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Generating large-scale details : altering soil surface and structure with tracks

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Abstract

Tracks, footsteps and ruts are common elements we can observe in natural sceneries. Wherever there is human activity, we can find marks on the ground left by vehicles or walkers. These marks can be directly dug from a soft ground or left on a hard ground by objects that carry materials on their surfaces. This paper presents a method to alter soil with such tracks. These tracks are defined with a trajectory and the shape of the mark left by the moving object (vehicle or walker) and is then applied to the soil for each step of the trajectory. Using our method we can obtain tracks on a loose soil and the deposit of material through the track on the desired length. This method allows us to produce a great number of large-scale details on terrains.

Categories and Subject Descriptors (according to ACM CCS): I.3.8 [Computer Graphics]: Applications—I.3.6 [Computer Graphics]: Methodology and Techniques—I.3.5 [Computer Graphics]: Boundary representations—

1. Introduction

The main characteristics of soil deformation are a lowering in soil surface according to the shape of the object, the presence of material below the object that will mark the ground and the deposit of material from a previous step. In computer graphics such tracks are hard to produce. They are often made without taking into account the soil structure and are applied to the ground regardless of the materials carried by the object making the marks. A basic stamping algorithm is used to perform a full track of marks, therefore it can include a lack of realism due to marks not matching the moving object and the soil structure.

Modeling such details requires knowing exactly what is the material below the mark and generating a fine resolution for the ground. Most of the existing methods use a simulation model relying on physics interactions between an object and the ground. In this paper, we present a method based on topology and textures to produce realistic tracks on a heterogeneous soil. A tracks is defined by a trajectory providing the path of the track and textures describing the shape of each mark on the ground.

We use a topological model of ground based on stacks of materials presented in [PGMG09] and two-dimensional generalized maps (2-G-Maps) described in [Lie94] and [Lie89]. Topological operations are used through each step of the track to apply the deformation on the ground according to the given trajectory and shape. Our method provides an efficient way to:

- Generate tracks on any surface.
- Alter soil with the desired stamp shape and transport material through the track.
- Create large number of mark with LOD.

This generic approach enables us to generate a large amount of marks and details on any surface such as mud, snow or rock. Material transport can also be applied to liquids which do not keep any mark on their surface.

2. Related Works

Ground generation and alteration is a subject barely addressed. In contrast, research fields such as fractal terrains, erosion and soil structure are abounded in the literature. Soil alterations are often done using physics models which require a heavy computational cost. Moreover, the object altering the soil structure must be animated in order to imprint tracks on the model. Furthermore, the underlying structure must have a fine resolution to obtain relevant results.

Aquilio *et al.* proposed a GPU-based algorithm for

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deforming terrains [ABZO06]. Their model relies on a dynamically-displaced height map handled on the GPU. This height map is then modified with a vertex shader matching the object depth information. The terrain is finally rendered using a hardware-based displacement mapping.

Chanclou *et al.* have developed a particle-based model for soil [CLH96], which handles objects moving on it. They have simulated both action on the ground from the object and action from the soil on the object's movement. Therefore, this method can deal with tire tracks and all the behaviour of the soil underneath.

Li *et al.* proposed a model for soil manipulation [LM93]. They use the interaction between the soil and tools such as a bulldozer's blade or a bucket. In order to deform the ground the soil structure is partitioned into slices on which forces are computed. Bulldozer's blade is also used by Holz *et al.* in [HBK09] in order to deform an hybrid model of soil. This model combines particles and a grid based representation of the ground. The soil is treated as a non-homogeneous material and can be compacted and eroded.

Sumner *et al.* in [SOH99] introduced a physical-based model for the interaction between the ground and a moving object. This method discretizes the soil into a uniform grid representing the surface of the ground. Then, a rigid geometric object is used to displace columns (compression or distribution) and to deform the soil structure. Finally several erosion steps are used to smooth the ground where the trace occurred.

In [CLS07], Cai *et al.* described a precomputed model of terrain partitioned in several blocks. They used a physical model in order to deform the ground surface, then, procedural textures are generated and the terrain is rendered using strip masks. The tire-soil interaction model of this method uses the bekker's soil-deformation model [Bek69].

