

# 3D orientation of archaeological fragments coming from a Gothic spire

C. Laugerotte and N. Warzée

Systèmes Logiques et Numériques, Université Libre de Bruxelles, Belgium

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## Abstract

*In this paper, we address the problem of 3D mesh orientation applied to archaeological fragments coming from a Gothic spire. Initially, these fragments are handled leading to their deterioration while the accuracy of measures (distances, molding shape, . . . ) is not always relevant. To facilitate their study, a 3D acquisition process is applied on these fragments to work on their virtual representations and computerised tools can be proposed. One of them consists in finding the orientation of a 3D mesh. Unfortunately, existing methods provide unexpected results. This is mainly due to the presence of ornamentalions we can assimilate to outliers. Therefore, we propose a new method based on the identification of significant areas on the 3D mesh and on the determination of their associated orientation by considering differential parameters. Then, a voting method is performed to identify the orientation appearing the most frequently. Applied on the fragments, we obtain successful results allowing to go further in their analysis.*

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## 1. Introduction

For several years, 3D acquisition systems allow to capture geometry of objects from different origins with an increasing accuracy. Afterwards, treatments can be applied on the resulting data to analyze them and to improve their understanding. Applied to the cultural heritage, a better knowledge of archaeological objects can be expected through 3D acquisition: this approach was already used and developed in different projects across the world.

The case of a plaque was treated by the Forma Urbis Romae project aiming the assemblage of the 1163 fragments coming from a giant map of ancient Rome carved onto marble slabs [Lev00].

For fragments coming from potteries, tools are developed to extract characteristics from sherds [SK02b, CM02], to classify them [SK02a] and to propose assemblages [CWA\*01, WC04, KS04].

Two examples of statues can be cited. The first one is the 3D acquisition of the Michelangelo's Florentine Pietà [RBMT98] where a highly-accurate model is used to study the different steps of work and the toolmarks in the unfinished parts [BMM\*02]. The second one is the Digital Michelangelo project [LPC\*00] which realized the acquisition of a collection of statuary by Michelangelo. For one

of them, the famous David, tools have been developed in order to help restorers in the evaluation of the surface exposure to falling contaminants or to investigate the statue's statics [CCG\*04].

Concerning architecture, the analysis of a fragment coming from a Gallo-Roman temple column allowed to deduce its diameter and the building's general shape was confirmed by the analysis of a fragment coming from the dentil [BPF03].

As we can see, computer is not only used to propose visualizations of virtual reconstitutions but provide tools to help archaeologists, historians and restorers in their work of study, analysis and restoration.

## 2. Overview

In 1991, archaeological excavation at the Brussels Saints Michel-et-Gudule collegiate church, realized under the floor of the choir, has revealed 11600 stone sculpture fragments from the XVth and XVIth centuries [BF98]. They represent the mixed remains of several monuments, most probably from around the choir.

By following a comparative approach, the archaeologists already identified several monuments: a Gothic rood-screen,



**Figure 1:** Some fragments coming from a Gothic spire and the top on the upper right corner.

a Gothic tabernacle and a Renaissance niche. Unfortunately, their tasks are long due to the great number of fragments and their fragility, leading to limit their handling and raising conservation problems.

To assist them in their tasks, computerised tools can be proposed. Concerning the handling of the fragments, they can be reduced by working with their virtual representations. Moreover, the 3D models analysis is facilitated and provides more accurate information. At last, automation of some treatments can be performed to speed up the work.

In this paper, we treat a specific element identified by the archaeologists: a Gothic spire (some fragments are given in Fig. 1). This latter is a distinctive feature of Gothic architecture which occurs on public and religious buildings (an example is given in Fig. 2) but also on smaller size construction such as tabernacles. The number and size of the preserved fragments show that they belong to this latter category.

