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Deconstructing the VR - Data Transparency, Quantified Uncertainty and Reliability of 3D Models

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Abstract

The paper discusses two key concepts required for the use of Virtual Reality and 3D modelling as a research tool for the humanities: data transparency -what is the type and nature of the archaeological/historical/ethnographical material on which the 3D model is based, and reliability - how the user can scientifically analyse the model. In this article, we will present a solution to these issues based on concepts deriving from fuzzy logic and fuzzy sets. Taking into consideration the "real nature " of humanities data, more often fuzzy than crisp, a different logic (fuzzy logic) should be applied when attempting to reconstruct a past reality. This will enable a quantifiable visualization of possible scenarios, otherwise discarded in traditional representations. Each scenario is accompanied with a "reliability index", estimated by the researcher according to his/her certainty on the existence of the modelled part and the "importance" of each component of the model. This approach will allow the user to reconstruct the "cognitive process" and the step-by-step "decision making" of the researcher that built the 3D model, and to open the model to a scientific analysis from a humanities point of view.

Categories and Subject Descriptors (according to ACM CCS): I.6.5 [Model Development]:

1. Introduction

Virtual Reality and 3D modelling applications to humanities and social sciences have a already long history of more than two decades, traceable to the early eighties of the last century. Though mostly applied to education or dissemination to the general public, theoretical and methodological issues regarding these kinds of applications to humanities have been raised and largely discussed in the past 15 years [Rei89], [RS89], [Sim97], [FS97], [BFS00], [Nic02]. The potential of VR as a communication tool has been highlighted by Ryan [Rya96], while accuracy of final product [Kan00], [FAF*00] and the need for data transparency [For00] are still commonly discussed issues.

Despite the increasing number of VR and 3D models of humanities data produced and reported [Bur02], [DS03], [NH06], most papers report issues related to communicating Cultural Heritage to the public or related technological aspects and improvements [ACN03], [CCSN04], rather successful stories related to the use of VR as a scientific tool in the humanities, a "pitfall" already pointed out a decade ago [Rya96], and since then rarely raised into the debate [Vat02], [BP04], [HND05]. An emerging legitimate question, after almost a quarter of century of work, is: "why should humanities and social sciences invest effort in VR and 3D modelling?". A more restricted version of the previous question is "how can humanities and social sciences exploit for research VR and 3D modelling applications?". Even though this paper is focusing on the use of VR and 3D modelling in the archaeological research, it is our belief that same arguments raised here are applicable to other fields of humanities and social sciences as well.

The article is proposing a methodology, based on the foundations of fuzzy sets theory, to evaluate the reliability of 3D reconstructions and thus open them to "archaeological scientific criticism". Its main steps are identification of basic components of the model, assignment of an index reflecting the reliability and importance to each one of them, computation of an overall reliability index of the entire model and an evaluation of the reliabilities of other possible reconstructions.

2. VR and 3D modelling and archaeology

Computer-based reconstruction of ancient monuments are increasingly confirmed to be extraordinary tools for representing the past reality, being archaeology a destructive technique employed on largely damaged contexts. One can visually express alpha-numeric data and graphically express thoughts and ideas, translating "...empirical phenomena into geometric language..." [FNRB02]. This aspect was well sinthesized by Niccolucci among others [Nic02]: "...since interpretation, explanation and communication involve reasoning, Virtual Archaeology can provide virtual creations to organize and synthesize known facts, showing them with greater clarity to others or to one's "inner eye", or virtual substitutes of physical objects...". The positive relationship between visualization tools and the way we perceive information has been demonstrated also by numerous researches in education, cognitive psychology and related disciplines [EFH76].

In order to accept as scientifically valid a 3D model, its resources and the criteria on which they are based should be explicitly presented [All98], and when possible, alternate reconstructions should be made available to the user [RR97]. Moreover, a 3D model can be considered scientifically reliable (the term "scientific" implying the Galilean approach, that can be repeated with the same result, beyond experimental errors) when it can be de-constructed and its data made transparent for a critical evaluation.

