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Progress in Detection of Axis-Aligned Planes to Aid in Interpreting Line Drawings of Engineering Objects

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Abstract

Freehand sketching is an important part of the conceptual design process, and the increasing number of recent sketching applications shows a developing awareness of this importance. We aim to provide an automated tool to turn engineers' freehand sketches into CAD models. This would allow engineers to spend their time more productively and to be more creative.
One natural component of such a tool would be a process for identifying axially-aligned planes implied by a natural line drawing. We present an algorithm for identifying such planes. We illustrate its utility by presenting two uses: identifying planes of mirror symmetry in objects and constructing the hidden topology of objects.

Categories and Subject Descriptors (according to ACM CCS): J.6 [Computer Aided Engineering]: Computer Aided Design

1. Introduction

Freehand sketching is an important part of the conceptual design process, and the increasing number of recent sketching applications [Gri97,Lip98,Mit99,SG00,Var03,FBC*04, FOM*04,JLA04,SC04] shows a developing awareness of this importance. Previous studies such as Jenkins [Jen92] have shown that engineers and architects, when creating a new design, start by sketching ideas freehand on paper, and once a satisfactory concept has been found, manually enter the design into a CAD package. Automating this process would remove a bottleneck.

Thus, our eventual aim is an automated tool to turn engineers' freehand sketches into CAD models—ideally, boundary representation solid models of the most plausible 3D interpretations of the sketches. This would allow engineers to spend their time more productively and to be more creative.

Reliance on manual intervention, using e.g. menus, is undesirable. Firstly, the details of using a complex computer interface distract the designer from creating an idea. Secondly, some applications of sketch understanding are unsuited to such intervention—for example, the cameraphone application of Ferrugia et al [FBC*04].

Two important stages in the process of constructing a three-dimensional object from a two-dimensional *natural line drawing* (i.e. hidden lines not shown) are *(i)* inferring as much as possible about the three-dimensional structure of the part of the object visible in the drawing, and *(ii)* deducing a structure for the hidden part of the object. These processes are based on *clues*: heuristics derived from the drawing which lead to hypotheses about the object. Clues may be small inferences, such as 'this edge is likely to be axis-aligned', or larger inferences, such as 'there is probably a plane of mirror symmetry here'; they may be derived directly from the drawing or indirectly via other clues. Clues may support one another or contradict one another. In general it is those clues which support one another which are used to reach conclusions about the object.

In previous papers, we have described various techniques which can provide such clues, including heuristics inspired by human perception [VMS05] and various assumptions concerning perpendicularity [MVS05]. This paper describes another such clue: the presence of *axially-aligned planes*.

One motivation for considering axially-aligned planes can be illustrated by an example. See Figure 1. Previous work has considered axially-aligned *edges*, but considering only edges, not planes, can lead to incorrect hypotheses. It is ev-

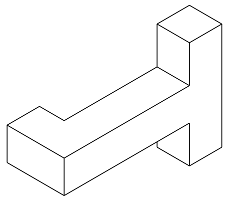


Figure 1: *Line Drawing*

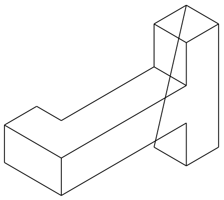


Figure 2: *Bad Hypothesis*

ident that the very short line terminating in a T-junction is aligned with the k-axis (see Figure 3). It is also evident that the uppermost vertex has existing edges aligned with the i- and j-axes. It is certain that there is at least one hidden edge at this vertex, and it is almost certain that this hidden edge (or one of them) is aligned with the k-axis.

If we consider *only* edges, not planes, we might reason as follows: we already know of one partially-visible edge aligned with the k-axis, and we are reasonably certain that we know of a vertex with a hidden edge also aligned with the k-axes. Might these be one and the same? See Figure 2. We could test this hypothesis by examining the geometry, and if we did so, we might discover that the fit was not particularly good (its 2D angle differs from those of known k-aligned edges by nearly 13°). However, we must allow for sketching errors, and although the fit does not look particularly good, it may well be (and, in our implementation, is) within the permitted tolerance—accumulated sketching errors from one side of the object to the other could distort things by this amount.

It is evident to a human that this hypothesis is unacceptable. The two vertices joined by our hypothetical edge lie in different ik-planes. Determining the axially-aligned planes gives us an algorithmic method for rejecting this hypothesis: a k-axis-aligned edge cannot join vertices lying in different ik-planes.

A second motivation for considering axis-aligned planes is the need for identifying symmetry in natural line drawings. Recent symmetry detection algorithms, such as that of Piquer et al [PCM03], start by identifying lines of skewed 2D symmetry in faces. Adapting such an algorithm for use with natural line drawings is problematic when parts of a face are occluded, and entirely inappropriate when (as is the case with Figures 16 and 26) two or more regions correspond to a single face. We present here an entirely different approach which does not require knowledge of the correspondence between regions and faces.

1.1. Glossary

We first define terminology we need in the rest of the paper. These and similar CAD/CAE concepts are described in more detail in textbooks such as [Lee99].

