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A Parametric Exploration of the Lighting Method of the Hagia Sophia Dome

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Abstract
Byzantine church design depended heavily on natural light for the generation of evocative effects supportive of the liturgical acts taking place in it. It appears that there were several effects coordinated in order to strengthen the impression of divine presence within the church. The method of ritual lighting reveals a sophisticated level of design in Byzantine churches involving a number of issues which must have formed a complete and integrated system. Specifically, one of these effects concerns the generation of light shafts within the church apse and a derivative apse geometry; a second effect concerns the lighting of the dome; a third effect concerns the church proportions in respect to the way its interior spaces ought to be viewed as well as a number of other relevant issues. Some of this work has been presented in various papers, conferences, and speeches. However, recently we have employed parametric modeling as a tool that helps us comprehend more fully and accurately the design strategies and methods involved as well as revise erroneous assumptions made in the initial stages of this research. The present paper aims at exploring the effect of the luminous dome through the aid of a 3D model focusing specifically in the system developed for the initial dome of Hagia Sophia in Constantinople. While this idea has been previously explored theoretically, the 3D model provides us with more accurate results, verifies some of our conclusions and refutes others becoming in this way not only an indispensable tool that allows us to reach a more detailed and clear exploration of our initial assumptions.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 Computational Geometry and Object Modeling and I.3.7 Three-Dimensional Graphics and Realism

1. Introduction

Byzantine church design depended heavily on natural light for the generation of evocative effects supportive of the liturgical acts taking place in it. It appears that there were several effects coordinated in order to strengthen the impression of divine presence within the church. The method of ritual lighting was quite complicated and reveals the quite sophisticated level of design in Byzantine churches involving a number of issues which must have formed a complete and integrated system. Specifically, one of these effects concerns the generation of light shafts within the church apse [POT1996] and the dependence of the apse geometry on these [PJ2006]; a second effect concerns the lighting of the dome [POT2004]; a third effect concerns the church proportions in respect to the way its interior spaces ought to be viewed [PJT1995] as well as a number of other relevant issues [POT1996]. Some of this work has been presented in various papers, conferences, and speeches. However, recently we have resorted to the aid of parametric modeling which provides us with a tool which helps us comprehend more fully and accurately the design strategies and methods involved as well as revise erroneous assumptions made in the initial stages of this research. The present paper aims at exploring the effect of the luminous dome through the aid of a 3D model focusing specifically in the system developed for the initial dome of Hagia Sophia. While this idea has been explored theoretically before [POT2004] the 3D model provides us with more accurate results, verifies some of our conclusions and refutes others becoming in this way not only an indispensable tool but also pressing us to achieve a more detailed and clear exploration of our initial assumptions. For instance, the accuracy achieved in a recent exploration of the light shafts generated in the church apse was impressive. The parametric model we created helped us discover the heptagonal geometry of the Hagia Sophia apse in plan and the reasons for its adoption, an issue that has never been undertaken before by architectural historians [PJ2006]

The focus of this paper concerns the manner in which the central dome was lit in domed church types. Even

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today, Byzantine church domes often appear constantly radiant especially around the apex area. The radiance of the dome is enhanced by the shape of the pendentives that are also lit and appear to dynamically lift the dome off the ground (Figure 1). It imparts the impression of a luminous complex launched from four points at the base and stretching over the interior space as if it were a weightless membrane not subjected to the law of earthly gravity, a notion carrying clear symbolic connotations. The image of Pantocrator, often placed at the apex, is not lit directly by sunlight but, since he is considered to be the distributor of "true light" to the world, is made to appear to emit constant light. Practically, such an impression could not occur by chance. Instead it would be the result of a specific study by Byzantine architects.