A typical 3D model (or VR) of an archaeological subject is build upon data of various sources (historical, linguistic, archaeological or history of art researches, just to name a few). Comparative and ethnographic studies, as well as the accumulated knowledge of the researcher are other factors that play a determinant role in the final shape of the 3D product. It is clear that without a clear and detailed description of the data and their sources a 3D model cannot be considered scientifically reliable. However, by merely presenting this metadata is only one step forward towards the use of a 3D model as a source of scientific information.

Incompleteness is a concept implicit in archaeological studies. Archaeology, and any other discipline that integrates archaeological data aiming at creating a virtual model, de facto do not provide enough data to establish a scientifically unquestionable model. To a certain point, all reconstructions remain "speculative". We should therefore consider as scientifically valid a model that allows us to quantify the "degree of speculation" and express it through established rules.

In other words, an important addition for turning a 3D model into a valuable source of scientific information is the possibility to evaluate the decision - making process of the researcher building the model, particularly in cases when (s)he is faced with several reconstruction options. This is mostly pertinent when data sets are not crisp (the window might have been square, or circular, or possibly elongated, and located at 2, 3 or 5 m from ground). Commonly, by arbitrarily deciding which window to model, and at which position in space, all other possible alternatives are discarded and disappear as soon as the model is completed.

It is proposed to apply concepts of fuzzy logic whenever fuzziness is identified in the data sets used for the 3D model, in order to: a. quantify reliability and uncertainty of reconstruction and b. introduce the concept of "fuzzy reasoning" to visualize "possible existences", each with its reliability index [HN03], [HND05]. In this way, not only the decision-making process becomes more transparent, but the user is provided with alternative choices for the 3D reconstructed elements, each with an index reflecting its possible existence. We will exemplify how these concepts were applied by us to the 3D model of a house from the archaeological site of Qalqal Asba.

3. Description of the case - study

The 3D model used as a case-study in this paper was created by a team of the University "l'Orientale" of Naples, as part of a wide research project at Aksum/Bieta Giyorgis, Ethiopia [PMP]. It represents the main building structure at the site of Qalqal Asba, an Aksumite site dated between 350 and 550 A.D. [BDF*02].

Translating site information into visual images required a complete and throughout analysis of all site data, in particular a throughout spatial analysis of archaeological features [PMP], [MPPF03]. Moreover, the virtual reconstruction of the structure (Figure 1) compelled the 3D model's authors to fill gaps and inconsistencies in available data and thus to critically analyse their archaeological data and cope with new questions arisen during the process of 3D modelling.

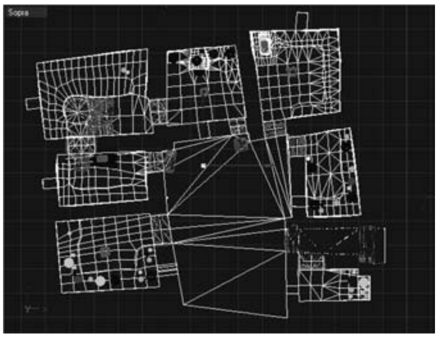


Figure 1: *General plan of the structure*

The Aksumite construction technique, with walls built of loose undressed stones, caused the almost complete collapse of the structure; only 2-3 lower wall courses were preserved. The building has a "U" plan, with a central courtyard open on the west side and nine rooms, all with access from the courtyard except for one, which had only internal access from the rooms beside.

Rooms are square or sub-rectangular, with stone benches along the walls. Occasionally, the roof was sustained by a wood pillar (not found) on a stone base. The characteristics of the rooms and an inventory of material culture allowed an estimation of their original function (Figure 2).