A solid model of a 3D *object* describes the *topology* and *geometry* of its *faces*, *edges* and *vertices*. *Topology* records connectivity between e.g. vertices and edges; *geometry* gives shape and positions e.g. the spatial coordinates of vertices.

A *natural line drawing* [Sug86] is a 2D drawing which represents the object as viewed from some viewpoint, and comprises *lines* (corresponding to visible or partially-visible edges) and *junctions* (where lines meet—most, but not all, junctions correspond to visible vertices of the object). Loops of lines and junctions form *regions*, which correspond to visible or partially-visible faces of the object. Note the careful distinction between 2D ideas (drawings, regions, lines, junctions) and 3D ideas (objects, faces, edges, vertices).

The *frontal geometry* is an intermediate concept between 2D drawing and 3D object (and thus is sometimes called '$2\frac{1}{2}$D'). In a frontal geometry, everything visible in the natural line drawing is given a position in 3D space, but the occluded part of the object, not visible from the chosen viewpoint, is not present.

Junctions of different shapes are identified by letter: junctions where two lines meet are *L-junctions*, junctions of three lines may be *T-junctions*, *W-junctions* or *Y-junctions*, and junctions of four lines may be *K-junctions*, *M-junctions* or *X-junctions*. Vertex shapes follow a similar convention: for example, when all four edges of a *K-vertex* are visible, the drawing has four lines meeting at a K-junction.

When reconstructing an object from a drawing, we take the *correct object* to be the one which a human would decide to be the most plausible interpretation of the drawing.

1.2. Assumptions

We make several assumptions which are standard for sketch interpretation:

- The drawing is of one object only;
- The object is physically realisable (not a trick drawing);
- The object is polyhedral (no curved edges or faces);
- The object is manifold (there are no degenerate vertices or edges);
- There are no 'cracks' (material discontinuities);
- The drawing is a natural line drawing (hidden edges are not shown);
- The drawing shows *only* edges (no shading or shadows);
- The drawing is topologically correct;
- The object is drawn from its most informative viewpoint;
- The drawing is drawn from a general viewpoint (no accidental coincidences);
- The drawing approximates to a parallel projection.

We do *not* make the following assumptions, sometimes made elsewhere, but which we regard as overly restrictive:

- The drawing is geometrically perfect;
- The object is *trihedral*, i.e. exactly three faces meet at each vertex.

We also assume that the drawing is the only information that we have. In particular, there is no opportunity to request later user intervention (for example to select between alternative possible interpretations).

1.3. Structure of Paper

Section 2 describes axially-aligned planes and how they may be identified. Section 3 describes some uses to which they may be put. Section 4 shows some experimental results, and Section 5 presents our conclusions to date and plans for further work.

2. Axis-Aligned Planes

In typical engineering objects we often find that several vertices lie in the same axially-aligned plane, so we have an empirical reason for considering them to be a common and important design feature. In this Section, we discuss axially-aligned planes, and how we identify them.

2.1. Illustration

Using the assumptions that the drawing approximates a parallel projection from a general viewpoint, there exist three object-relative perpendicular axes (i, j and k), none of which is aligned with the 2D drawing x- and y-axes or the perpendicular z-axis added by inflation to $2\frac{1}{2}$D. See Figure 3, noting in particular that the k-axis has a z-component and is not aligned in 3D with the y-axis.

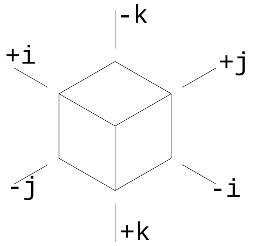


Figure 3: *ijk-Axes*

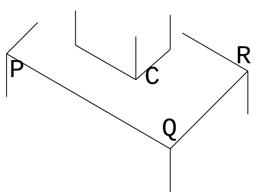


Figure 4: *A Boss*

Provided (i) that we can identify which lines correspond to edges aligned with these axes, and (ii) there are sufficient such edges (which, in practice, there usually are), we can obtain z-coordinates for each vertex of the drawing [LB90,VMS04].

Extending the concept of axis-alignment from edges to planes is straightforward. An axially-aligned plane is a plane whose normal is aligned with one of the three object-relative axes (ijk). Thus, e.g. the normal of an ij-aligned plane is parallel to k-aligned edges. The colour figure shows the axially-aligned planes implied by the drawings in Figures 21 and 26: in the latter, vertices of the same colour in the leftmost figure are in the same ik-aligned plane, in the middle figure the same jk-aligned plane, and in the rightmost figure the same ij-aligned plane.

It is clear from these examples that the existence of axially-aligned planes is something which the human eye rapidly detects when looking at drawings. For this reason, we can consider them a useful clue to the meaning of the drawing.

2.2. Identifying Axis-Aligned Planes

This Section describes an algorithmic approach for identifying axis-aligned planes implied by drawings.