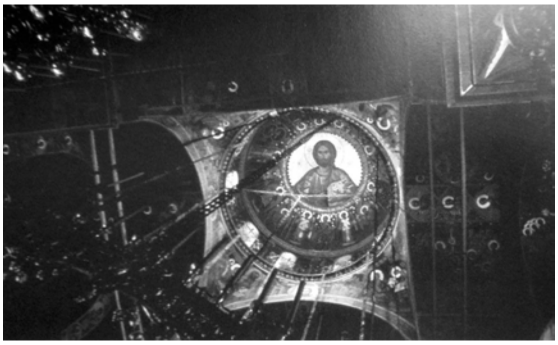

Figure 1: *Iveron Monastery Church dome and pendentives, Mount Athos, Greece.*

A study of this kind seems to have been undertaken in Hagia Sophia resulting in an impressive solution. Written evidence to this is provided in the extant writings authored by its architect Anthemius of Tralles in the form of solid geometry propositions for the construction of reflectors.

One of these reflectors was designed for the purpose of directing solar light at a single point within a building, taking into account the varying positions of the sun throughout the day and the seasons (Note 1) (Fig. 2).

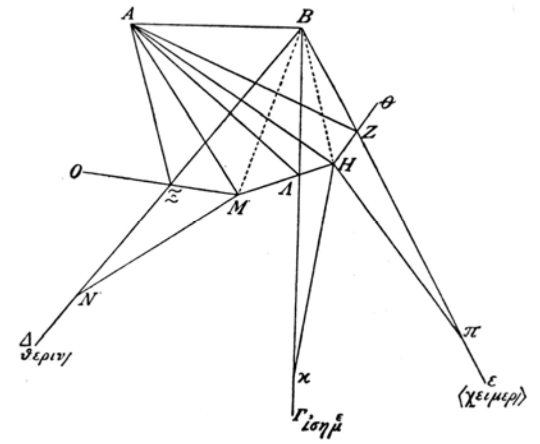

Figure 2: *Anthemius' reflector ΘZHΛMNΞO reflects sunlight coming from either Δ, Γ, or ε, and passing through B, to A. Δ represents summer solstice, ε winter solstice and Γ the equinox.*

As it is revealed by these written sources, Anthemius was an expert in the handling of light, being able to direct, focus, and stabilize it. It should be expected, thereby, to use his capacities if necessary in designing Hagia Sophia, his life masterpiece. The manner of the admission of light, as it appears, was critical to the new church, a fact which not only led Anthemius to creating reflectors of local importance but, as we shall see, played a significant role in the shaping of major formal elements as well. In fact, the handling of light became a decisive generative force of the geometry and an integral part of the built form.

Regarding this particular reflector, there are clear indications both in the drawing and the accompanying text that it was intended for Hagia Sophia. These indications have been thoroughly analyzed elsewhere (Note 2).

Anthemius states: "It is required to cause a ray of the sun to fall in a given position, without moving away, at any hour or season." (Note 3)

In order to satisfy the conditions posed, the reflector would have to be elliptical in three dimensions. It would have to possess, that is, a double curvature. Any mathematician could ascertain that the form of an elliptical reflector of this kind would be independent of latitude and as long as light penetrated through the center (B) of the prescribed opening, i.e. the one focus of the ellipse, would end up to the second focus (A). Although the reflective properties of the ellipse were known to Anthemius (Note 4) he resorted to a new solution which he related to the solstices and the equinox. Instead of using the simplest and most direct method for constructing an ellipse by means of a string attached to its two foci (A & B), Anthemius

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resorted to a quite complicated process of resolution by means of tangents, which was used here for the first time (Note 5). The reason was, as it seems, that this resolution presented certain practical advantages (Note 6). The "practicality" in which the problem was solved was strongly criticized by modern historians of science as evidence of Anthemius' limited mathematical expertise (Note 7).