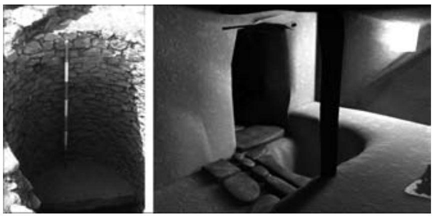


Figure 2: *The store-room: architectonic details and virtual reconstruction*

Remains of roof and second floor were almost impossible to recognize; one of the reasons for this is that these features of Aksumite buildings were constructed by layers of small stones and mud supported by a lattice network of wooden beams and smaller branches. Missing information from archaeology was counterbalanced by the ethnographic research, which provided information on the traditional architecture techniques and materials, spatial organization, and remains of material culture (Figure 3). Visits at a number of abandoned houses gave insights into the residuals, time and dynamics of collapse and depositional processes [BDF*02].

Figure 3: *Etnographic survey and the 3D model*

4. Evaluation criteria

The evaluation criteria of the reliability of the 3D model were based on a combination of two indices: importance and reliability, for each of the 3D model's features. Since the aim of the discussed 3D model was to understand the original architecture of the main building, the values of the above mentioned indices reflect the contribution of each analyzed component to its general shape.

Architectural units, such as walls, roofs or pavements were important for a general representation of the structure, while location of doors and windows, the interior design and furniture were significant in understanding aspects of social organisation, labour division and function of particular features. Moreover, by choosing to model particular decorations for doors, or a particular type of roof, style and cultural aspects of the ancient inhabitants of the site were evidenced. These were grouped under the term "conceptual categories" (see below) in order to express concepts that could contribute to the vision of the whole structure and as a sort of "bonding agent" for all the above categories.

Thus, components were divided into four main groups: components that directly affect the frame of the structure (shape and location of foundation, walls, roof, etc.), components with a secondary architectural role (shape of windows, doors, etc.), components of interior design (furniture and other material culture objects, such as grinding stones, fireplaces, etc.) and finally components that depict conceptual ideas (division and use of social space, etc.).

A reliability and an importance indices defined by values deriving from fuzzy logic concepts - that is belonging to the whole [0,1] interval - were assigned to each reconstructed component (Figure 4). The factors taken into consideration when assigning the reliability index were based on the source of data: primary (excavation), secondary (analogies), and then assumptions made on scientific deductions or pure imagination. The importance index was estimated according to the aim of the model, as emphasized above. Translating text concepts to numerical values, we obtained value intervals as follows: very important = value 1-0,8; significant = 0,7-0,5; scarcely important = 0,4-0,2; insignificant = 0,1-0.

5. Basic concepts of fuzzy logic and fuzzy sets theory

Fuzzy logic [Zad65], [Zim84], [Nov89], [Kos93], [MF93] parts from the observation that the world is composed by shades of gray, rather than sets of blacks or whites. The argument of the Sorites Paradox (what is a heap) may be used as an example for the recognition of vague concepts [FIS00], as opposed to Boolean ones (fuzzy against crisp). An important advantage of fuzzy logic is that it operates in the grey field of the uncertainty and ambiguity; using fuzzy sets one can handle imprecision. Moreover, fuzzy logic aims to extend ordinary deductive methods to situations in which the information available may be only partially or approximately true.

Since its introduction some half a century ago, fuzzy logic has been applied in natural, social and human studies, engineering and computer sciences, covering practically most fields of modern research, a large part of our modern daily life depending on fuzzy logic based systems and machinery, and the list is still long. Fuzzy logic was applied in social sciences as well [Rag00], anthropology [Rui99], databases [Cho03], economy [Dol94], religious studies [Rap93], vegetation science [Hal97], [Mor93], [Muc97], [OLGE98], [Rob86], [Rob89], [Sat96], or geography [Fis00], [HSB93], to name just a few.

