the contour gage are transformed into the object coordinate system using simple matrix multiplication. We use the Delaunay triangulation in MATLAB to form a mesh from the points marked on paper.

Figure 2: *Models from calipers: A computer mouse is modeled with 10 points found with caliper distances (top row). More complex examples of models from caliper measurements are a handle of a bottle (middle row) and a glass squirrel (lower row.)*

Examples of building models with calipers alone are shown in Fig. 2. For all the caliper results shown in this paper (either calipers alone or calipers with other methods) the number of points used in our models range from 10 to 39. Solution times in MATLAB using *mdsscale* for the first 10 points and *fminunc* for subsequent points range from one to two seconds. Solution times in Excel range from less than a second to just under 30 seconds depending on whether an additional individual point location is being computed, or a global solution for 30 or more points with a poor starting point. All timings are from calculations on a Pentium M 1.6 GHz processor.

In the top row of Fig. 2 we show a computer mouse with points marked on painter's tape. We took 30 linear measurements for 10 points (six measurements to define the first four points, and four measurements each for the other six points) to form the model on the right. The points define a mesh that is extremely simple, but is water tight and is defined by points specifically chosen by the user. The middle row of Figure 2 shows modeling another object , a plastic water bottle, with a different topology. Thirty points were computed in MATLAB from 125 distances. The bottom row of Figure 2 shows an example of a glass object that is difficult to scan optically. The photographs show the object with measured points marked on painters tape, and views of the model are shown in red. Thirty-nine points were computed on the object in Excel using 167 measured distances. The minimum value of the total objective function f() (sum of square of errors for 39 points) was $1.41cm^2$ for the 15 cm tall figure.

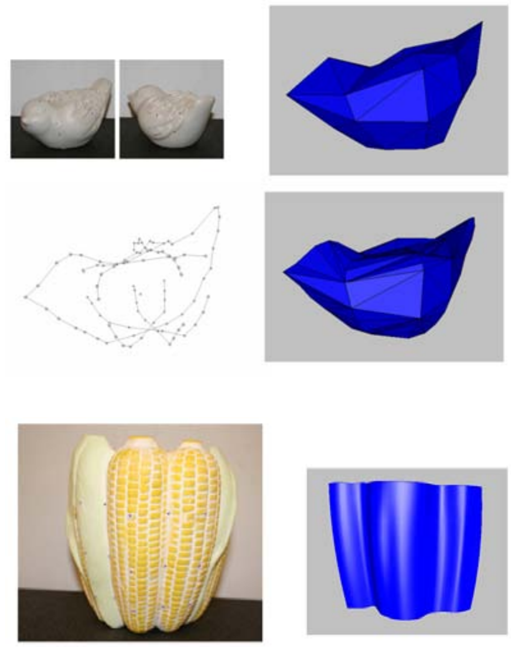

Figure 3: *Contour gage results: A small bird-shaped object (upper left) is approximated with caliper measured points (upper right) and then a set of measured contours (middle left) are used to produce a refined model (middle right.) A section of a vase (lower right) is approximated by a series of measured contours (lower right).*

Figure 3 shows examples of using the contour gage. In the upper left, two views of a small figure of a bird with points temporarily marked on it are shown. Using just points found with the calipers, the model in the upper right is obtained. Contours were then obtained with the gage, and positioned using caliper-located points, as shown in the middle left. The resulting digital model including the contour data is shown in the middle right. The bottom row of Fig. 3 shows modeling a section of a vase shaped like a set of corn ears (this same vase was shown wrapped in paper in Fig. 1). Three contours were measured and positioned using the 15 points marked on the object.

Figure 4 shows the results of using marking on paper. In the upper row, the image on the left is the result of triangulating the points shown in the sketch in the bottom row of Fig. 1. The connectivity between the points is applied to the 3D coordinates of the points to form the 3D shape shown on the upper right. In the lower row, the results of marking on the paper wrapped on the object on the lower right of Fig. 1 are shown. The image shown in the lower left of Fig. 4 shows the paper marked with the points (in red) and with surface relief (from rubbing with crayon), after it was flattened. The image shown in the lower right of Fig. 4 show the crayon markings applied as a texture map to the 3D model formed of the vase that was shown in the bottom row of Fig. 3.

4. Quantitative Assessment

The quality of the results of calipers and contour gages depends fundamentally on the accuracy and resolution of the physical instruments. Beyond the inherent limitations of the instruments, in this section we consider the effect of redundant measurements, and comparison to values obtained with a relatively inexpensive image-based method that produces similar sparsely sampled models.