The usefulness of the ellipsoidal reflector becomes clear when one reads the only description of Hagia Sophia available from that era, contained in the work of Procopius, Justinian's court historian (Note 8). This account provides evidence that light did not penetrate uncontrollably into the space but entered through deep windows lending an "incessant gleam" to the dome. This is precisely what would have happened in case the ellipsoidal reflectors had been placed at the window sills. The apex of the dome would have been constantly and brilliantly illuminated. Furthermore, if these three dimensional reflectors did not occupy only the window sills but continued up on the window reveals on both sides of the window, then the effect of a brilliant apex would have remained constant throughout the day commencing at sunrise and lasting until the sunset. In order for this effect to be achieved, the window sills and reveals would have to have acquired a certain depth. In this way no direct light would have been able to enter but only indirect or reflected. As far as its construction is concerned this reflector would consist of flat mirrors subdivided into ever smaller parts until they reached the nearest possible approximation to an ellipse, that is, by golden, glazed mosaic laid out on a preset mortar base. This method was frequently used at the time and had reached a level of unprecedented refinement (Note 9).

But since by using these reflectors virtually the apex of any dome could be brightly and constantly lit no explanation is provided regarding the form of the original dome itself which, as reported by contemporary historians, was of an unusual and structurally vulnerable shape. All contemporary accounts agree that it had a precariously flat curvature which, as modern scholars have repeatedly speculated, might have caused its subsequent downfall. At least one of them asserts that this was due to the absence of sound practices of structural engineering [WAR1976]. It would seem reasonable, however, that the architect's choice should be attributed not to his lack of knowledge, in which case he would have followed time-honored antecedents, but, instead, to his will to defy technological limitations in order to achieve a highly desired effect.

While the original dome no longer survives, we have historical descriptions of it and thus certain theories have been proposed regarding its form. Although these vary somewhat regarding its plan, they agree on its vertical profile (Note 10). The impact of the slight plan variations would not have been considerable in respect to the effect of light. On the contrary, the shallow profile, of which we are certain, would have had a decisive impact on the way light would be captured and reflected by its surface (Note 11).

The explanation regarding the vertical profile of the dome is probably found in another reflector design by Anthemius (Note 12). The significance of this reflector design lies on its similarity to the original dome profile, on the one hand, and to a peculiarity observed in the process of its solution, on the other. This problem deals with the issue of the reflection of light on the concave side of a spherical catoptrical surface (Figure 3).

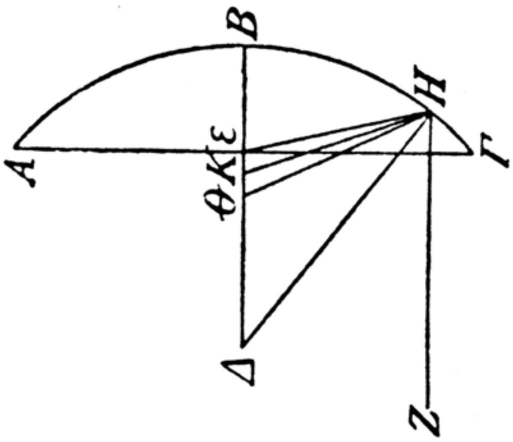

Figure 3: *Anthemius' reflector ΑΒΗΓ reflects any ray parallel to ΔΒ, such as ZH, to K.*

The reflector ΑΒΗΓ is given the shape of the circular segment circumscribed around a square of side ΑΓ. In this circular segment the solar rays that fall with a direction perpendicular to the chord ΑΓ will be reflected toward point K.

Discussing some preliminary issues of historical interest about the problem at hand, Anthemius mentions that Apollonius had already discovered the focal point but that he himself had some additional suppositions to prove. While Anthemius examines the focusing capabilities of the spherical reflector in relation to those of the parabolic, which he has dealt with in the preceding problem he offers no reason for choosing this particular circular segment. Obviously, the focal point would remain the same regardless of the size of the circular segment. But if the segment were larger, that is, if it were a semicircle then

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due to its greater size the focusing strength of the reflector would be doubled. But no such thing is mentioned.