In previous work [VMS04,VMS05] we described methods for inflating natural line drawings to $2\frac{1}{2}$D by finding and using the three object-relative axes as in Figure 3. As part of this process, we attempted to identify which lines in the drawing correspond to edges aligned with the three object-relative major axes (ijk). In this section, we do not discuss the relative merits of the various methods we considered, but just assume that this information can be determined with reasonable reliability and present a self-contained technique which makes use of it.

Given this information, identifying axis-aligned planes is, in principle, straightforward. For example, two vertices joined by an edge aligned with the i-axis must lie in a single ij-plane and also a single ik-plane, but in different jk-planes. Similarly observations hold for edges aligned with the j-axis or k-axis.

Allowing for hole loops, where in the completed object one loop of edges is contained entirely within another (such as in the object portrayed in Figure 10), is straightforward provided that they can be identified as such: the two loops of edges lie in the same plane. See, for example, the boss in Figure 4, where the loop of edges comprising the base of the boss is contained within the loop of edges defining the coplanar face. Clearly C is coplanar with PQR, so if e.g. the edge PQ is i-aligned and the edge QR is j-aligned, the four vertices $CPQR$ all lie in the same ij-aligned plane.

A reasonably successful method for identifying hole loops, and classifying them as bosses or pockets/holes, is by means of *cofacial configurations* as presented in [Var03]. Briefly, we first identify the disjoint subgraphs in the initial drawing, treating T-junctions as subgraph boundaries, and then look for pairs of vertices from different subgraphs which are separated only by whitespace (such as C and Q in Figure 4).

Our starting condition for identifying axially-aligned planes is thus that we have the necessary information concerning axially-aligned edges and possible cofacial configurations.

In our algorithm we represent axially-aligned planes as unordered lists of vertices; each vertex appears in exactly three lists, one each for its ij-aligned plane, its ik-aligned plane, and its *jk*-aligned plane. Initially, we have no knowledge concerning axially-aligned planes, so there are $3n$ unordered lists (where n is the number of vertices), each containing just one entry. The algorithm proceeds by merging lists whenever it can be deduced that vertices lie in the same axially-aligned plane. The algorithm for detecting axially-aligned planes is thus:

- Inputs:
  - A line drawing, inflated to $2\frac{1}{2}$D,
  - A list of edges identified as unequivocally aligned with one of the object-relative (ijk) axes,
  - A list of candidate cofacial configurations compatible with the results of inflation;
- Initially, create $3n$ unordered lists (where n is the number of vertices), allocating each vertex to its own ij-aligned, *ik*-aligned and *jk*-aligned list;
- For each axis-aligned edge, determine the appropriate lists (two of ij-aligned, *ik*-aligned and *jk*-aligned, depending on the edge alignment as described above) corresponding to the start and end vertices and merge them;
- For each cofacial configuration, determine the appropriate lists containing vertices Q and C (ij-aligned, *ik*-aligned or *jk*-aligned, depending on the alignment of edges PQ and QR as described above) and merge them;
- Output the resulting lists of vertices.

We now consider three questions concerning how this algorithm interacts with other components of a line drawing interpretation system:

- Is it worthwhile repeating the inflation process after knowledge of axially-aligned planes has been gained?
- What is the best approach to identifying coplanar faces which are not linked topologically?
- What should be done with T-junctions? Although the human eye distinguishies easily between occluding and non-occluding T-junctions (labelled O and N respectively in Figure 8), no fully-reliable algorithm for distinguishing them is known.

2.3. Repeating Inflation

Clearly, the knowledge that a group of vertices lies in the same axis-aligned planes would be a useful input to the inflation process. Is it worthwhile repeating the inflation process with this new knowledge added? If so, is a second iteration sufficient or should the process be repeated until the results converge?

The results presented in Section 4 assume just one iteration of the inflation process. Detection of axially-aligned planes follows this. The analysis in Section 4 includes a discussion of the utility of adding extra iterations of inflation.

2.4. Geometric Coincidence

What is the best approach to identifying coplanar faces which are not linked topologically? The instances where this happens can be divided into two groups.

The first is where a face contains a hole loop—there is no graph connection between the outer loop and inner loop(s) of the face. Hole loops usually, but not always, correspond to bosses or holes/pockets. For example, Figure 21 contains a boss, and Figure 10 contains a pocket (or possibly a through hole).

The second is where, although the coplanar faces are graph-connected, coplanarity can only be deduced from geometric criteria, not from topological criteria. For example, Figure 21 contains one such pair of faces (brown in the rightmost drawing of the colour figure) and Figure 26 contains two such pairs of faces (green and brown in the central drawing of the colour slide). Recall that in a drawing derived from a sketch, the geometric coincidence will not be exact. If separate groups of vertices are identified as coplanar because they are so to within any particular geometric tolerance, it is possible that groups of vertices which a human could see cannot be coplanar would be identified as coplanar because they too are within that tolerance.

The results presented in Section 4 detect cofacial hole loops by initialising cofacial hypotheses as described in [Var03] and updating the merit of these hypotheses during the inflation process as described in [VMS05]. We have not, as yet, investigated methods for evaluating whether geometric coincidence (or near-coincidence) of planes is or is not intentional.