The problematic is the following. According to the moldings shape, different spires have been identified by the archaeologists. Moreover, the spires geometric characteristics (height, dimensions of the basis, number of crockets on a spire) are unknown. Therefore, computerised tools would be useful to extract and analyze the moldings and crockets from 3D models. As a first treatment before obtaining further information, it is necessary to find a valuable orientation. Unfortunately, traditional methods to find a valuable orientation for a 3D mesh provide unexpected results. Therefore, we address this first step by proposing a new method to determine a 3D mesh orientation.

The paper is organized as follows. Previous work concerning 3D normalization is treated in section 3. In section 4, we describe the method before presenting the results in section



**Figure 2:** Town hall in Brussels: entire spire (left) and detail of the top (right).

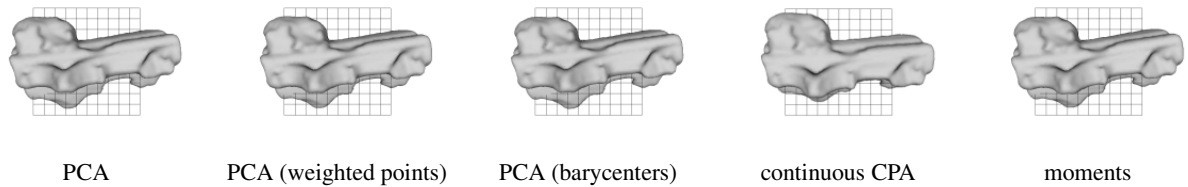
5. Finally, conclusion and future work are considered in section 6.

### 3. Previous Work

Finding the orientation of an object is part of what is generally called 3D normalization. It is a process used for different purposes such as model retrieval and shape analysis [SV01, VS00, NK01, TV03]. It is composed of three transforms - translation, rotation and scaling - to place a 3D mesh into a canonical coordinate frame. In our case, we are only interested by the rotation allowing to identify an orientation.

The common method used is the PCA [GW92] which determines the principal axes from a point set (*e.g.* vertices, centroids of faces [PMN\*00], weighted vertices [VS00]) by solving an eigensystem. An amelioration is proposed in [VSR01] by introducing the CPCA (continuous PCA) that takes into account all points inside the polygons of the meshes.

Other approaches are inspired by the dynamics of rigid bodies [MK93] and consider the higher order moments gathered in the inertia tensor  $I$  composed by integrals over the volume of an object. Then, solving the eigensystem associated to  $I$  provides the axes for which the products of inertia vanish. This approach was applied in [NK01] by using a method to solve the integration developed in [Mir96]. In comparison to the PCA, this method uses volumetric properties instead of a simple distribution of points all over the surface of an object. However, it only treats closed surfaces.



**Figure 3:** Orientation of a fragment with existing methods.

Both of these approaches are sensitive against outliers since the contribution of vertices increases quadratically according to their distance from the principal axis [KCD\*02, KFR03]. Therefore, when outliers are far away from the principal axes, they may perturb considerably the determination of its orientation. Moreover, if the eigenvalues are similar, the choice of the principal axis is ambiguous and axes may switch [NK01].

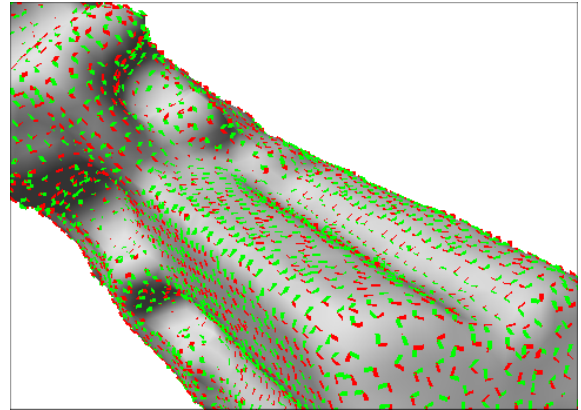
Different approaches have been investigated to cope with these problems in particular situations. In the case of symmetric objects, a better and more robust approximation of the orientation can be computed by analyzing a reflective symmetry descriptor [KCD\*02]. In [CO02], silhouettes of two 3D objects are compared to place both of them into a common canonical coordinate frame. Note that the aim of this method was the comparison of two objects rather than the orientation of an object in an independent way.

