Layered Dynamic Control for Interactive Character Swimming

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Abstract

This paper proposes a layered strategy for controlling character motion in a dynamically varying environment. We illustrate this approach in the context of a physically simulated human swimmer. The swimmer attempts to follow a dynamic target by augmenting cyclic stroke control with a set of pre-specified variations, based on the current state of the character and its environment. Control of a given swim stroke is decomposed into three layers: a basic stroke sequence, a set of per-stroke control variations, and a set of continuously applied control variations. Interactive control of the swimmer is possible as a result of an efficient physical simulation using a simplified fluid model. Our results show layered dynamic control to be an effective adaptive control technique in well conditioned physical simulations such as swimming, where simulation states resulting from control errors are recoverable.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Interaction, Physical simulation, Fluids, Character animation

1. Introduction

Computer animation research has produced a number of models for synthesizing motion of virtual humans. Much of this research has addressed the control of specific forms of motion such as walking [LvF96], running, cycling [HWB*95], and acrobatics [WH96]. Character motion in fluids is a relatively unexplored area, with prior work addressing locomotion of non-human characters such as evolving creatures [SIM94] and fish in water [vF93] [TT94] or birds in air [WP03]. Character state during swimming is stable and changes smoothly compared to motions such as walking that take place on the fringes of highly unstable character state. Swimming thus has a graceful unique visual quality unlike other motions to an extent that the term "fluid motion" is often used to describe the visual appearance of any graceful movement. Inspired by the desire to generate "fluid character motion", at a rate suitable for interactive control, this paper addresses the control and animation of swimming human figures as shown in Figure 1.

Our approach is based on the physical simulation of a dy-

namic human figure model in a simplified fluid model. The simulated swimmer follows a target under interactive user control. Swim strokes are cyclically defined using a pose control graph (PCG) and applied as input to proportional-derivative (PD) controllers within a physical simulation. The smooth transitions of character state when swimming, relative to say, walking, allows us to develop a layered control algorithm to make such goal directed physical simulations more efficient. The abstraction of various aspects of character control into three independent layers shown in Figure 2 makes interactive control of the overall simulation tractable.

In addition to the basic stroke simulation layer, a per-cycle control layer applies stroke variations on a per cycle basis to achieve a desired change in the character state, such as turning. These variations are defined heuristically, based on techniques developed by swimming coaches, observing real swimmers or through pre-simulation motion tests of the virtual swimmer. A continuous perturbation layer continuously adapts the character pose locally to the current state of the environment, such as adjusting the wrists based on the local relative fluid velocity to increase thrust. At a higher level the selected stroke with which to follow a target is typically specified by the animator.

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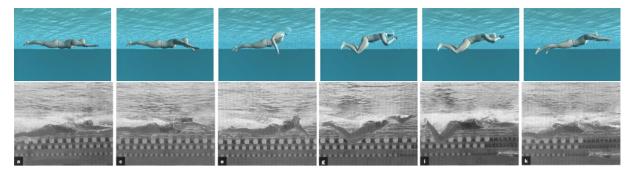


Figure 1: Comparison between a synthesized breast stroke and a real swim sequence (from [MAG03]).

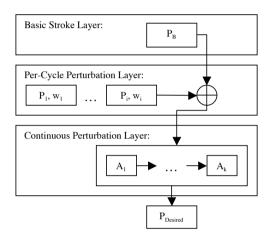


Figure 2: Swimmer architecture: The basic stroke layer cycles through a PCG to generate a pose P_B . The per-cycle perturbation layer blends a set of pose perturbations P_i , with P_B . The continuous perturbation layer makes local pose changes, adapting blended pose to the dynamic environment.

This paper thus contributes a system to generate visually realistic swimming characters under interactive control. The animation algorithm is based on independent layers of dynamic control, which is shown to be an effective strategy for simulated characters in well conditioned environments (where errors in control input are not irreversible). The generated swimming motion finds applications in animation, games, for swimmer education or as an analysis tool for designing more effective swimming strokes.

The rest of the paper is organized as follows: Section 2 reviews related work. Section 3 describes the swimmer, fluid and their interaction model used to perform the physical simulation. Section 4 describes our layered dynamic control algorithm. Section 5 provides implementation details and discusses the resulting animations created by our system. Section 6 concludes with directions for future research.

2. Related work

Our work is related to four areas of research: dynamic control for humanoids, dynamic control in a fluid environment, fluid solid interaction, and the biomechanics of swimming.