2.5. T-Junctions

How, if at all, should T-junctions be grouped in axially-aligned planes?

A T-junction occurs when one line terminates at a mid-point of another line; the resulting shape resembles a capital T. Essentially, there are two types of T-junctions: non-occluding T-junctions, which correspond to the three visible edges meeting at a vertex with four or more edges, and occluding T-junctions, which are simply the points at which one edge occludes another; the latter do not correspond to vertices at all. The implications of non-occluding T-junctions are no different from any other junction type, and need no separate discussion.

The implications of occluding T-junctions are different. The *tail* of the T-junction, the point on the occluded line where it disappears from view, does not have the same z-coordinate as the same 2D point on the occluding line. It is possible (and not uncommon) for the occluding and occluded faces to be parallel, and when this is the case they obviously cannot be coplanar.

The question arises: is it better to treat T-junctions as potentially lying in either one or two planes, to attempt to distinguish occluding and non-occluding T-junctions and make use of the corresponding inferences, or to ignore them altogether, when determining axis-aligned planes?

To illustrate the problem, consider, first, the ij-planes (object-relative horizontal planes) in Figure 12. Clearly, there are three such planes, and any algorithm for finding planes should find three of them. However, if we follow previous work (such as the algorithm in [Var03] for determining subgraphs) by assuming that T-junction tails are propagation boundaries, we have a problem. Either we consider only the through lines at the T-junction, in which case we find that we have five ij-planes—the points on the occluding edges at the two T-junctions are not in the same ij-plane as any true vertex, or we consider only the T-junction tails, in which case we find that we have five ik-planes and five jk-planes—the points on the occluded edges also do not correspond to true vertices, and introduce spurious planes. In neither case do we obtain the result that we want, and this suggests that we must approach T-junctions in some other way.

Consider next the non-occluding T-junctions at the tops of the bosses in Figures 22 and 16. These correspond to true vertices, and ideally we should wish to include them in their appropriate axially-aligned planes. These two non-occluding T-junctions are particularly useful as they are an important clue to the mirror symmetry of the two objects. If we knew which T-junctions were occluding and which non-occluding, we could make a sensible choice on this basis, but unfortunately we know of no reliable method for determining this (neither the labelling method of [VMS04] nor the geometric method of [VMS05] can determine this reliably for these two drawings).

Since the algorithm we present in Section 3.1 should be able to cope even without important vertices such as the non-occluding T-junctions in Figures 22 and 16 being included in its input, on the principle of doing least harm, it seems best to ignore T-junctions entirely. Thus the results presented in Section 4 effectively treat all T-junctions as if they were occluding—axis-aligned planes are groups of vertices, and occluding T-junctions do not correspond to vertices.

3. Using Axis-Aligned Planes

Having suggested that axis-aligned planes correspond to something that humans readily identify in drawings, and having given an algorithm for automatic identification of such planes, we now consider what hypotheses can be made on the basis of these clues. We suggest here two uses of axially-aligned planes:

- Detection of planes of object mirror symmetry,
- Direct use in constructing hidden object topology.

3.1. Finding Planes of Mirror Symmetry

Martin and Dutta [MD94] identify several reasons why new designs are often in some way symmetric. Aesthetically, people find symmetrical shapes pleasing, and expect presence of asymmetries only when there is a need for them. Practically, symmetrical components are often easier (and thus cheaper) to make.

Identification of planes of mirror symmetry from a drawing can be a particularly useful clue to the structure of the entire object [Var03]. For example, if the plane of topological mirror symmetry in Figure 21 can be identified correctly, the entire topology of the object becomes known.

In those engineering objects where planes aligned with the three orthogonal axes predominate, planes of mirror symmetry are also often axis-aligned. It should be possible to identify such planes of mirror symmetry given knowledge of visible axis-aligned planes.

Conceptually, we distinguish *topological mirror symmetry* from *geometric mirror symmetry*. Formally, a polyhedron has topological mirror symmetry if there is a non-trivial one-to-one mapping of its vertices with isomorphic edge graphs; less formally, a drawing has topological mirror symmetry if its natural interpretation is an object with topological mirror symmetry. Geometric mirror symmetry necessarily requires topological mirror symmetry, but the converse is not true—topological mirror symmetry can exist even without geometric mirror symmetry. Note, for example, that the objects in Figures 21 and 26 have topological mirror symmetry but not geometric mirror symmetry, and that the topological mirror symmetry implied by the drawing is a useful clue to the topology (and even the geometry) of the complete object (simply adding those extra vertices required for a complete one-to-one mapping enables us to construct most of the hidden topology). This is true whether the absence of geometric mirror symmetry is intentional (as it clearly is in Figure 26), or results from freehand sketching errors—indeed, in Figure 21 it is not clear whether the absence of geometric mirror symmetry is intentional. In general, we are only interested in those topological mirror symmetries for which an approximate geometric mirror symmetry exists.

In designing our algorithm, we make the assumption that if a plane of geometric mirror symmetry exists, more of the near side of the object is visible; hidden parts of the object are mostly on the far side of the mirror plane. This assumption can be used in two ways.