Zeng *et al.* proposed a physical model of deformation for granular material interacting with rigid bodies [ZTT*07]. This granular model relies on a height map where the material can be distributed to its neighbours. The movement of the rigid body is also affected by the granular material beneath.

In [CZXY07], Chen *et al.* proposed to visualize tire tracks with LOD. They used a hierarchical structure for handling large scale dynamic terrain. Tire tracks are then applied to the model via a kinematics' simulation of a moving vehicle.

3. Mesh conversion and track generation

Refining terrains at a small scale requires the ability to easily access vertices and edges neighbours. This possibility is given by a traditional mesh but is extremely costly. Therefore, we use 2-G-map that is a topological model which allows to represent the topology of any two-dimensional subdivisions. This model is similar to quad-edge data structure from [GS85] and to the extensions of combinatorial maps [Tut84]. It relies on using a single element: the dart, on which 3 bijections (assembly operations) are defined. Therefore, this model makes it easier to design data structures and algorithms.

Bijections are used to easily travel through neighbours at a minimal cost. Thus, with a given dart we can reach any dart in it's neighbourhood and apply modification to it. Figure 1 shows the three bijections used in our model to acces neighbours. $\alpha 0$ is the link between two darts and provides their common edge. $\alpha 1$ is used to sew edges and form faces. In a similar way, $\alpha 2$ sews faces to obtain the final mesh. In addition we included a new link, $\beta 1$, between darts and stacks of materials.

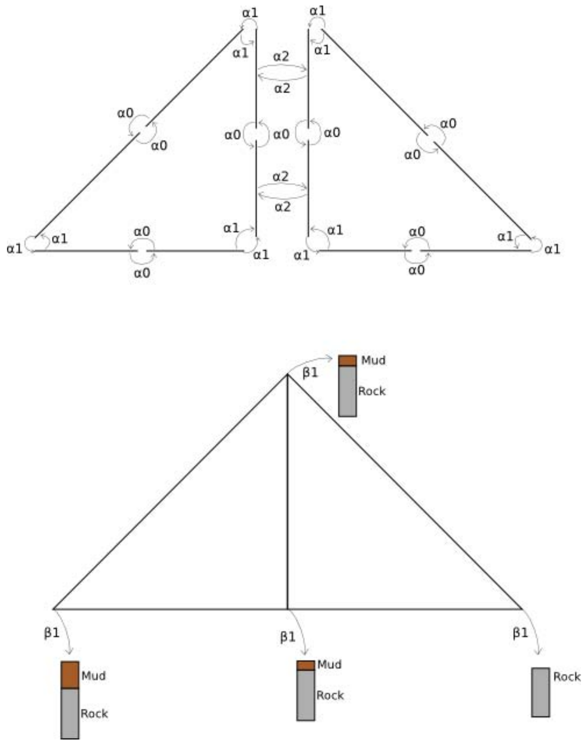

Figure 1: *Bijections in the 2-G-Maps model,* $\alpha 0$ *sews two darts into an edge,* $\alpha 1$ *sews two edges and* $\alpha 2$ *sews a face to another. We can see on top a mesh composed of two triangles and below the topological representation of this mesh with the corresponding* β_1.

We also add the posibility of loading a mesh composed of vertices in order to convert it to our topological structure. This conversion is done by matching vertices and their neighbourhood and then sewing them using assembly operations. In addition, the user provides a depth and a material for each vertex which represent the soil structure underneath. Using vertex painting and masks the user can provide various and complex soil structures under the whole mesh.

Our model's pipeline is described in Figure 2. The mesh and material stacks are provided along with the stamp shape and height-map. The topological model is then created using these data and can be altered with tracks. We provide generic data after applying tracks to the soil structure. These data are the altered mesh and opacity textures describing materials on the ground surface. The rendering processes can be done afterwards using any renderer.

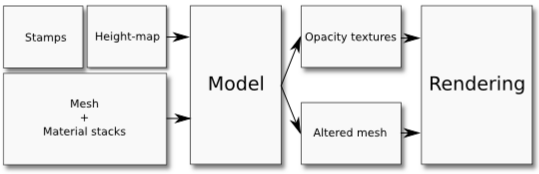

Figure 2: *Pipeline of our model.*

We need several tools to alter our topological model with tracks. First of all, we must convert the mesh and stacks data, then we have to provide generic and easy to use tools for describing tracks and finally we have to apply the desired trajectory to the model.