Controlling Human Motion

Algorithms for physics-based running, cycling, vaulting and platform diving were described in [HWB*95] and [WH96]. The character is modeled using a system of rigid bodies and muscle force using PD controllers. Biped walking has also received a lot of attention in the research community. [LvF96] described a dynamic control algorithm for physically based walking animation based on the idea of limitcycle. [FvT01] showed the benefits of solving complex dynamic control problems by sequentially applying various simple dynamic controllers. This paper illustrates that composing various simple controllers in a layered fashion in well conditioned simulation environments can also provide an effective solution to complex dynamic control problems. Our system like the body of research above, is based on physical simulation without which achieving visual realism of complex figures in a dynamic environment is a difficult problem.

Dynamic Control in a Fluid Environment

In prior research on controlling human motion, the physical environment does not influence a character in the immersive way in which a fluid field does. The interaction between a swimmer and fluid for has been addressed by [vF93] and [TT94] for locomotor control of fish in 2D and 3D respectively. In [vF93], a simple 2D "fish" character learns robust target following through stochastic optimization of controller parameters. In [TT94] manually-designed locomotion controllers provide the foundation for rich behavioural repertoires of fishes living and interacting in a virtual marine world. Modeled as spring-mass systems with a simple sinusoidal actuator propelling the fish through water, low-level controllers were directed by a behavioral model to make the fish "swim" or "turn" and perform higher-level functions such as schooling. Such a behavioral system could be used to specify the target followed by our virtual swimmer. More complex swimming figures were seen in [SIM94], virtual creatures evolved to be better swimmers using genetic algorithms to modify skeletal structure and muscle forces. Fitness evaluations (measures of success) act as an optimization process in order to drive the evolution towards the desired behavior. Research on flight [WH91] [WP03] looks at the effects of aerodynamic forces. [WH91] linearized the Navier-Stokes equation to provide a set of primitives to define flow fields for animating aerodynamics. [WP03] parameterized bird flight using a small set of wing beat parameters and proposed a generalized optimization algorithm to solve for the optimal parameters for different bird models to perform various maneuvers by attempting to satisfy a trajectory defined the user.

Fluid Solid Interaction

The hydrodynamic or aerodynamic models used in the previous section are highly simplified. More sophisticated models for animating fluid solid interaction is an active area of research [GHD03] [CMT04]. [GHD03] uses a spring mass system to model the interaction between deformable objects and fluid. [CMT04] solves the problem of rigid body and fluid interaction by incorporating the rigid body constraints into the boundary condition in the fluid.

Biomechanics of Swimming

A wealth of research can also be found on human swimming in the biomechanics and athletic training community. Research on swimming biomechanics is primarily focused on analyzing drag and lift acting on a swimmer's body, a survey of which can be found in [MAG03]. [BdH*95] and [PKH*00] use mechanical models to measure the drag and lift on hands, arms, legs and feet of a swimmer while [BR02] uses CFD (computational fluid dynamics) to simulate the flow around a static arm mesh. [SH03] analyzes the forces on an arm model during motion captured from a real swimmer. We use the observations in [SH03] to motivate the segmentation of swimmer control into the three independent layers seen in Section 4.

3. Swimming Dynamics Simulation

Our system simulates the dynamics of swimming using rigid bodies and a simple fluid mechanics model.

3.1. Swimmer Model

The character is modeled by 16 segments of rectangular rigid bodies connected by 16 joints and 39 degrees of freedom (DOF) to form a humanoid swimmer. A mixture of Euler angle and quaternion (ball joint) representations are used. Figure 3 shows the DOF of our virtual character. Parameters for the rigid bodies are obtained from [WH96]. The total weight of the swimmer is 89.57kg and the density is 1.044g/mL making the swimmer sink in water. Breathing has a significant impact on the torso volume and thus on the buoyancy of the swimmer. These parameters in [WH96] are measured on cadaveric data, where the measured volume of

the torso is lower than that of a live swimmer with air filled lungs. We increase the torso volume by up to 3L empirically, which results in a buoyancy of 91kg in water, transitioning our swimmer from a sinker to a floater. The muscles of the virtual swimmer are modeled by PD controllers.

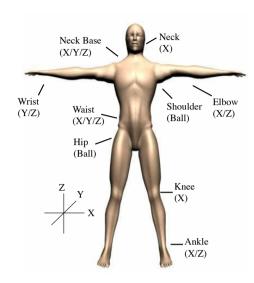


Figure 3: Joint DOF in the swimmer model. The character is initialized to lie on its stomach in the pose shown.

3.2. Fluid Model

We use a simple fluid model in our simulation. The hydrodynamic force is based on a thin plate model from fluid dynamics literature [HOE65][HB75]. Buoyancy is approximated using bounding ellipsoids. The flow field does not dynamically interact with the character.