Firstly, it could determine which candidate planes we should examine: we should start at the mid-point of known planes, and work away from the viewer, analysing whether or not such planes can be planes of mirror symmetry. Consider Figure 24, where, if all axially-aligned planes are found, there should be four along two of the axes and three along the third. Since all axially-aligned planes are visible, the mid-point of the known planes is indeed the plane of symmetry: for the three-plane axis, it is the second plane, and for the four-plane axes, it is the mid-plane of (plane parallel to, and mid-way between) the second and third planes. In other drawings, to allow for possible invisible planes, the candidate planes we must examine are all planes and mid-planes from the mid-point onwards.

In practice, the input data produced by Section 2.2 is often not suitable for this purpose—in particular, planes at the rear of the object are commonly not fully-grouped, so there are more known planes at the rear than at the front, and the mid-point of known planes is thus behind the plane of mirror symmetry. To guard against such possible problems, we avoid the short-cut offered by the idea of the previous paragraph and check the mid-planes of all possible pairs of planes. Although this increases the time complexity of the algorithm, this is not a problem in practice as the algorithm takes a small fraction of a second in all cases we have tried. However, it does add to the likelihood of false positives, as discussed below.

Secondly, the assumption that more of the near side of the object is visible gives us a terminating condition for the inner loop of our algorithm. When every vertex on the far side of a candidate mirror plane has been paired with a vertex on the near side, leaving only vertices on the near side unpaired, we have identified as much correspondence between the two as is possible.

The algorithm we use is thus as follows:

- Identify axially-aligned planes (as described earlier)
- Group axially-aligned planes by axes (treat *i*-aligned, *j*-aligned and *k*-aligned planes separately)
- Identify candidate planes and mid-planes for each group of axially-aligned planes
- For each candidate central plane:
  - Start with all vertices unpaired
  - Repeat: find the best pair of vertices (nearest geometrically to mirror-images of one another as reflected through the central plane) and pair them
  - Until: all unpaired vertices are on near side of central plane
  - Assess the candidate central plane: how good is the mirror symmetry?

Even though the planes of symmetry are usually axially-aligned, the things which are symmetric need not be, and often are not. Our algorithm should also identify mirror planes in drawings of such objects. We do not attempt to identify diagonal planes of mirror symmetry such as that in Figure 17.

In principle, our ideas could also be extended to identify non-axially-aligned mirror planes, but these are less common in engineering objects and we have not pursued this.

When considering our assessment criterion for candidate planes of mirror symmetry (how good is the mirror symmetry?), we must beware of *false positives*—mappings which, although not corresponding to topological mirror symmetry, achieve a high score. Consider, for example, Figures 5 and 6. Each figure shows a possible vertex mapping, in which vertices labelled with the same letter are interchanged. Figure 5 illustrates a 'good' plane of mirror symmetry—one that a human would agree represents genuine mirror symmetry—running along the object. Figure 6 illustrates a false positive—one where it is apparent to the human eye that there is no mirror symmetry—running across the object (between the two faces labelled *JHGI*).

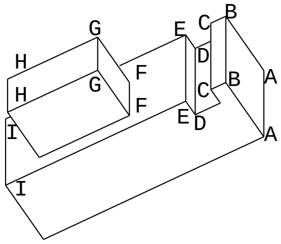


Figure 5: *Symmetry Plane*

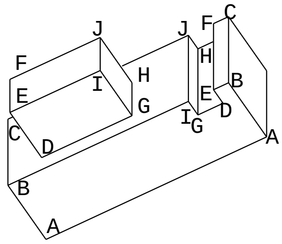


Figure 6: *False Positive*

Note that the final stage of the algorithm (assessment of mirror symmetry) is not straightforward. In particular, it cannot simply use the number of paired vertices: in our examples above, the false positive pairs more vertices than the 'good' plane of mirror symmetry! Neither can we rely entirely on the strict definition of topological mirror symmetry (edge graph isomorphism) since edges can be hidden even when their start and end vertices are both visible. Thus, there must be some geometric input to the assessment criterion. However, it cannot be purely geometric either, since we must allow for freehand sketching errors, and we want to find topological mirror symmetries such as those in Figures 21 and 26 even where there is no corresponding geometric mirror symmetry. Assessment must therefore use both topological and geometric clues, and finding the best balance between the two is not straightforward.

The simplest assessment criterion comes from the algorithm given above. When performing the mapping, we wish paired vertices to be as close as possible to one another's mirror images, and we implement a scoring function based on this distance. It can be argued that this choice is suboptimal, as it could lead to topological mirror symmetries being undervalued (although it can also be argued that topological mirror symmetries which diverge strongly from geometric mirror symmetry should not be considered valuable). Further investigation is needed: other possibilities which should be investigated include (i) how close a 3D line joining paired vertices is to being parallel to the mirror plane normal; (ii) how close the 3D lines joining paired vertices are to being parallel to one another.