3.1. Our topological model

The topological model can import any triangle mesh. Moreover, material stacks embedded in the mesh are also loaded. This feature allows to have a coherent model for the surface and the soil structure. While loading mesh data, we topologically sew bijections needed by the 2-G-map between darts, edges and faces. Also, stacks are applied to each dart with their depth and material type.

Another feature embedded in the model is the topological refinement. Each face can be divided into 4 triangles using primal triangle quadrisection (Fig. 3). In this refinement we have to keep up to date every topological link used by the 2-G-Map. Neighbours are then automatically sewn using existing bijections. In the case a neighbour triangle is not selected for being refined we have to divide it into two triangles for keeping links between darts. This topological refinement is then used to add details where the tracks will be imprinted. Stacks of material are also kept during the refinement process. We use interpolation to create stacks for each new dart in the model.

3.2. Track generation

With our model, we can have vast varieties of details altering the soil structure. Moreover, we can add thousand of them on any soil. Tracks consist of stamps arranged through a trajectory. Several parameters such as weight, speed and acceleration must be taken into account to model them. Thus, we provide interactive tools in order to finely define a trajectory and the associated track.

Trajectory generation: A trajectory is defined by the user by drawing a line on the model surface. The location of the stamps are then automatically computed using two parameters, the desired number of stamps and the width between a stamp and the trajectory line. The user can also define if a walker begins with the right or the left foot. Stamps size is also a parameter taken into account. Moreover, the size of the stamps can be adjusted to fit the desired trajectory.

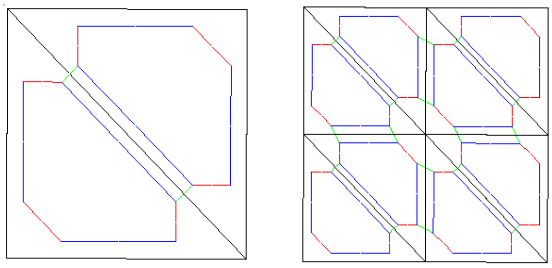

Figure 3: *Primal triangle quadrisection, the topological way. Both triangles on the left are subdivided on the right figure. We can see every bijections between darts (blue :* $\alpha 0$*, red :* $\alpha 1$*, green :* $\alpha 2$*).*

Stamps are provided as two textures (Fig. 4). The first one is the stamp shape giving the general shape of a stamp projected on the surface, the second one is a height-map used to know how the soil structure will be impacted by the stamping process. This height-map can be designed to fit the desired mark volume or automatically computed. In Figure 4 we can see that height maps can be very similar to the provided shape or quite different. For example, the horseshoe shape is the same as mainly hard part of the horseshoe hit the ground. On the contrary, the human footstep shown on the bottom right corner has a height map rather different than its shape. This is due to the differents points of support of the foot. We can also see on upper left corner that the height map provided for a wolf step can be design to keep some part hard, here claws are not blur in order to mark the ground more clearly.

Figure 4: *The stamps provided for the trajectory generation. We can see differents kind of shapes and their corresponding heigh-maps on the right.*

Figure 5 shows two different shapes of trajectory. Moreover, trajectories can be defined with evolving speed as we can see on the bottom track in Figure 5. These trajectories show two kind of tracks that can be applied to different grounds.

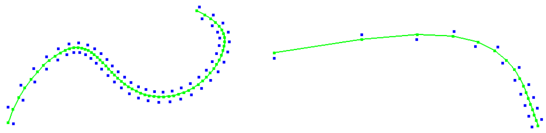

Figure 5: *Possible trajectory definition with our model. The right trajectory simulate walker evolving speed from right to left, the lenght between each mark increase according to the speed.*

Track stamping: Marks are then applied to the ground. We use the shape texture to select faces used by the whole track and refine them. All the faces are selected at once in order to narrow neighbours refinement. This provides a uniform subdivision of the entire track. Each mark can be refined on the whole area or only on border. This allows us to choose if details inside a mark are relevant for the generation or not.