The peculiarity noticed in the problem solution is the following. After Anthemius has proved that the focal point is K, curiously enough, he returns to dealing with the subject in order to prove the equality of the angle that the incident ray creates with the curvature ZHΓ to the corresponding angle between the reflected ray and the curvature KHB. This proof does not add anything to the preceding one but it clarifies the equality of the angles of incidence and reflection the rays form with the curvature *per se* rather than with the tangent. In this manner he interrelates the motion of light rays with the curvature without any intermediaries such as the centre, the radius or the tangent in order to ensure that whatever the angle of incidence, the corresponding angle of reflection will be governed by a simple and constant rule. There is only a short distance to the observation that the more obliquely light comes in the larger the number of successive reflections will occur within the circular segment (Figure 4). This probably constitutes the additional hypothesis that Anthemius is referring to, but unfortunately the text stops before the final conclusions are presented.

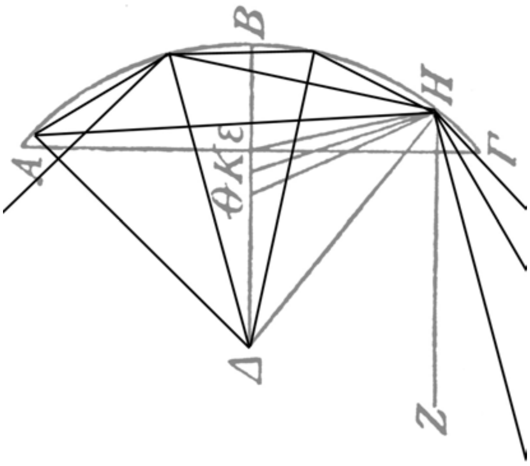

Figure 4: *An illustration of light behavior when incident rays are oblique (superimposed over Anthemius' original reflector diagram).*

By giving the dome such a shape, it would be possible to receive intensely oblique light rays and create multiple reflections within the dome lighting both the southern and the northern areas of it. In achieving this oblique penetration of light the role of the ellipsoidal reflector would be decisive. For instance, the rays that would present 15º-30º divergence from the perpendicular to the chord would generate a second reflection within the curvature of the reflector while with a divergence greater than 30º would generate a third reflection. If one considered that a large number of rays would be introduced with an even greater obliqueness, that is, around 45º, this complex of reflectors would cause an entrapment of light which would generate a uniform illumination of the dome.

The curvature profile of the original dome (Note 13) in transverse section is identical with the arc shape described by Anthemius' problem (Note 14). It is especially remarkable that the base of the inscribed square coincides exactly with the lower cornice while the upper horizontal line of the square coincides precisely with the cornice at the base of the dome windows (Figure 5).

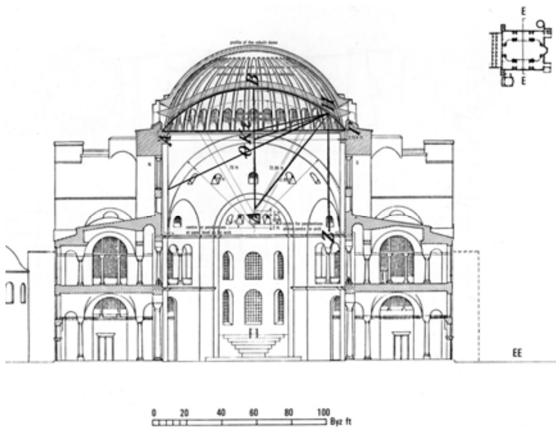

Figure 5: *Anthemius' reflector diagram superimposed over a cross-section of Hagia Sophia (with original and current domes). (Source: Anthemius and Mainstone).*

By accurately placing Anthemius' proposition for a spherical reflector over the two overlaid drawings of the initial dome and the present one by Mainstone, we discovered some additional relationships. First, that Δ by coinciding with one of the possible centers of the initial dome (since there are several theories about its exact height) indicates the most probable dome profile. Second, that HΘ, where Θ is the middle of ΔB determines the limiting case for the reflection of rays coming parallel to ΔB and at the same time determines the lowest point of the pendentive. This means that the light reflected from the church floor as the most extreme case would be reflected first onto the dome and from there no lower than the springing of the pendentive, a principle which would ensure a gradual increase of light intensity from the pendentives toward the dome. Finally, no reflections from the floor would exceed Hε , i.e. the level of the cornice, therefore there would be no disturbance of the illuminated dome by any motion taking place on the church floor.