There is much work still to be done on this idea. Firstly, where identification of axis-aligned planes is incomplete, there are so many planes (and so many ways of arranging them) that it is almost always possible to find a false positive which achieves a high score using our assessment criteria. Additional work, both on improving the detection of axis-aligned planes and on refining the scoring mechanism, will help to overcome this problem. Secondly, there are occasions where a suggested mirror plane is contradicted by the most probable interpretation of the frontal geometry. Figure 17 is a good example of this: the most probable interpretation of the central feature is that it is entirely concave, and if this is the case the topmost ij-aligned plane is fully-visible and demonstrably not bisected by an axis-aligned mirror plane. Additional work, combining line labelling (to detect occlusion and convexity/concavity) with the inflation techniques of [VMS05], will make assessment of mirror symmetry in such cases more reliable.

3.2. Constructing Hidden Topology

Our preferred approach to constructing the hidden topology of an object is essentially a search through the space of possible completions. This space can be expressed as the terminal nodes (representing completed objects) of a directed acyclic graph in which non-terminal nodes represent partial objects, and links between nodes represent addition of one or more items of topology. A graph link might, for example, represent:

- Addition of a new edge joining two existing vertices;
- Addition of a new vertex and two or more edges joining it to existing vertices;
- Reconstruction of all that hidden topology which can be deduced on the assumption that a topological mirror plane exists.

Graph links represent hypotheses about the current partial object. To make the search efficient, these hypotheses are assigned merit figures, and at any node, the hypothesis with the highest merit figure is investigated first.

The previous section described how axially-aligned planes can be used to hypothesise planes of object mirror symmetry. Determining the corresponding topology to be added for a hypothesis is straightforward. Depending on how much of the object is visible, this may be enough on its own to complete the topology of the object.

There are other ways in which knowledge of axially-aligned planes can be used either to create hypotheses about hidden object topology, or to reinforce hypotheses generated by other heuristics (thus increasing their merit figures and making it more likely that they will be investigated).

Consider, for example, the three blue planes in the colour figure. It is clear that hypothesising that there is a vertex where these planes intersect is reasonable, and would be a sensible step towards realising the topology of the compete object. Similarly, it would be reasonable to hypothesise the presence of a vertex where the planes coloured (from left to right) brown, cyan and blue intersect.

As well as being useful for making hypotheses about hidden object topology, knowledge of axially-aligned planes can also be used to reject bad hypotheses. We have already given an example of this in Section 1 (Figure 2).

4. Results

In this section, we present results concerning: detection of axially-aligned planes (Section 2.2). We intend to provide examples of the successful use of these techniques in detection of planes of mirror symmetry (Section 3.1) and constructing hidden topology (Section 3.2) in following papers, in which it will be integrated with other techniques for hypothesising hidden geometry.

To test our ideas, we evaluated them by constructing a system which identifies candidate cofacial configurations as described in [Var03], performs inflation as described in [VMS05], and then attempts to identify axially-aligned planes and assess candidate planes of mirror symmetry as described in this paper. It should be noted that [VMS05] is itself experimental, and not an optimal approach to inflation—in particular, it makes no use of line labelling or any technique resembling line labelling.

As test data, we used the test set of twenty line drawings [VMS04], Figures 7–26, believed to be typical of engineering concept drawings (this set of drawings is available on-line as the Second Test Set at `//ralph.cs.cf.ac.uk/Data/Sketch.html`).

We discuss the results of processing the test drawings individually, as presenting the results in tabular format could be misleading. For example, there are cases where the correct *number* of axially-aligned planes is identified but these are the wrong planes, while there are also cases where more than the correct number are identified but this can be readily explained and is unproblematic (i.e. no errors will be introduced in subsequent stages of processing). We start with the successes and working in generally decreasing order of success, and focus our analysis on those places where the algorithm has failed in some way—from the point of view of developing and extending our ideas, these are more interesting than successes. We finish with a few general conclusions.

In two of the figures, Figures 7 and 8, construction of axially-aligned planes is entirely successful, generating the correct planes in each of the three axes.

In Figure 9, the four ij-planes and the four jk-planes are identified correctly. Five ik-planes are identified where a human would see three—some edges have not been identified as axis-aligned. There are no misgroupings (occasions where axially-aligned planes which should remain distinct have been merged), and geometric ordering is correct (the distance coordinates of each plane equation, when ordered numerically, give the expected results).

In Figure 10, the three ij-planes are identified correctly.

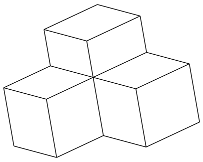
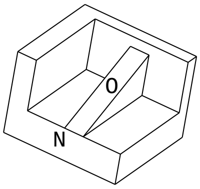


Figure 7:



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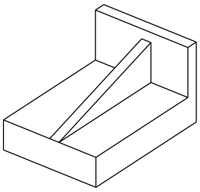


Figure 9:

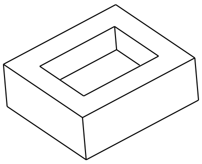


Figure 10:

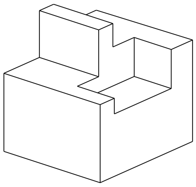
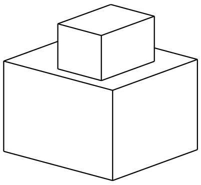