Furthermore, we use Level of Detail controlled by the camera position and a refinement parameter set by the user. This ensures that the details are only applied where it's relevant (Fig. 6). These levels of details are achieved by taking into account the size of each face projected on the screen. If a face's size is equal or lower than the desired minimum size, then we don't include it in the refinement process so that it will keep it size unvaried.

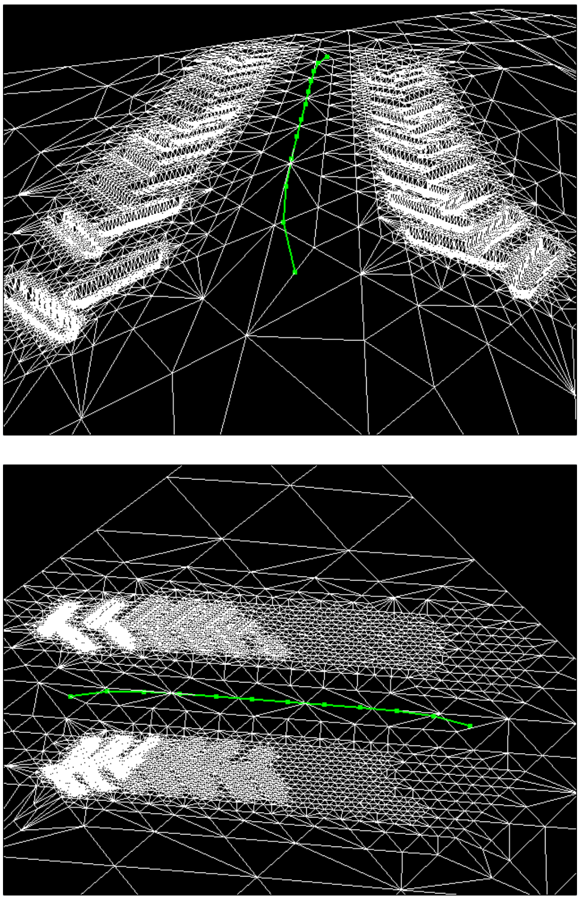

Figure 6: *Level of detail in our method. On top the camera view and below the perpendicular view where we can see details scale along the trajectory.*

3.3. Track imprinting

The last part of our generation process consists in using our topological model to lower the soil structure where the marks occurred. This step takes into account the parameters of the moving object such as weight and speed. At each steps, material can be taken or left on the soil.

In order to proceed with soil lowering and controlling borders of a mark, we provided to our model two different methods. The first one only affects the height of each dart, a lowering parameter is computed under the simulation criteria and then, is applied to each dart in order to determine its new position. This first method gives us the ability to create soft borders for the chosen marks. This kind of border is often seen on soft and dry ground such as earth or dry sand.

The second process uses involutions of our model and is relaying on a topological extrusion which is achieved in two main steps, a border detection helped by the topology embedded in our model and adding new faces to build the new extruded border.

Border detection : To extrude a mark, we have to know exactly which darts define its border. For that matter, during the track stamping each face is tagged in order to know to which mark it belongs to. Then, using the $\alpha 2$ involution, we can easily reach each neighbour of these faces using their darts. If the neighbour doesn't have the same tag as the considered face then, we are on face belonging to the border of the mark. We save the dart that allowed us to find a different tag to reuse it in the next step.

Extrusion : Now that we have recorded every darts representing the border of the mark, they can be used to make an extrusion while the mark is lowered. Darts under the mark is lowered according to the imprinting parameters and then the border is processed. New faces are sewed all along the border to complete the extrusion. Each dart recorded as belonging to the border is used to create and sew these new faces. We can see in Figure 7 the main steps of our method.

The first Figure (7.a) shows two faces sewed with $\alpha 2$. The first step of the extrusion is the lowering of darts (7.b). Position of the darts changes but not the seams so that the two faces are still sewed with α2. New faces are then created (7.c) and added to the model. We can easily find the positions of the newly created darts of each faces which are the positions of the dart belonging to the border. Each position is found using the border dart or its direct neighbour reached with the α2 we have kept (7.b). Finally, the new faces are sewed to the old ones with α2. All along this process of extrusion, each new faces which are created are also sewed between them around the border of the mark.