4.1. Measurement Redundancy

We used the data for the vase model shown in Fig. 3 to examine the effect of our measurement strategy. We made exhaustive pairwise measurements for the 15 points. We compare the results of running *mdscale* on the exhaustive measurements ($M = 105$) versus running *mdscale* on exhaustive measurements for the first 10 points, followed by adding the subsequent points using only four distance measurements per point ($M = 65$). Table 1 shows the statistics for the computed distances d^* relative to the measured distance d for the two methods. The reduced measurements give acceptable results, but noticeable improvement is obtained from additional data.

Table 1: *Impact of Strategy on $|d^* - d|$ for points measured for the corn example in Fig. 3*

Quantity (mm)	M=105	M=65
Maximum Difference	1.7	2.8
Average Difference	0.6	0.7
Standard Deviation	0.4	0.7

4.2. Comparison to Inexpensive Photogrammetry

Sparse points can also be measured with inexpensive photogrammetry. In this section we compare the results used with the two methods, and how a caliper/gage method can complement inexpensive photogrammetry.

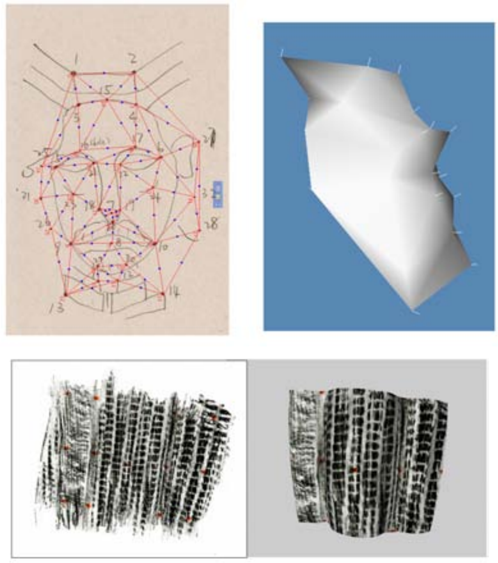

Figure 4: *Results of using paper: A triangulation formed from an approximate sketch (upper row), Details marked on wrapped paper applied as texture map (lower row).*

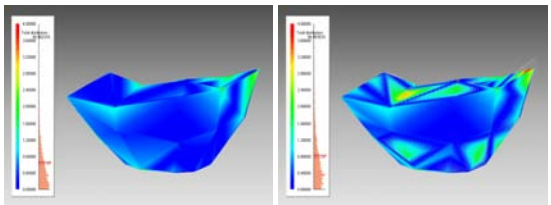

Figure 5: *Error visualization (Unit: mm). (Left) The error between the caliper model of the bird and the PhotoModeler model. (Right) Visualizing the error between PhotoModeler model and the caliper-contour model. The colored surface is the PhotoModeler model and the white dots are sampled from the caliper-contour model.*

We reconstructed the bird model with PhotoModeler (www.photomodeler.com), a commercial photogrammetry package, as a comparison to the simple tool measurement. We used 21 images of the bird taken with a calibrated Canon Digital Rebel XT camera. We manually specified 34 feature points and matched them between different images. These feature points are the same as those used in caliper measurement, therefore we have a simple mapping between the model reconstructed in PhotoModeler and the caliper model. This mapping is used to compute a rigid transformation matrix to align the two models. The error of those two models are shown in the top row of Figure 5, where the minimum distance between two surfaces are visualized. The two surfaces are very similar, with an average distance between corresponding vertices of 0.6 mm. This demonstrates the feasibility of caliper reconstruction method acting as a backup for optical methods which capture only sparse points. We note that the total time for the human labor capturing the overlapping images and marking points in the PhotoModeler package was comparable to the human interaction time required for the caliper model.

Furthermore, the contour gage data provides complementary information to refine the rough model. We illustrate the refinement obtained with the contour measurements relative to the PhotoModeler result in the bottom row of Figure 5.

The caliper measurement may also be used to provide an independent way to assess the accuracy of a model reconstructed in optical methods. In the bird case, the radius of the feature points marked on the object is about 1 mm and the resolution of the caliper used is 0.1mm, therefore the error of the measured distances should be bounded within 2.1 mm. The RMS error between the measured distances and the distances on the model built from caliper measurements was 0.5mm. The RMS error between the measured distances and the computed distances on the PhotoModeler model is 1.21 mm, which gives an upper bound on the error of the point positions in the PhotoModeler model. We could of course compare linear distances without the model built from calipers. However, with the model from caliper measurements we can confirm that all of the individual measurements have been spatially organized properly, and visualize the comparison of the measurements in the style shown in Figure 5.