Figure 11:



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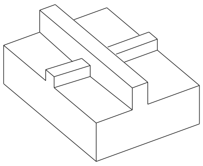


Figure 13:

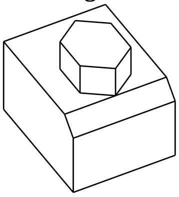


Figure 14:

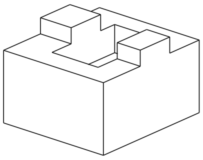


Figure 15:

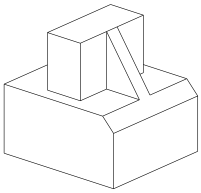
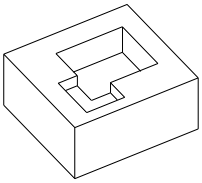


Figure 16:



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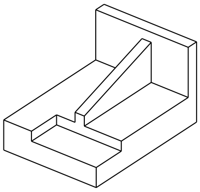

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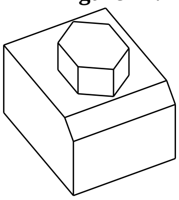

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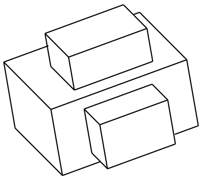


Figure 20:

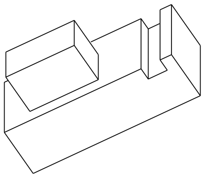
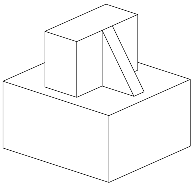


Figure 21:



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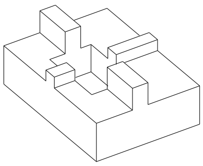

Figure 23:

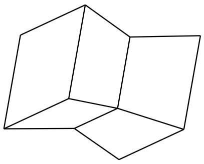

Figure 24:

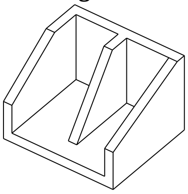

Figure 25:

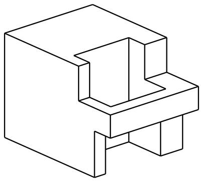

Figure 26:

Test Drawings

Five *ik*-planes and five *jk*-planes are identified where a human would see four of each—the edge joining the base of the pocket to its rim has not been identified as unequivocally axis-aligned. The geometric ordering of planes is correct.

In Figure 11, the five ij-planes (horizontal planes) are identified correctly. Seven *ik*-planes are identified where there should be six–a few edges were not identified as unequivocally axis-aligned. There are no misgroupings, and geometric ordering is correct, but the ungrouped vertex is geometrically much closer to a neighbouring plane than to the one it should be grouped with. Five *jk*-planes are identified where a human would see three. In view of the length of the graph-connected path between one of the rearmost vertices and the other five, it is to be expected that these will not be grouped together (visually, although they look coplanar, they are not unequivocally coplanar). There are no misgroupings, and geometric ordering is correct.

In Figure 12, four ij-planes are identified where a human would see three—the hole loop hypothesis, although correct, was rejected by the inflation process. Seven *ik*-planes and six *jk*-planes are identified where a human would see four of each—some edges were not identified as axis-aligned.

In Figure 13, eight ij-planes are identified where a human would see four. The extraneous planes arise where faces appear coplanar but cannot be proved to be so. Geometric ordering is correct. Five *ik*-planes and eight *jk*-planes are identified where a human would see four of each. There are no misgroupings, and geometric ordering is correct. One of the extraneous *jk*-planes is added because an edge has not been identified as axis-aligned; the others arise because planes which appear coplanar cannot be proved to be so.

In Figure 14, eight ij-planes are identified where a human would see four. The rim edges of the boss are not axis-aligned, and our approach has not been able to make any deductions about axis-aligned planes from them. The ij-planes of the base of the object are identified correctly. Ten *ik*-planes and nine *jk*-planes are identified where a human would see five and six respectively. Again, our approach has been able to deduce little about the boss. The results, although not misleading as such, are of little use.

In Figure 15, seven ij-planes are identified where a human would see four. This is mostly because unconnected planes which are 'obviously' (to a human) coplanar cannot be identified as such by our algorithms. There are serious geometric errors, with the two planes corresponding to the rearmost horizontal face both being misordered geometrically. Note that the *cofacial configuration* technique does not help here—it is not used in this drawing, as all vertices are graph-connected. It is the length of the graph-connection between the front and back vertices of this (and the resulting accumulation of errors) which is the source of the problem. There is very little grouping of *ik*-planes and *jk*-planes (fourteen *ik*-planes and thirteen *jk*-planes, instead of the six and four respectively a human would see). Again, the inflation process did not identify all edges as axially-aligned.