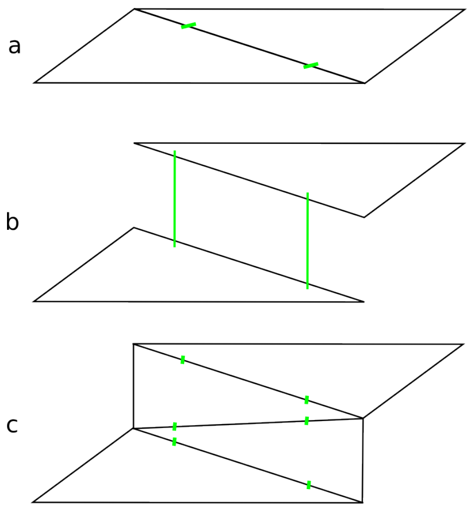

Figure 7: *Main steps of the topological extrusion. α2 are represented in green, the other involutions are not shown here.*

These methods allow us to simulate a hard or a wet soil which has clear-cut marks. In contrast, with no extrusion and only a lowering of vertices, marks can be simulated on soft grounds such as dry sand or soft earth. In addition our method can elevate the external border of any mark in order to provide a more realistic alteration. We can see in Figure 12 several tracks of gulls on the beach, stamps provided to our model to produce marks and two focused zoom. The first zoom (upper left corner) shows how borders are handled for simulating a dry sand. The other zoom shows clear-cut borders of a wet sand.

The weight of the subject is the main parameter used to lower the ground. According to the weight and the height-map provided, darts are lowered more or less. To apply this lowering, the weight is used to modify the height-map levels. Then we can apply the deformation to every dart below the considered print. We also take into account the depth of the stacks beneath the considered dart, in order to obtain its maximum depth. If this limit is reached we stop the lowering as we are on a hard and undeformable ground. Figure 8 shows how weight affects our imprinting on a soil.

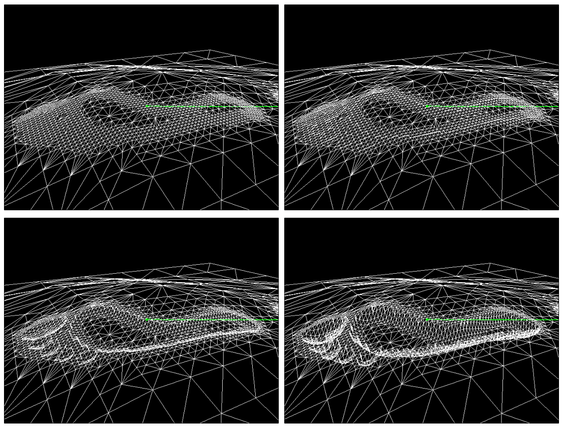

Figure 8: *Influence of the weight in our simulation. The subject weight increases from top left to bottom right figure. We can observe that the ground is more and more lowered at each step.*

Speed is also used as a parameter, as it influences the type of each mark. An object which moves fast will produce marks that are more lowered to the front, whereas low speed will give homogeneous marks on the ground. For vehicles simulation, we also use the size of the wheel, which in our case corresponds to the number of patterns on the wheel. In addition to knowing how many patterns will be imprinted, this parameter is also used to control material deposit through each pattern and to modify material stacks.

3.4. Material deposit

Our model also simulates material transport and deposits through the track. This feature relies on texture based methods dependent on the stamps shape and permits to manage material hung beneath a foot or a wheel.

Each time the soil is hit by a step, the stacks bellow are affected. First of all, stacks allow us to know exactly if the soil can be lowered. In addition, marks can be done at the frontier of a totally hard material and a soft one. These marks will only lower the soft material. When a stamp is applied, material is added to the texture, according to the soil structure and the stacks below the model. Each storage texture represents the area under the object stamping on a soil. For example, a walker will have two storage textures, one for each foot. A vehicle will have as many textures as he has patterns on its wheels. Every handled pattern has a single storage texture linked to it.