To summarize, none of these methods always produce a satisfying result. In the case of the use of PCA and tensor of inertia, this is mainly due to the presence of outliers which perturb significantly the localization of principal axis. In other approaches, the simplification of the problem according to the framework of the application restrains their use.

#### 4. Fragments Orientation

Applied to the archaeological fragments we have, the existing methods to find a valuable orientation fail (as illustrated in Fig. 3). Indeed, a Gothic spire is composed of an elongated core, generated by the extrusion of a molding, along which equally spaced ornamentations appear. The expected orientation is associated to the direction of the fragment core and the crockets are considered as outliers perturbing the principal axis determination. Therefore, a criterion is needed to distinguish crockets and the elongated core.

An approach consists in selecting significant areas on the surface and in associating a direction for each of them. Afterwards, an average can be performed on the selected orientations to determine the principal orientation. The main difficulty is the identification of these significant areas on the surface and the determination of an orientation. For this purpose, a criterion should be defined precisely.



**Figure 4:** 3D surface with principal directions computed on each vertex.

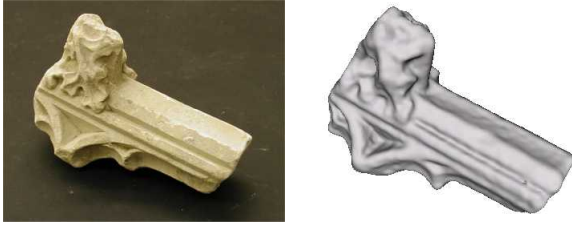
#### 4.1. Method Description

The geometric nature of the core allows us to consider it such as a developable surface with particular geometric properties [dC76]. Indeed, all the points on this kind of surface have, at least, one principal curvature equal to zero (the case where the two principal curvatures are equal to zero appears when the point is on a planar surface). Moreover, when only one principal curvature is equal to zero, its corresponding principal direction indicates the orientation of the extrusion. The method we propose takes into account these properties and is used as explained in the following sections.

#### 4.2. Significant Areas

We define a significant area when one of its principal curvatures is close to 0 while the other is not equal to 0 (to discard planar areas).

On a surface represented by a mesh, we consider vertices to identify these areas. Let  $v$  be a vertex associated to a finite surface element. It is characterized by its two principal curvatures  $\kappa_1$  and  $\kappa_2$  and its two respective principal directions (these latter are visualized on a 3D mesh in Fig. 4). In this case, a vertex belongs to a significant area if one of its principal curvature  $\kappa_1(v)$  is close to 0 and if the absolute value of



**Figure 5:** Example of an archaeological fragment (left) and its virtual representation (right).

the second curvature  $\kappa_2(v)$  is greater than a threshold. Moreover, the principal direction associated to  $\kappa_1$  is considered as a potential orientation for the object.

### 4.3. Voting

Once all the potential orientations are identified, we compute their histogram and deduce a principal orientation from the analysis of their distribution.

Practically, an orientation owns two directions. To avoid any confusion, we reduce the domain of possible orientations to a half sphere. For this purpose, we flip each principal direction  $\vec{d} = (d_x \ d_y \ d_z)^t$  when its  $x$  component is negative.