In Figure 16, seven ij-planes are identified where a human would see four. One of the extraneous planes is added because although two regions apparently correspond to the same face this cannot be determined by our algorithms; the others are added because edges have not been unequivocally identified as axis-aligned. There are no misgroupings. More seriously, the geometric ordering is incorrect: the two vertices at the bottom of the non-axially-aligned face have been placed lower than the three vertices at the base. This is a problem in the output of the inflation process, but not one which can be solved simply by feeding the axially-aligned planes back as input into the inflation process. Seven *ik*-planes are identified where a human would see six. As with ij-planes, there are no misgroupings, but the geometric ordering is seriously incorrect. Six *jk*-planes are identified where a human would see five. Again, there are no misgroupings, but the geometric ordering is seriously incorrect: two of the 'front' vertices are behind the boss.

In Figure 17, five ij-planes are identified where a human would see four, as (somewhat surprisingly) the inflation process did not identify all edges as axis-aligned. More serious is a geometric error: the base of the pocket has been placed higher than some vertices in the top face. Ten *ik*-planes and ten *jk*-planes are identified where a human would see five of each. Although unproblematic, this is unhelpful.

In Figure 18, six ij-planes are identified; a human would also see six, but not exactly the same ones. The 'obviously' (to a human) non-axially-aligned face has been misidentified as ij-aligned, and the vertices at either end grouped in the same ij-plane; the two apparently coplanar faces cannot be proved to be coplanar, so are not grouped together. There is also a geometric misordering of the bottom two ij-planes. Six *ik*-planes have been identified where a human would see four, and there are misgroupings. The five *jk*-planes have been identified correctly. There is a minor problem: the geometric distance between two of these planes is small, so a naive attempt to merge planes which are geometrically close could introduce a serious error here.

Figure 19 differs from Figure 14 only in that the boss has been moved back slightly so that Figure 19 implies a hole loop where Figure 14 includes a non-trihedral vertex. However, the results a In Figure 19, seven ij-planes are identified where a human would see four. Again, our approach has been able to make little of the non-axially-aligned edges of the boss. The ij-planes of the base of the object are identified correctly. Seven *ik*-planes and nine *jk*-planes are identified where a human would see five and seven respectively; there are several misgroupings in both. Geometric ordering of the front two *jk*-planes is incorrect; this is an error which should be detectable using line labelling, particularly since all the vertices involved are trihedral.

Similarly disappointing results were obtained in Figures 20 and 21.

In Figure 22, five ij-planes are identified where a human would see three—there are some edges which the inflation process has been unable to identify as unequivocally axis-aligned. Geometric ordering is correct, the *cofacial configuration* technique correctly identifies that the base of the central boss is coplanar with the top of the supporting cuboid, and the geometric distance between the two pairs of planes which should be coplanar is small. There is a serious error in the *ik*-planes: several distinct planes have been conflated. There are six planes, as one would expect, but they are wrong. Nine *jk*-planes have been identified where a human would see five, again because some edges have not been unequivocally identified as axis-aligned. This is unhelpful but unproblematic. There are no misgroupings, and geometric ordering is correct.

In Figure 23, twelve ij-planes are identified where a human would see five. The base plane is identified correctly, but the rest include serious misgroupings. Twelve *ik*-planes and fourteen *jk*-planes are identified where a human would see six of each. There are no misgroupings, and geometric ordering is correct. In both cases, the front of the object has been grouped correctly, but little if any grouping has taken place at the rear of the object.

In Figures 24 and 25, the inflation process of [VMS05] makes incorrect decisions about edge axis alignment; the results obtained by feeding this incorrect input into our new idea are meaningless.

Our current implementation cannot process Figure 26—there is a region containing only one true vertex, and attempting to calculate its 3D plane results in an error.

The immediate conclusion to be drawn from these results is that most of the problems result from deficiencies in the input, not failures of the technique. We consider some possibilities for improving input quality in the next section.

The results obtained in Figures 14 and 19 show that the idea proposed in Section 2.4 of identifying candidate cofacial hypotheses as described in [Var03] and assessing these hypothesis during inflation using the techniques of [VMS05] is successful.

5. Conclusions and Further Work

Although the idea presented in this paper appears interesting and useful, it needs better-quality input from the inflation process before it can be considered reliable. In particular, the inflation techniques of [VMS05] should be combined with line labelling in order to provide knowledge of occlusion and convexity/concavity.

The results presented here would be improved significantly if we could be more certain about identifying axis-aligned edges. At the moment, we only use those edges which have been identified as certainly axis-aligned. It might be preferable to use all edges where the confidence of axis-alignment is higher than a threshold value (although since this could increase the number of false positives, the threshold value would have to be chosen with some care).

Extending our approach to make use of edges which, while not axis-aligned, are clearly in one of the axis-aligned planes ought to be straightforward—identifying these is a natural extension of the ideas of [VMS05]. This would enable correct interpretation of (for example) the boss in Figure 14.

Nevertheless, the ideas presented here is both natural, in that it identifies something humans also identify in drawings, and flexible, in that there are several alternative ways of implementing them. We plan, in future papers, to present an evaluation of these various alternatives, and to present an integrated approach to construction of hidden topology which includes the best of these.

6. Acknowledgements

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