These textures are used for storing material along the track. Then we automatically generate patterns according to the mark shape and use them with the storage texture to know exactly where material deposit occurs. We can see on Figure 9 how patterns are used to deposit material on the

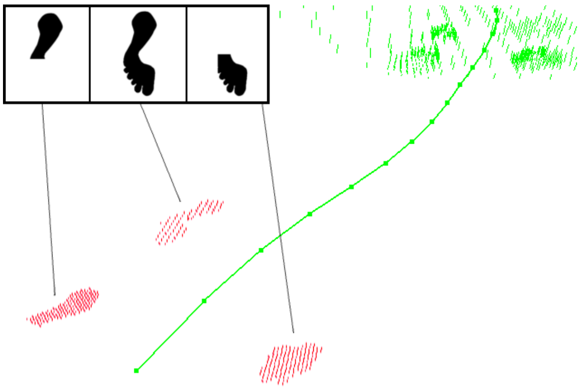

Figure 9: *We can see simple deposit patterns on top of this figure and the result of applying such patterns to the model. Red stacks represent material deposit along the trajectory.*

ground. The green line represents the trajectory, the green segments represent material which is present on a hard soil and the red segments represent the material disposed through the stamping process. Here we have three simple patterns which have disposed material on the ground. Each pattern is linked to the area where it disposed the material. These storage textures are then used all along the track to handle material deposit, as they correspond to the amount of material stored on the previous steps of the trajectory.

We consider that each new step can add or take away material from the stacks below. On the ground where there is no stack available, we only deposit stored material and new stacks are added to the model so that they could be altered on the next step of the simulation. As we only make slight modifications to the ground structure, material stacks keep nearly the same size. The material from a stack can only be modified during the imprinting and material transport methods. These two process can handle every material which sticks to an object and which can be transported to another surface.

4. Results

Using our method we obtain the altered mesh of the considered ground model and an opacity map describing the new soil structure. The corresponding image can be obtained using these two outputs and any available rendering software. The opacity map construction is done through the use of topological information provided by the 2-G-Map. It allows us to acces a dart neighbourhood with a minimal computational cost.

Our model can be efficiently used to produce realistic results. To this end, the user provides only a shape and a height map defined to match the desired object. The second step is to import a mesh into the model and to simply draw a trajectory on it. Finally, the user has to set a few parameters in order to obtain the desired results. In a few minutes a trajectory of marks is set and a few minutes later the track is computed. In comparison, a physically based model will have a lot more parameters to set and it will take more time to compute. Also, trying to create all the marks of desired trajectory by hand can lead to a very huge time of modeling.

For exemple, the sand scene represented on Figure 10 took us approximately 31 seconds. 1 second for importing the mesh, 20 seconds to design the trajectory and set the desired parameters and 10 seconds to compute the output mesh. The scene presented in Figure 12 took a longer time. The mesh is larger and was loaded in 2 seconds. We can see three trajectories of marks on the sand which took us 2 minutes to design. The final mesh has been computed with a very fine resolution in order to have a lot of details. This took us 38 seconds. As a result, this scene has taken 2 minutes and 40 seconds to be ready to import in any rendering program.