The determination of the principal orientation is realized in a  $(\phi, \theta)$  space where  $\phi$  and  $\theta$  are spherical coordinates. Since  $\vec{d}$  is normalized and  $d_x$  is positive, we have the following expressions

$$\begin{cases} \phi = \arctan\left(\frac{d_y}{d_x}\right) \\ \theta = \arccos(d_z) \end{cases}$$

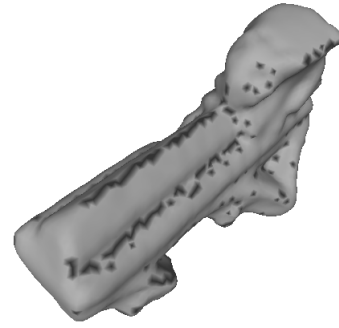
with  $\phi \in [-\frac{\pi}{2}, \frac{\pi}{2}]$  and  $\theta \in [0, \pi]$ .

An accumulator  $A[\phi, \theta]$  is used and all its values are first initialized to 0. Each principal direction  $\vec{d}$  corresponding to an area of interest is transformed into spherical coordinates  $(\phi, \theta)$ . Afterwards, it is discretized into  $(\phi_d, \theta_d)$  and the accumulator value  $A[\phi_d, \theta_d]$  is incremented.

### 4.4. Orientation Identification

Once all the principal directions have voted, the maximum value, located in  $A[\phi_M, \theta_M]$ , can be interpreted as the principal orientation of the object. This is a valuable estimation since a high accumulation of potential orientations denotes the presence of an elongated part in the object.

At last, to determine the complete canonical frame, we need to compute the two remaining principal axis. For this purpose, there are different possibilities. For example, looking for another maximum in the accumulator or applying a PCA on the vertices of points projected onto a plane perpendicular to the principal orientation previously computed.



**Figure 6:** Visualization of significant areas identified on a mesh with the parameters  $\epsilon = 0.05$  and  $\delta = 0.2$ .

In our implementation, we used this latter method to obtain three orthogonal axes. Therefore, the first axis is associated to the fragment orientation and the second axis expresses the orientation along which the variability of the points describing both the molding and the crocket shapes is maximal.

## 5. Results

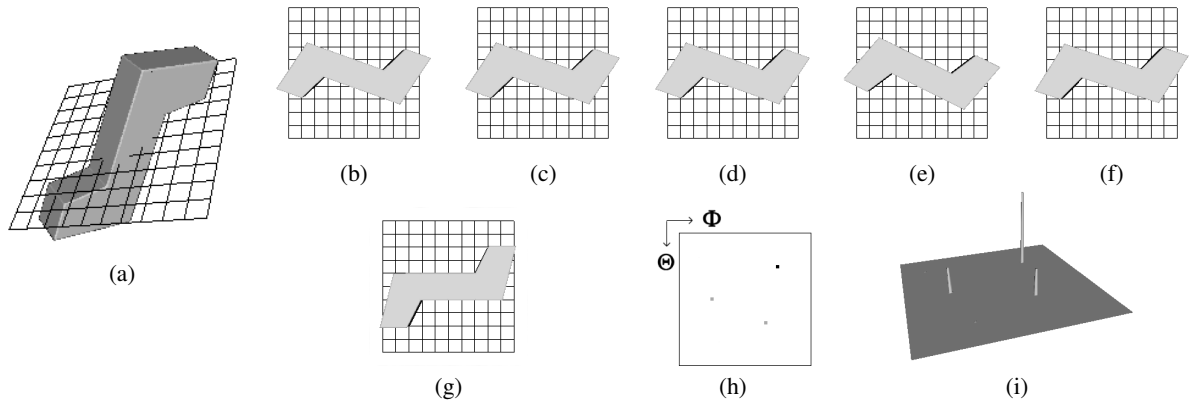
### 5.1. Data Acquisition

To work efficiently with virtual representations of the fragments, we needed to have highly-accurate 3D models since the level of detail on the fragments can be less than one millimeter. Moreover, due to the fragility of the fragments, the 3D acquisition system should use a non-contact technique. These constraints suggested a system based on the laser strip triangulation. The device we used is the 3D scanner *FastSCAN<sup>TM</sup>* commercialized by *Polhemus* [Pol] and we realized the acquisition of a selection of 54 fragments according to their interest for the archaeologists, resulting in 3D meshes with a vertices decimation close to 1mm. A fragment and its virtual representation is shown in Figure 5.