PhotoModeler and many other image based methods depend on a calibrated camera. If for some reason conditions change during acquisition, such as the camera focal length changing, it would not be possible to use the standard technique with the acquired images to model the object. In such a case the caliper model can be used as a known target to calibrate the camera parameters and recover the use of the images for computing a model.

5. Hybrid Methods

In the last section we examined building sparse models, and how these models compare to inexpensive photogrammetry. In this section we explore how sparse caliper/contour models can be combined with methods such as stereo vision for capturing dense model. In addition to acting as a backup and estimate of model accuracy as discussed in the last section, caliper and gage methods can be used in conjunction with systems for dense sample capture to address problems with registration and data holes.

Optical capture systems generally work with reflected light and produce range images – a height field represention of a portion of a surface. A general problem in scanning is registering height fields together to form a full object. Automatic initial registration methods such as spin images or harmonic images [ZH99] rely on the quality of scans to detect salient features. Manual methods can be used for pairwise registration but result in unacceptable accumulated error in an initial global registration. Registration techniques such as iterative closest point refinement (ICP) [BM92], work by drawing together overlapping range images. Registration is difficult or may fail for range images with little overlap, large holes, or a lot of noise. Range images obtained in the field with passive methods such as stereo vision typically have these characteristics. We can assist the registration using a network of measured points as a digital framework, similar to the use of points determined with theodolites to align range scans, as described in [GBCA03].

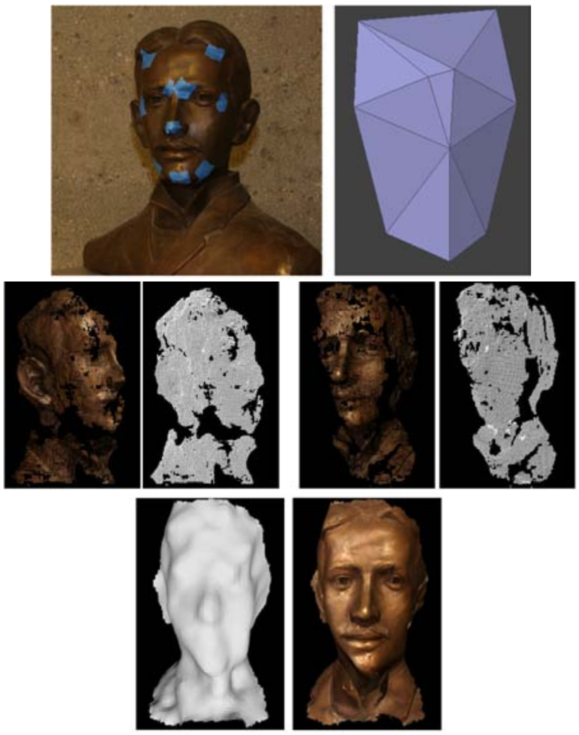

Figure 6: *Top row: An object and a sparse caliper-based model of the object, Middle row: Scans obtained with a stereo vision system, Lower row: A model generated by registering the scans on the sparse model.*

There are frequently holes in optically captured models. This may be due to the nature of the material, or the inability to position the camera to see the surface. Many methods have been developed to fill such holes plausibly, e.g. [SACO04], [DMGL02]. However, for studies it is desirable to have a model based on data, rather than plausible filling. Once scans are registered to a model based on caliper measurements, this model can sometime be used in the surface areas where optical capture left holes.

5.1. Results

We obtained optical scans of some objects using the version of the Small Vision System (SVS) sold by Videre Design for stereo vision. Aligned scans from the vision system are assembled into a single mesh using PlyMC [CCG*03] processing software. We compute final textures by combining multiple captured images projected onto the model after it is segmented into charts using our own implementation of standard methods described in references such as [BR02].

Figure 6 shows an object modeled with a sparse mesh of points located using caliper distances. Though sparse, the vertices in the mesh are at key identifiable points on the object such as the nose. Two range images from a stereo vision system were obtained with little overlap. They are registered in a common global coordinate system by identifying key points on the textured range image that are measured points in our sparse caliper-based mesh. This registration allows the scans to be merged into a single mesh that can be texture mapped without severe distortion.