Hydrodynamic forces

We separate the hydrodynamic forces into drag, lift and friction. Drag and lift are generated by displacing fluid molecules while the swimmer moves its body. Friction comes from the attraction between the body surface and the fluid molecules due to viscosity. Assuming the fluid has steady flow as does most biomechanics literature on swimming [TOU02], the magnitude of the drag and lift can be computed as:

$$f = \frac{1}{2}C(\alpha) \cdot \rho \cdot a_n \cdot |\nu_n|^2 \tag{1}$$

where $C(\alpha)$ is the drag/lift coefficient, ρ is the density of the fluid, a_n is the projected frontal area of the polygon, and v_n is the normal component of the relative velocity of the fluid. The magnitude of the friction force is computed by

$$f = \eta \cdot a_t \cdot v_t \tag{2}$$

where η is fluid viscosity, a_t is the projected tangential area and v_t is the tangential component of the viscosity. The forces are applied along vector D, L and T as shown in Figure 4. Given the normal of the polygon, N, and the relative velocity of the fluid, V, we consider the hydrodynamic model to determine D, L, T and $C(\alpha)$. The model we use is a thin rectangular plate with aspect ratio 0.2. We use an extra forward vector, F, to indicate the long side of the rectangle. D is the normalized projection of V on the plane spanned by N and F. L and T are defined as follows:

$$U = \frac{D \times F}{|D \times F|}, L = \frac{D \times U}{|D \times U|}, T = \frac{V - (V \cdot D)D}{|V - (V \cdot D)D|}$$
(3)

The angle of attack, α , is computed as:

$$\alpha = \cos^{-1}(D \cdot N) \tag{4}$$

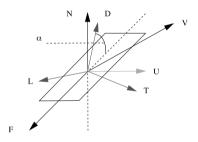


Figure 4: Applied fluid force direction vectors.

The drag and lift coefficient can be approximated by interpolating the results summarized in Figure 5. Only front facing polygons $(N \odot V > 0)$ contribute to force. Back facing polygons are assigned a force of magnitude zero.

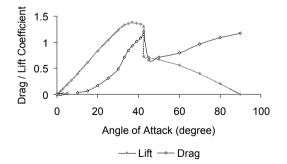


Figure 5: The drag and lift coefficient of a fully immersed thin plate at a given angle of attack for a low Reynolds number. The data is obtained from [HOE65] and [HB75].

We sum up forces on all polygons of a segment and apply

the external force and torque coming from the fluid to the corresponding rigid body.

Buoyancy

We approximate the effect of buoyancy based on the submerged ratio the ellipsoids representing each body part, computed as illustrated by Figure 6. Since the ratio of volume is preserved under affine transformation we compute this ratio by first finding the point of intersection I_E of the fluid surface with a principal axis of the ellipsoid. This point is transformed back to a point in the unit sphere I_C subtending an angle θ as shown in Figure 6. The ratio of submerged ellipsoid volume is then $(1 + \cos(\theta))/2$.

Swimmer-fluid interaction

The motion of the swimmer through the fluid flow field does not influence the fluid flow field in our simplified fluid model. We approximate the flow of the fluid around the character based on the swimmer's velocity and orientation. V_F , the velocity of the fluid near the swimmer, is set to be proportional to V_S , the swimmer's velocity. V_F is determined. We use sin^3 , the drop-off function of a cylinder's drag coefficient in [HOE65] to approximate the effect of profile drag at different angle of attack, specifically:

$$V_F = ((r_{Max} - r_{Min})\sin^3(\alpha) + r_{Min})V_S$$
 (5)

where r_{Max} and r_{Min} are user defined maximum and minimum ratios and α is the angle between V_S and the z-axis of the swimmer's pelvic frame. In addition, the user can create any desired flow field using primitives defined in [WH91].

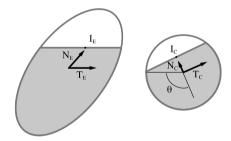


Figure 6: The ratio of volume of partially submerged ellipsoid is computed by transforming it to a unit sphere.

4. Dynamic Control

Given the physical models of the swimmer, fluid and their interaction, we now describe the layered dynamic control algorithm that drives the swimmer to follow an interactively specified 3D target. There are three layers in our system as shown in Figure 2. Each layer operates independently and the combined results of the three layers result in desired swimmer pose that forms the PD control input to the physical swimming simulation. Figure 8 summarizes the different effects of each layer on the resulting motion.