### 5.2. Implementation

The determination of the principal orientation of a 3D mesh depends on the principal curvatures and the principal directions. Therefore, finding a correct approximation for these differential parameters is essential. In practice, to compute these parameters on a polygonal surface, methods consider a vertex  $v$  and its adjacent vertices to approximate the shape of the surface around  $v$  from which the evaluation is done [Ham93]. Others methods propose discrete versions of differential geometry properties [Tau95, MDSB03, CSM03]. A comparative study of these different methods is proposed in [GI04]. In our implementation, we use the algorithm proposed in [Tau95].

The method has two input parameters  $\epsilon$  and  $\delta$  necessary



**Figure 7:** Orientation of a synthetic mesh: (a) original mesh and orientations obtained by (b) PCA (c) PCA with weighted vertices (d) PCA with weighted barycenters (e) continuous PCA (f) moments (g) our approach. Concerning our method, we show (h) the accumulator produced from the mesh (to make the figure readable, the dots were enlarged). A better visualization is proposed in (i) where the accumulator is represented by an elevation map.

to define how significant a vertex is. In the case of a vertex  $v$  corresponding to a significant area, let's note  $\kappa_1(v)$  the curvature associated to the value close to 0 and  $\kappa_2(v)$  the other one. Then,  $v$  is significant iff the absolute value of  $\kappa_1(v)$  is inferior to  $\epsilon$  and the absolute value of  $\kappa_2(v)$  is greater than  $\delta$ . An example of significant areas on a mesh is illustrated in Fig. 6.

Concerning the principal orientation identification in the accumulator, the research of the maximal value is not always relevant due to the presence of noise in the models. Therefore, a better choice consists in considering a neighborhood defined by a disk, its radius  $r$  being another parameter of the algorithm and its center being the expected orientation.

### 5.3. Synthetic Data

The synthetic data we used is an elongated object on which outliers appear at their extremities (Fig. 7(a)) to perturb the evaluation of its orientation. In order to work with relevant approximation of the curvatures and principal directions, the mesh was subdivided to obtain a dense and homogeneous vertices distribution over the surface.

The results provided by existing methods and by our method are compared in Fig 7(b) to (g). As expected, they show the interest of our approach since it is the only one that allows to identify correctly the orientation associated to elongated parts.