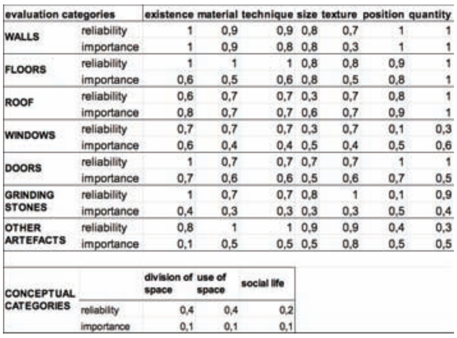
While fuzzy logic is mostly used to solve problems concerning "de-fuzzification" of processes or prediction of models with a fuzzy nature, the archaeological reality, where a definite "true" or "false" will be possible only with the invention of Well's time machine (most probably never!), is intrinsically fuzzy and therefore should be treated as such. Since many of our concepts in archaeology are best defined as vague (fuzzy), and the archaeological reasoning is more often based on approximations and evaluations rather than on crisp affirmations, fuzzy logic offers an alternative methodological framework and the fuzzy set theory the necessary research tools that fit better with the research subject (the archaeological material and its interpretation).

6. Quantifying uncertainty of a 3D model

In the following section some mathematical methods are proposed to summarise all the numerical data obtained as reliability and importance indices.

6.1. Assigning reliability and importance to the model's components

As already mentioned, the first step in quantifying the reliability of a 3D model is to build a table (Figure 4) in which all components of the model are detailed, together with their attributes, on the base of the established criteria (see above).



evaluation categories		existence	material	technique	size	texture	position	quantity
WALLS	reliability	1	0,9	0,9	0,8	0,7	1	1
	importance	1	0,9	0,8	0,8	0,3	1	1
FLOORS	reliability	1	1	1	0,8	0,8	0,9	1
	importance	0,6	0,5	0,6	0,8	0,5	0,8	1
ROOF	reliability	0,6	0,7	0,7	0,3	0,7	0,8	1
	importance	0,8	0,7	0,7	0,6	0,7	0,9	1
WINDOWS	reliability	0,7	0,7	0,7	0,3	0,7	0,1	0,3
	importance	0,6	0,4	0,4	0,5	0,4	0,5	0,6
DOORS	reliability	1	0,7	0,7	0,7	0,7	1	1
	importance	0,7	0,6	0,6	0,5	0,6	0,7	0,5
GRINDING STONES	reliability	1	0,7	0,7	0,8	1	0,1	0,9
	importance	0,4	0,3	0,3	0,3	0,3	0,5	0,4
OTHER ARTEFACTS	reliability	0,8	1	1	0,9	0,9	0,4	0,3
	importance	0,1	0,5	0,5	0,5	0,8	0,5	0,5

CONCEPTUAL CATEGORIES		division of space	use of space	social life
	reliability	0,4	0,4	0,2
	importance	0,1	0,1	0,1

Figure 4: *Model's components and subcategories with attributes*

A first view at the relationship between reliability and importance of the components may be achieved by plotting these values on a scattergram (Figure 5). The graphic visualization enables to see the tendency of the components' reliability and importance, enabling a quick visual evaluation of the 3D model's (un)certainty. Note upper left square indicating categories (components of the model) with a high reliability but a low importance, against the lower right corner, important components of the model but with a low reliability. These extremes might be re-evaluated and accordingly treated. The fact that in our case most values are concentrated in the upper right corner (high reliability and high importance) means that there is a consistency in the modeling process and information is generally reliable and open to a critic evaluation.

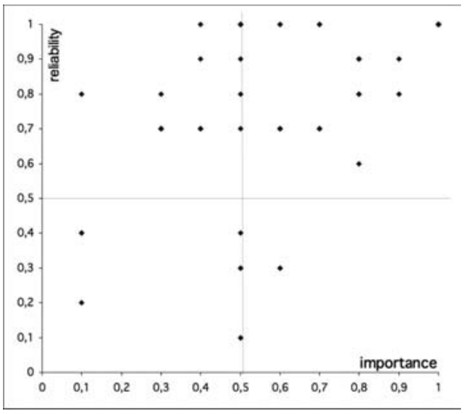


Figure 5: *Scattergram of importance/reliability indices*

6.2. Summarizing the categories

In the next step for each category one general value of reliability and one general value of importance might be computed. As an overall reliability index the average value is proposed because it should consider all the reliabilities of the subcategories. The importance index instead should indicate the detail's most important subcategory (or subcategories), so the maximum function will be used. As the main goal of this paper is to present a solution for evaluating the reliability of a 3D model and to enable its further visual analysis by means of layers defined by given parameters, it is convenient to combine these reliability and importance indices into one variable, so that we obtain one evaluation index for each category.