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The words of Procopius might be indicative of this entrapment of light when saying that it: "… is singularly full of light and sunshine; you would declare that the place is not lighted by the sun from without, but the rays are produced within itself, such an abundance of light is poured into this church." (Note 15)

Thus the effect of the combination of this dome profile with the ellipsoidal reflector would ensure that the dome would receive light from the greatest possible number of windows at any one time thus providing a constant and brilliant light at the apex throughout the day and the year while the shallow dome curvature would generate light reflections that would be trapped within it driving away all shadows and achieving an even illumination. The apex would be enveloped in a sort of a cloud of light. The specular surfaces of golden, glazed mosaic surrounding all window reveals and window sills would create a bleeding effect on the window sides so that the distance between the windows would be perceptually diminished disconnecting, in this way, the dome from its base.

Thus, the cupola would appear completely detached from the rest of the church, creating the impression of an area of light existing on its own accord. This light would be perceived as an unnatural, unearthly light, entirely detached both physically and metaphorically from the present world being isolated, suspended and hovering above, unconnected to the events and occurrences of the world below. It would be a wonderful representation and a most convincing impression – in spatial terms – of that indescribable divine light, the "superessential" light, the light "that casts no shadows".

2. The construction of the parametric model and the lighting model

In order to verify the hypotheses put forward above, we made use of sophisticated parametric software to reconstruct and verify the geometric proofs and relationships put forward by Anthemius. Additionally, to test the light behavior, we reconstructed the basic geometry of the original dome and used an accurate lighting model to observe its behavior.

2.1 The parametric model

In order to verify that Anthemius' reflector (as shown in Figure 2 above) actually works, his textual instructions were followed to create a parametric model using Bentley's Generative Components (GC) software. The GC software allows the user to define global parameters (variables) and establish complex geometrical relationships among entities in a 3D environment. We used GC to construct the mechanics of Anthemius' reflector with the ability to modify the angles and the distances at will. The model behaved as expected and does verify that light passing through point B will indeed be reflected to point A (Figure 6). The next step was to superimpose the parametric model over the cross-section of the Hagia Sophia dome by Mainstone to observe if the reflector geometry is consistent with the architectural environment. It is important to note here that because the original dome has collapsed the exact geometry of the dome as well as the parapet and cornice underneath it is unknown (Figure 7).

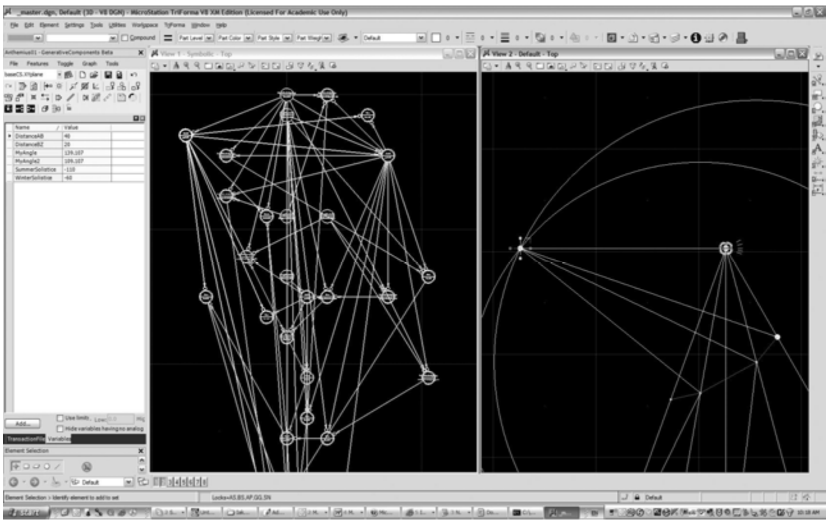

Figure 6: *Screen capture of the parametric model reconstructing Anthemius' reflector.*

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Thus, the exact parapet geometry as shown in Mainstone is to a large extent conjectural as is the one we are proposing. However, as the reader will notice in the next section, Mainstone's parapet proposal creates an uneven lighting condition that is not consistent with the descriptions of the dome from that period.