Figures 7 and 8 show a carved stone segments from an historic building. (Figure 7 is the object associated with the sketch in the lower left of Fig. 1.) We received permission from the building's preservation staff to place painters tape on the stone to make caliper measurements of these architectural details. We also captured range images using a stereo camera rig (two Canon Digital Rebel XT cameras), calibrated with the targets provided with SVS.

Figure 7 shows the sparse model obtained for the figure of the face with our caliper measurements. As in Fig. 6, the sparse model locates key features of the object. Six patches of geometry were captured with the vision system. The patches were noisy. We aligned each of the patches with our sparse caliper-based model. The resulting merged mesh formed a reasonable model, but the shape of the nose was not captured in any of the stereo-based patches. We adjusted the nose position using our simple caliper-based model which did capture this feature. The final model we computed is shown in the bottom rows of images.

Figure 8 shows the sparse models and geometry merged from stereo for the second architectural feature. In this case the patches from stereo are much better due to texture in the stone. However, a hole is left in the model because of the limitations on where the stereo rig could be physically located. The hole contains a face with a nose that is important to model for undistorted texture mapping. We added geometry for this hole using a model computed from caliper measurements made just in the hole region. Filling the hole in this manner the head of the figure has a correct profile, and texture mapping on the face of the small figure is less distorted.

Figure 7: *(Top row) An architectural detail and the sparse model produced with calipers. (Middle row) Patches of geometry from the passive stereo vision system. (Lower two rows) The patches are successfully aligned and modified using the sparse caliper-based model to produce a final model.*

6. Conclusion and Future Work

We have presented novel uses of simple tools to create digital models from existing physical objects. Locating a set of points in three dimensions from a set of caliper measurements between points forms the basis for these models. The models can be improved using data from contour gages positioned with the points from caliber measurments. We have shown how these techniques can be used in combination of optical capture methods.

We are pursuing additional ideas to make this approach more useful. In some cases a user may be not be able to specify the set of pairwise measurements taken, and so may

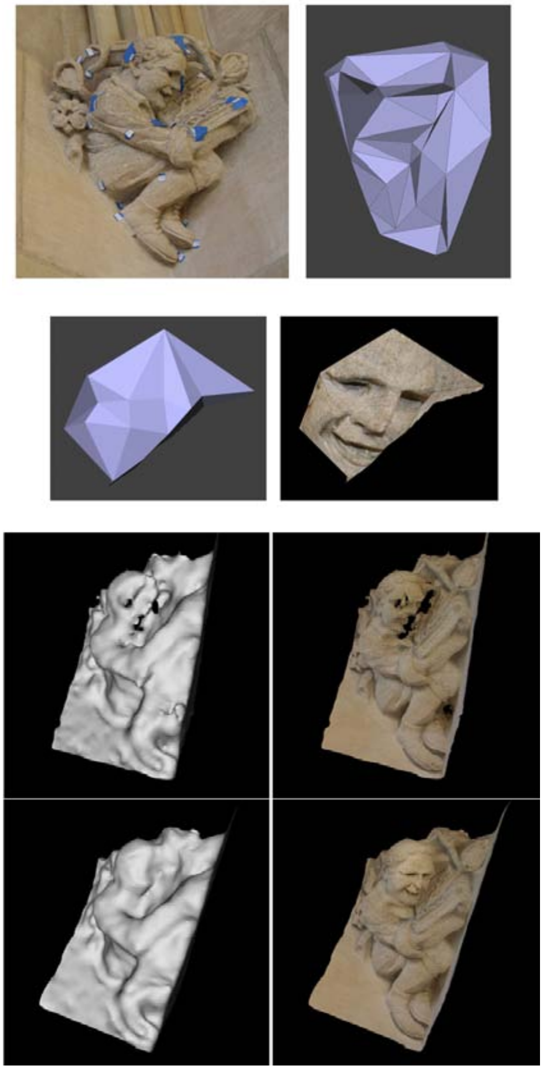

Figure 8: *(Top row) An architectural detail and the sparse model produced with calipers. (Second row) A caliper-based model made just of the small face, shown untextured and textured. (Third row) Model from stereo vision with a hole. (Bottom row) Model with the hole filed with calipers-based geometry.*

not be able to tell if a set is complete. We would like a method that determines what subset of measurements determine a rigid structure when the test fails. We are also exploring the incorporation of other traditional instruments besides calipers and gages, such as tools for measuring angles, into building models.

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