A required function to compute this had to satisfy two main conditions: a. the values should belong to the $[0,1]$ interval and b. it should distinguish reliability and important indices as variables, so that a cross - change between them would lead to a different result. We propose a following function:

$$R_n = r_n(\sin\frac{\pi}{2}i_n), \tag{1}$$

where r_n is the overall reliability and i_n is the overall importance of the category, computed in the previous paragraph.

A character of the proposed function causes more attention to be paid to the reliability, than the importance index, that is, categories with a high reliability index generally get higher overall value than those of a low reliability even when the latter being of high importance, which is demonstrated on the following graph (Figure 6).

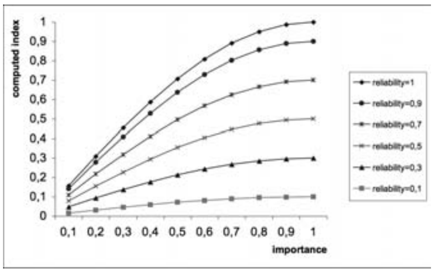
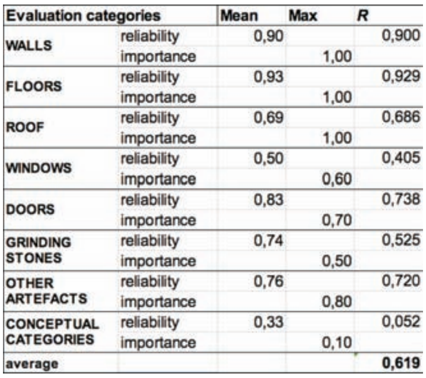


Figure 6: *Example values of the R_n function*

The focus was put on reliability because of the goal of our work, which was to create a reliable model, even when having a lower number of details. Of course exchanging r_n with i_n in the formula would inverse the interpretation. In the following table computed category indices for our model are presented (Figure 7).



Evaluation categories		Mean	Max	R
WALLS	reliability	0,90		0,900
	importance		1,00	
FLOORS	reliability	0,93		0,929
	importance		1,00	
ROOF	reliability	0,69		0,686
	importance		1,00	
WINDOWS	reliability	0,50		0,405
	importance		0,60	
DOORS	reliability	0,83		0,738
	importance		0,70	
GRINDING STONES	reliability	0,74		0,525
	importance		0,50	
OTHER ARTEFACTS	reliability	0,76		0,720
	importance		0,80	
CONCEPTUAL CATEGORIES	reliability	0,33		0,052
	importance		0,10	
average				0,619

Figure 7: *Computed indices of model's components*

6.3. Computing the overall value of the model

Since the above-computed indices already contain all the information about details' reliability and importance, we can simply compute their average and assume it being the value index for the model. So the overall index R was defined as follows:

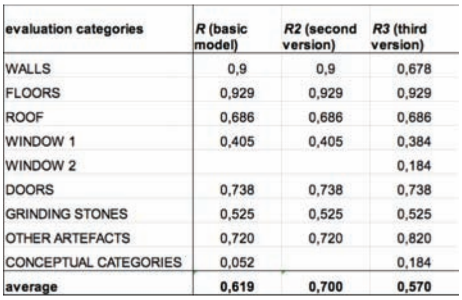
$$R = \frac{\sum_{n=1}^{N} R_n}{N}, \tag{2}$$

where N is the total number of categories in the model. In our case $R = 0{,}619$, which should be interpreted as a good result for our research objectives. Future evaluations of other models will enable us to define the minimum acceptable value of R of scientifically reliable a 3D model.

6.4. Defining other versions of chosen details

Let's say we want to analyse other versions of the model checking how it goes with different details chosen. Of course these exchanged details may have different parameters - reliability and importance.