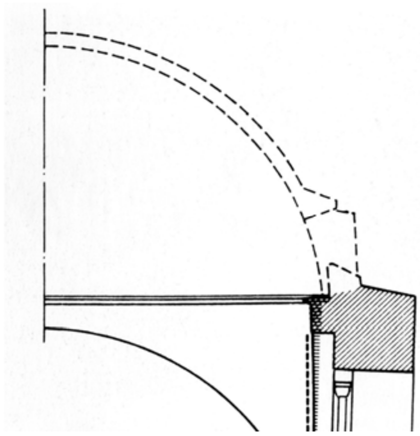

Figure 7: *Illustration showing that no parapet remained after the collapse of the dome. (source: Mainstone)*

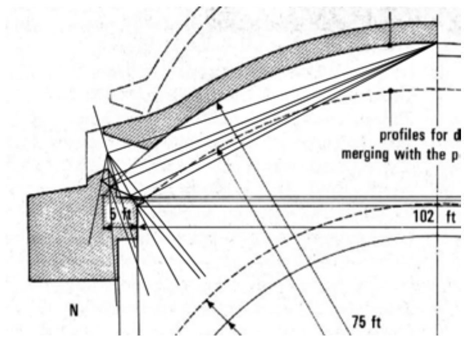

Figure 8: *Parametric model of Anthemius' reflector superimposed over the cross-section of the original dome (source: Mainstone).*

2.2 The lighting model

To test the hypothesis that the flatter geometry of the original dome combined with the geometry of an ellipsoidal mirror at the base of the dome created a more uniformly lit dome with a continuous ring, we modeled Mainstone's proposal for the original dome. Computer-based lighting models have been shown to be accurate tools [GTG1984]. We used AutoDesk's 3D Studio MAX and applied the most accurate advanced lighting method (Radiosity combined with ray-tracing with high-accuracy settings). As an initial test, we constructed a simple room with a window and rendered it with and without a mirrored surface (Figure 9). We used this model to ensure that the recipient surfaces are indeed affected by the simple addition of a reflective surface into the scene. We then used the same materials and applied them to the dome geometry. We also used the same lighting model (Accurate IES Sky and IES Sun light). Given the limitations of time and scope of this paper, we did not vary the hour and date. We are planning a more complete investigation in the future.

Next we constructed the geometry of the dome and two versions of the parapet (the one proposed by Mainstone) and the one proposed by us as illustrated in Figure 8 above (Figure 10). We then rendered each, viewing the dome from the bottom looking up at the dome (Figure 11). Using Adobe Photoshop, we applied the same curve distortion to both original images to better discern the light distribution in the dome (Figure 12). It is clear that the ellipsoidal model created a brighter and more uniform light distribution. The geometry of the curved mirror and the flat geometry of the dome combined to create an effect of an "incessant gleam" as suggested by accounts from that period. The solution proposed by Mainstone provided a much more uneven lighting condition with artifacts due to the sudden change from an inclined cornice to a vertical parapet.