4.1. Well-conditioned environment

The layered strategy for interactive dynamic control is based on an implicit assumption that the forces in our fluid environment are continuous and that there are no character states in system from which the controller cannot recover. This allows us to separate global control of the character into independent layers that are applied greedily over a current window of time, instead of using global optimization given a pre-specified trajectory as in [WP03]. While general fluid environments can be quite turbulent our simplified steady flow model produces smooth interaction forces between the fluid and the swimmer. As an example Figure 7 shows the external forces on the swimmer's left hand during one breast stroke cycle. Such a control strategy is harder to apply to motion such as walking, where contact forces result in sharp discontinuities in the profile of forces over time. This paper exploits this well-conditioned property of a fluid environment and proposes a continuously adapting scheme in the continuous perturbation layer to augment the stroke.

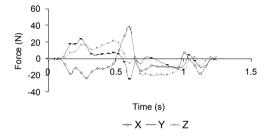


Figure 7: External force on the left hand of the swimmer expressed in world space. One cycle of the breast stroke with the character starting at rest is shown.

4.2. Basic Stroke Layer

The basic stroke layer takes a sequence of poses defining a stroke cycle and interpolates the appropriate pose for the swimmer at the current time. Figure 9 illustrates how the desired pose drives the motion of the character. This layer determines the basic form of a stroke used by the swimmer. The pose cycles for strokes in our system are defined empirically based on their verbal and visual description in [MAG03]. Key poses are defined for each joint separately, reducing redundant poses in our sequence. In the breast stroke, for example, the legs are not actively moving during the sweeping phase of the arms and need not to be explicitly specified. Independently keying joints also makes the incorporation of pose perturbations from other layers straightforward.

4.3. Per-Cycle Perturbation Layer

This layer defines perturbations to the basic stroke cycle as motion modifiers to empirically accomplish specific tasks like turning while swimming. These perturbations, like the basic stroke layer are empirically defined. In our system we use this layer to change the direction of the swimmer using six perturbation sequences. These sequences correspond to the modifications to a basic stroke that the swimmer would employ to achieve pitch, yaw and roll in either direction. We define these perturbations relative to the basic stroke.

The input to this layer is the current simulation time, target position, and the interpolated pose from basic stroke layer. At beginning of each stoke cycle, a weight is determined for each perturbation sequence and this weight scales the values of perturbation to be applied for that cycle. The weights are computed based on the position and orientation of the swimmer relative to the target. Through preprocessed simulations of the swimmer at various velocities in a static fluid, we determine the weight value needed to turn the swimmer through a given angle. The weights scale and blend the six perturbation sequences with pose sequence output from the basic stroke layer. Figure 10 illustrates one cycle of turning left while treading water using this layer. Figure 11 shows the change in character orientation over time.

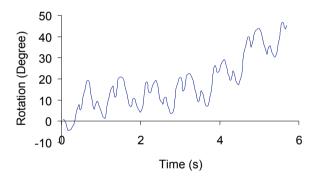


Figure 11: Turning left while treading water. Rotation is measured about the character's yaw (Y) axis.

4.4. Continuous Perturbation Layer

The continuous perturbation layer allows us to dynamically vary the motion of a character in a local fashion based the current environment and a set of heuristics that are generally valid in well-conditioned environments. The input to this layer is the target position, the blended pose from the per-cycle perturbation layer and a set of rules. These rules are based on the intuitive strategies people take when swimming and are applied to modify the blended pose continuously. Currently, we apply two rules in our system seen in Figure 12:

Palm Orientation The palms should face into the direction of the fluid if more force in required in the desired direction. Conversely, the palms should slice through the fluid if less force is desired.

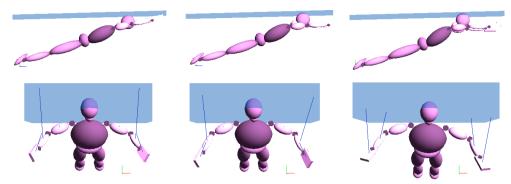


Figure 8: The effect of subsequent layers on the resulting motion. a) Basic stroke layer drives the character's desired pose. b) A per-cycle perturbation aligns the character's arms with the fluid surface. c) A continuous perturbation reorients the palms to maximize forward thrust on the palm.

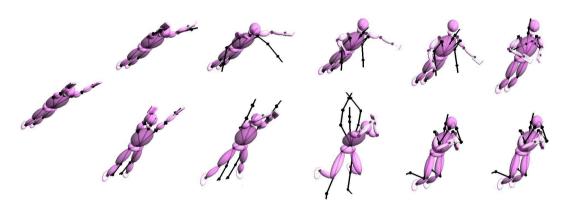


Figure 9: The difference between desired (skeletal line) and the actual poses in simulation. One cycle of the breast stroke is shown (clockwise from left).