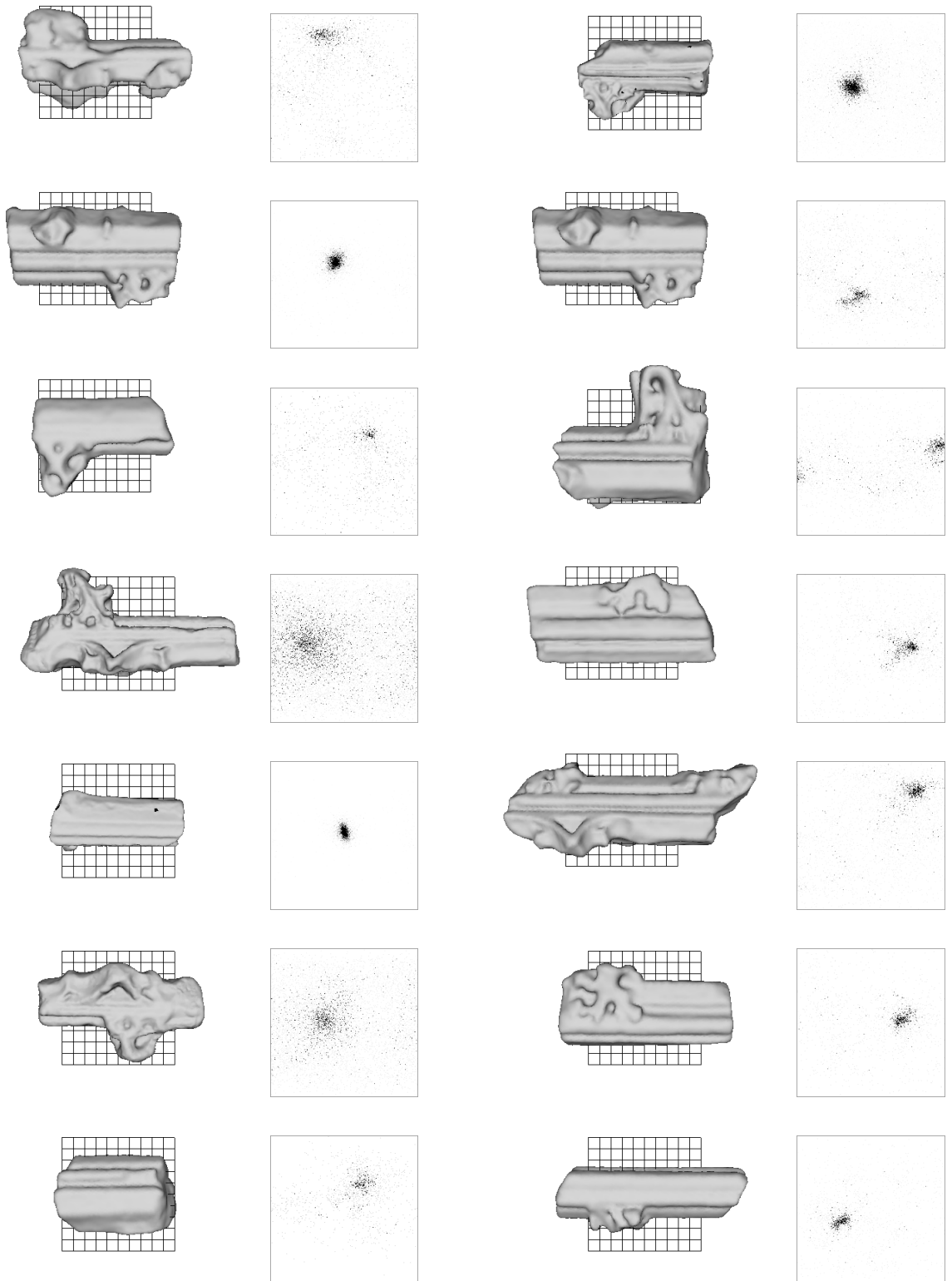
### 5.4. Archaeological Fragments

Concerning real data, since noise is present and can perturb the differential parameters approximation, an important task

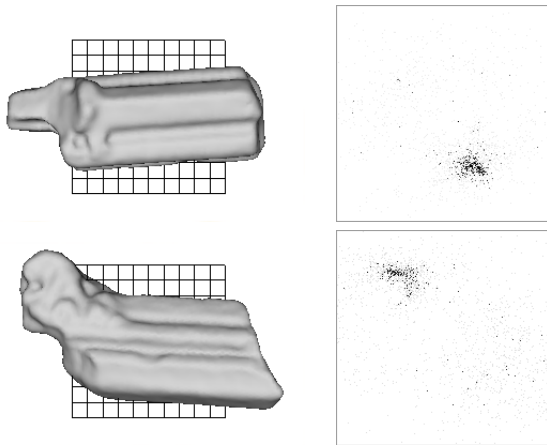
is the choice of the parameters  $\epsilon$ ,  $\delta$  and  $r$ . In the following, we have chosen experimentally  $\epsilon = 0.05$ ,  $\delta = 0.2$  and  $r = 5$ .