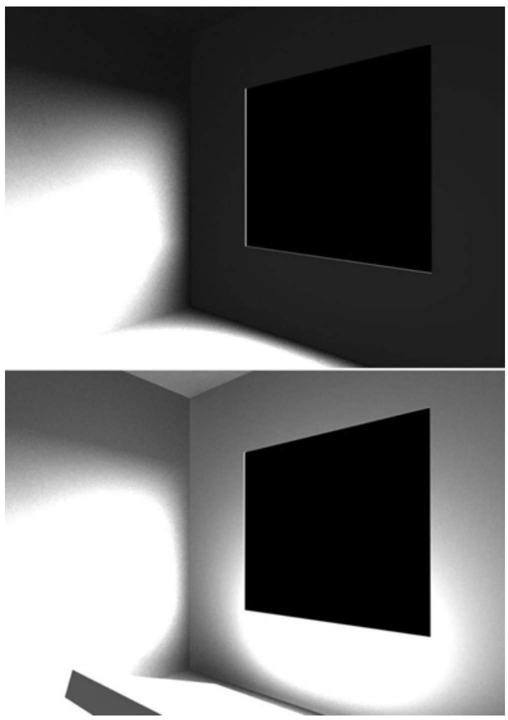

Figure 9: *Lighting model test.*

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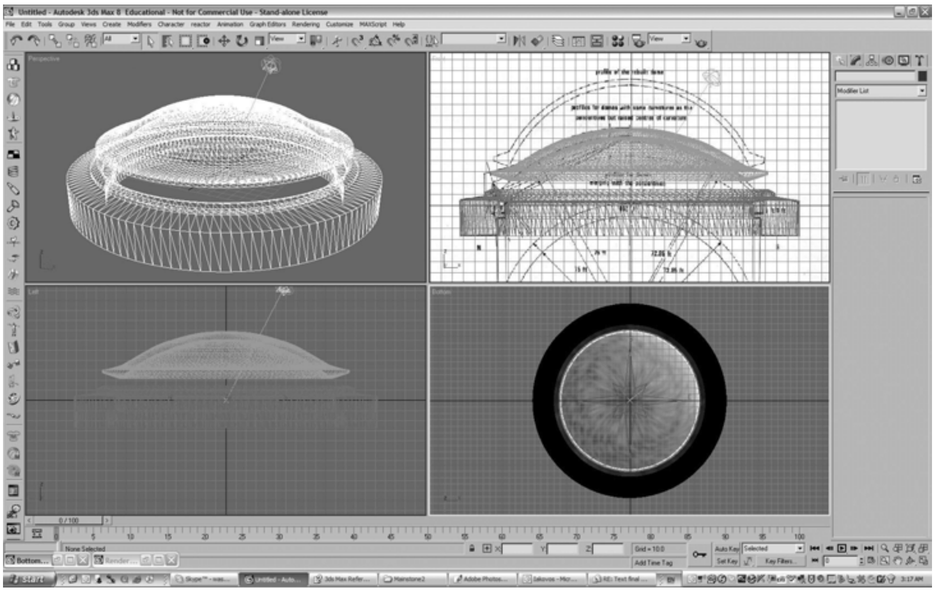

Figure 10: *Screen capture of the 3D Studio MAX model of the dome.*

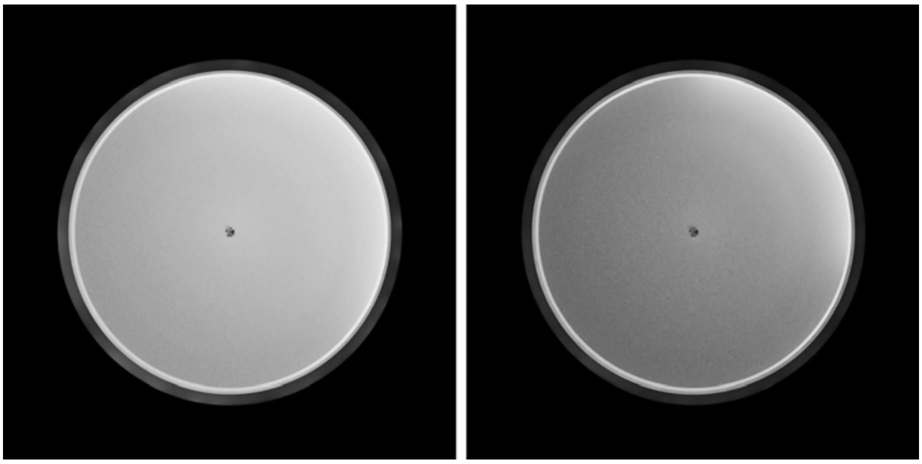

Figure 11: *Lit domes with ellipsoidal mirror (left) and slanted cornice (right).*