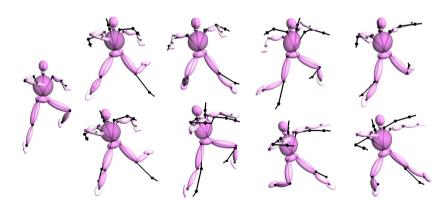


Figure 10: The actual and desired pose of a character turning left while treading water. One cycle is show going clockwise from the left.

 Arm Reach The arms should reach out further if more force is desired in the current desired direction and the arms should be closer to the torso for less force.

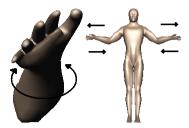


Figure 12: Continuous perturbations: a) palm orientation and b) arm reach.

Figure 13 demonstrates that applying the palm orientation rule to our basic treading water sequence improves forward propulsion of the swimmer.

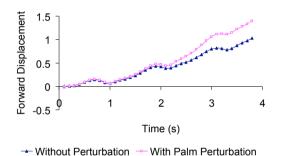


Figure 13: Effect of palm orientation control on the forward displacement of character's root (pelvis).

5. Implementation and Results

The layered algorithm was implemented using the commercial toolkit *SD fast*. Strokes are defined as keyframed joint angles exported from commercial animation systems. The target is interactively specified by the user with a mouse or read as a trajectory from a file.



Figure 14: Tracking a figure of eight.

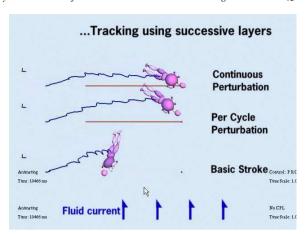


Figure 15: Effect of layers on target tracking in a strong current.

To test the capability of our system, we created several different target trajectories for the virtual swimmer to follow. Figure 14 shows the swimmer tracking a figure of eight. Figure 15 shows the effects of subsequent layers on tracking a target with a cross current in the fluid. Note that the basic stroke layer leaves the swimmer entirely at the mercy of envirnonmental forces and unable to follow the target. The percycle perturbation layer is able to cause the character to face into the current every cycle to maintain a horizontal heading. Finally the continuous perturbation layer is able to produce greater thrust as needed and allows the swimmer to track the target more closely.

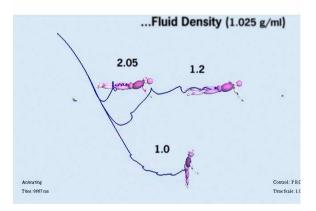


Figure 16: *Varying the density of the fluid.*

We also find the approach to be robust to the manipulation of various swimmer and fluid parameters as shown in Figure 16, which shows the swimmer in varying fluid density. Note how the swimmer sinks less with increasing fluid density. The swimmer struggles to recover and follow the horizontally moving target in the least dense fluid due to a lack of sufficient thrust. The swimmer similarly struggles in a very dense fluid on account of a greater drag force.

We implemented three strokes to test the versatility of our approach and visually compared them with video footage of real swimmers. Figure 1 shows the breast stroke, Figure 17 the front crawl and Figure 18 shows the swimmer treading water.

6. Discussion and Future Work

The main contribution of this paper is to propose a continuously adaptive layered algorithm for interactively controlling a virtual swimmer in a dynamic environment. The figures in the paper and the accompanying video show the approach produce visually pleasing results and to be robust to changes in the environment and model parameters.

Currently, our fluid model is unaffected by the motion of the swimmer through it. This is an oversimplification of reality and its biggest drawback is that the thrust generated is lower than that observed in real swimmers. One possible future direction is to incorporate more accurate fluid forces while maintaining interactivity of the simulation using, for example, the panel method [KAT01]. A full CFD simulation with unsteady flow is, however, necessary to truly capture the motion of swimming characters.

There are a large number of unexplored questions in controlling the motion of characters in fluids. At a higher level the selected stroke with which to follow a target is typically specified by the animator. Automating control over stroke selection best capable of following a given target, in a manner similar in spirit to Faloutsos et. al [FvT01] is one future direction. Higher level strategies for the control of multiple swimmers, such as in a water polo game, is a subsequent step. Most importantly, unlike motion on land which has been well studied and can be analyzed using motion capture, movement in water is less explored and its capture using vision techniques is difficult. We hope that this paper will stimulate future work in this area and serve as a tool to coaches and athletes for swimming education and stroke analysis and improvement.

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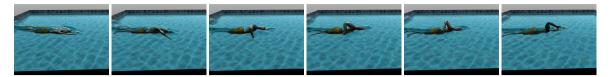


Figure 17: Front crawl.



Figure 18: Treading water.

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