For the 54 fragments we tested, we obtained satisfying results. Some examples are given in Fig. 8 and they show that, even though the 3D meshes contain more or less important crockets, the concentration of similar principal directions in the accumulator allows us to find correctly the orientation of the objects without any ambiguity. The only problem we met is the potential orientations distribution in the accumulator when it is too spread. But generally, the parameter  $r = 5$  was enough to detect it correctly. We mainly encountered problems with only two fragments we didn't succeed to treat. The results we obtained are given in Fig. 9. In these two cases, the potential orientations distribution is too spread, due to the break surface shape which is more or less elongated in the direction of the fragment.

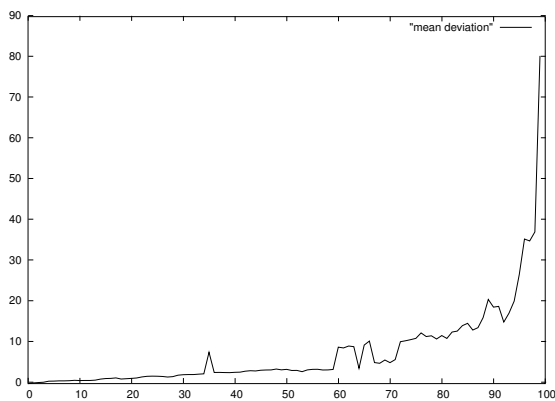
In parallel with these results, it is interesting to test the robustness of our approach against simplification because the size of the models is often reduced by a simplification process to ease their storage. For this purpose, we treat all the meshes as follows. For each mesh, we first consider the orientation found with our method as the correct orientation. Then, we apply a series of  $k$  simplification processes where, from the  $n$  vertices contained in the original mesh,  $\frac{in}{k}$  vertices were removed at the  $i$ -th simplification process (with  $0 < i < k$ ). For this simplification process, the software QSlim [Gar] was used. Finally, we compute the deviation between the orientation deduced from the original mesh and the orientation found for the successive simplified meshes. In Fig. 10, we show the average of the deviations according to the simplification process level. It appears that the results are stable until a simplification process implying the deletion



**Figure 8:** Examples of 3D orientations for the archaeological fragments and the associated accumulator. For a better visualization of the accumulator, the contrast has been enhanced.



**Figure 9:** Examples of fragments on which our method fails.

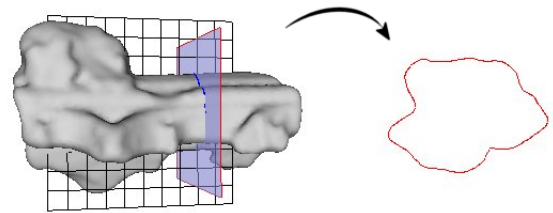


**Figure 10:** Mean deviation (in degrees) of the orientation according to the percentage of vertices removed (on the  $x$ -axis) during the simplification process applied on the original mesh. The mean is realized on a selection of 33 fragments and we chose  $k = 100$ .

of 10% of the initial vertices. The degradation of the results can be explained by the fact that decimation in the initial mesh allows to have a relevant surface description. A simplification process, which implies vertices deletion, coarsens the object approximation and the differential parameters approximation became less and less accurate as simplification occurs.

## 6. Conclusion and Future Work

We introduced a new method to find the orientation of 3D objects. It is based on the assumption that the principal orientation is in relation with the principal directions, associated to a principal curvature close to zero, appearing the most frequently on the surface and characterizing its elongated parts.



**Figure 11:** Molding extraction by cutting virtually the fragment with a plane perpendicular to its orientation in an area where an ornamentation is not present.

The results we obtained with data produced by a 3D acquisition system show its efficiency in a practical case compared with existing methods. Moreover, they allow to go further to get new information from the fragments such as the molding shape (which can be simply deduced as illustrated in Fig. 11) since it is an important attribute for this kind of fragment.

Moreover, this method can be used for other kind of objects where the orientation of surfaces generated by an extrusion should be detected. Nevertheless, we have just shown the feasibility and relevance of this approach. Although convincing results have been shown, a stronger validation would be needed to show its extensibility to other classes of objects.

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