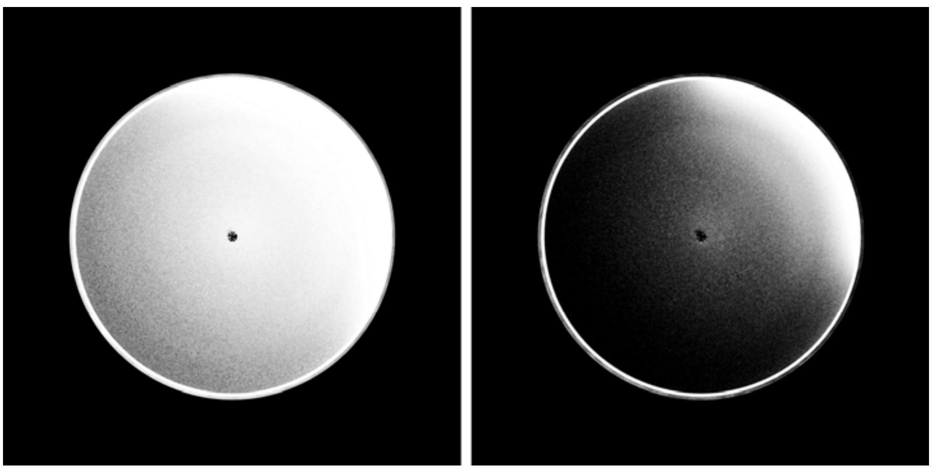

Figure 12: *Curve distortion applied to lit domes with ellipsoidal mirror (left) and slanted cornice (right).*

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3. Conclusion and future work

Light and geometry played an important role in the design of Byzantine churches and in the design of Hagia Sophia in particular. The advent of sophisticated parametric digital tools and advanced lighting models has allowed us to test many of the theoretical propositions and discover new findings regarding the behavior of architectural constructions. The model allowed us to synthesize Anthemius' research on reflectors with the original geometry of the window sills and the dome of Hagia Sophia. Furthermore, it also provided an accurate visualization of the lighting conditions within the dome given two possible profiles for the parapet and cornice. This visualization could then be compared to historical documentation and other evidence.

Many of the results presented in this paper require further verification and elaboration. Future work will include the design of more sophisticated 3D parametric models and the testing of other churches and noted works of architecture. The dome of Hagia Sophia requires further elaboration of the pendetives to fully account for their role and lighting. The geometry of the windows at the base of the model is largely unknown and requires further investigation to determine its attributes and resulting effect on the lighting of the dome.

The parametric tools are of value in of themselves for analysis as well as design. We intend to develop generalized parametric tools that will help the designer specify desired conditions and explore alternatives created through the manipulation of parameters and geometric relationships. Such digital tools allow us to think algorithmically and parametrically about our designs such that we can create spaces and solids that are constructed rigorously and exploit the full potential of geometry and light.

Notes

1) The geometrical solution to this problem appears in his excerpt entitled Peri Paradoxon Michanimaton (About Paradoxical Machines) reproduced and with an English translation in [HUX1959], pp. 6-9.

2) See [POT1996] and [POT2004].

3) See [HUX1959], p. 6.

4) See [HUX1959], p.10.

5) See [HUX1959], pp. 9-10.

6) These advantages are analyzed in [POT1996] and [POT2004].

7) See [CAM1990], p. 121 and [TOO1976], pp.187-20

8) See [PRO1961], I, I, 41-3. For a full analysis see [POT2004].

9) See [MAT1964], p. 29.

10) See [SWI1940], p. 153.

11) See [MAI1988], p. 127.

12) This design is contained in the excerpt Fragmentum Mathematicum Bobiense, also attributed to Anthemius in [HUX1959], pp. 22-3.

13) See [CON1939], p. 589 and [MIC1976], p. 37.

14) See [HUX1959], p. 22.

15) See [PRO1896], p. 6.

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