To illustrate the following step two hypothetical versions of our model were defined. In the first one all the features were kept as they were before, only excluding conceptual categories, which were very uncertain. In the second we change a bit the concept of the model, defining artefacts and conceptual categories in a more reliable way, but having lower reliability for the walls and windows (separated here into two categories - Window 1 and Window 2). We perform first three steps of the algorithm to all of the newly defined versions obtaining results as follows (Figure 8).



evaluation categories	*R* (basic model)	*R2* (second version)	*R3* (third version)
WALLS	0,9	0,9	0,678
FLOORS	0,929	0,929	0,929
ROOF	0,686	0,686	0,686
WINDOW 1	0,405	0,405	0,384
WINDOW 2			0,184
DOORS	0,738	0,738	0,738
GRINDING STONES	0,525	0,525	0,525
OTHER ARTEFACTS	0,720	0,720	0,820
CONCEPTUAL CATEGORIES	0,052		0,184
average	0,619	0,700	0,570

Figure 8: *Reliability of alternative models*

6.5. Final evaluation of the model

Since from the very beginning of the evaluation process fuzzy indices were used as a way of describing the model's details, they can now be used for defining a fuzzy set describing membership of reconstructed categories to the entire model. A formal definition of a fuzzy set M_i (a set of elements belonging to a particular i- version of the model) should be given as follows:

$$M_i = \{(c_n, \quad \mu_i(c_n)), \quad c_n \in M\}, \qquad n = 1, \ldots, N \tag{3}$$

where M is the set of all the categories defined in the model, N is a number of elements of M and $\mu_i : M \rightarrow [0,1]$,

$\mu_i(c_n) = R_n^i$ is a membership function (R_n^i is the R_n function given for the *i*- version of the model). Actually, three fuzzy sets: M_1, M_2, M_3 were already presented in (Figure 8), with elements of M listed in the category column and membership function for all of the sets M_1, M_2, M_3 defined in R, R_2, R_3 columns respectively. The sets can be also represented in a more easy-to-see way by means of 3D bars (Figure 9).

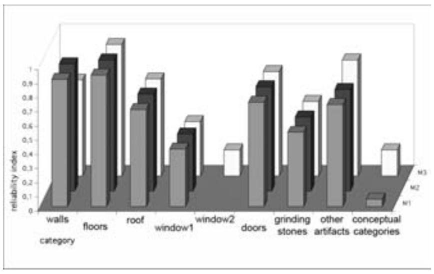


Figure 9: *Visual representation of the fuzzy sets M_1, M_2, M_3*

As one can observe, introduction of fuzzy sets into the process of evaluation of a 3D model provides some new possibilities to express its value by means of numbers and traditionally statistical tools like diagrams and plots. Although it is still only static (but meaningful) information, it is highly desired to improve its use by applying in dynamical visualisation not only of the metadata, but of the model itself.

7. Conclusions

3D visualization is an optimal tool for virtually re-creating and thus analysing archaeological material in its "real" context. It also allows to visually tackle the archaeological remains with the researcher's "mental model", expressed by the (3D) virtually reconstruction of these remains and their context. Accordingly, a 3D model may be viewed as a platform gathering outcomes of a multi-disciplinary research. Thus, a 3D model can be viewed as an expression of the sum up of the acquired knowledge regarding the investigated subject, combining both the perceptions (the archaeological data) with the inferences (the information derived from analysing this data), and thus visually communicating the cognitive process of the archaeological reasoning.

In order to analyse the model from a scientific point of view, aspects of data transparency and reliability of reconstruction should be clearly expressed. The article introduced two key elements that may advance these issues - reliability and importance indices. These indices, obtained by applying concepts deriving from the foundations of fuzzy sets theory, allow a numerical expression of the (un)certainty of a 3D model.

The theoretical discussion presented above will be continued in future work, by developing an algorithm and software that would implement a contingency threshold to a viewer and thus visualize only components passing desired reliability. In the case of Qalqal Asba, the dynamical visualization of the model would not only allow to present its other possible architectural reconstruction (based on known different features from other sources of data), but also give a possibility to show alternaive interpretation of the use of space and organization of the activities in the house.

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