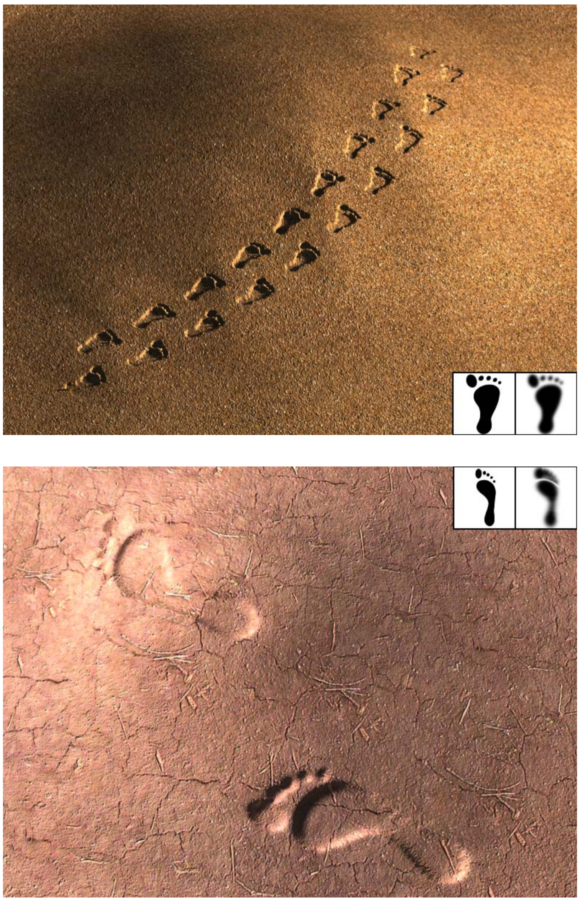

Figure 10: *Footprints on a sea shore sand and on a muddy ground. We can see for each figure the shape and the height map used to generate tracks.*

We can see on Figure 10 two kinds of marks on the ground. On the top we present a track of steps made in wet

Figure 11: *Hundreds of marks on the ground. We can observe three tracks crossing each other with a large number of footsteps along them. Such result cannot be obtain by disposing marks by hands.*

sand. We can observe the clear-cuted border on each mark. On the bottom figure two footsteps are done on a soft earth. The borders are made to simulate the softness of the ground. We can see on both pictures the stamps used to generate the marks. For the first trajectory on wet sand, the shape is just blured to obtain the height map used. In the second generation (soft earth) the height map is designed from the shape provided in order to keep a gap beween the toes and the sole.

Figure 11 presents a very large amount of marks. We have more than 500 marks representing 3 tracks and altering a ground made of sand. Each mark is well defined even if the camera is far from the considered ground. This detailed resolution can be set manually or automatically. By using displacement mapping or complex physical models, such amount of details become a highly challenging task. In contrast, our method easily achieves the desired results and only requires a minimum amount of parameters.

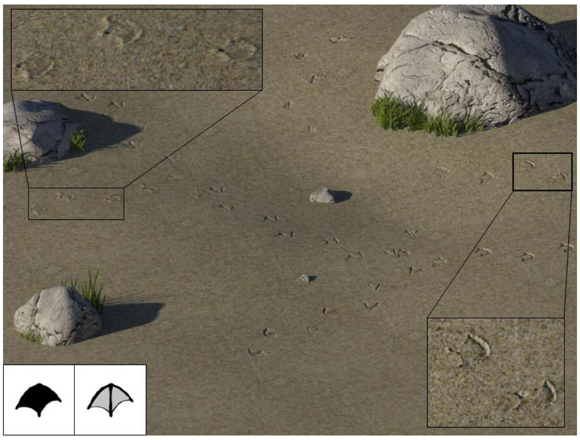

Figure 12: *Various tracks made by a gull on the seashore.*

Figure 12 presents the result of a simulation of tracks made by gulls on a seashore. Parts of the ground are zoomed in in order to show borders handling. We can see that the tracks are very realistic and that the sea gulls feet have accurate detail. Each step on the ground add between two thousand and three thousand faces. This value is depending on the distance between the step and the camera. This model has 288 faces without tracks and our method add 136507 faces for all the tracks imprinted on it. The upper left corner shows the the marks made in dry sand. The borders are soft and the external borders are elevated to simulate material displacement. The lower right corner shows a simulation of marks in wet sand. The borders are clear-cutted and there is no external border elevation.

Material deposit is illustrated in Figure 13. We can observe several tracks made by vehicles coming from the mud on both sides of the road. The vehicles are then driven on the road and leave material on the ground. This figure shows both aspects of our model, the tracks imprinting and lowering on soft ground and material deposit. We can see here a large quantity of material deposit on the road as the mud is transported through the tracks. The material on the road give a realistic result as more and more tracks are added. Moreover, thanks to stamps and adherence value, the mud is spread through the tracks in a realistic way. The more a vehicle travels on the road, less material is deposed. This is due to the fact that the material contained in the storage textures is deposed on the ground. This model has initially 1537 faces, we then add 86095 faces to generate all the details on the ground.

Our model can also simulate accurately tire tracks using a pattern. We can see in Figure 14 a track made by a vehicle on a different material. We can observe that the continuity is respected through this track even if the shape is complex. Also, the tires have left on the ground some material from the previous steps. The large number of details on this model represent an addition of 40383 faces. Between 20 and 30 faces are used for each tile depending on the camera position.

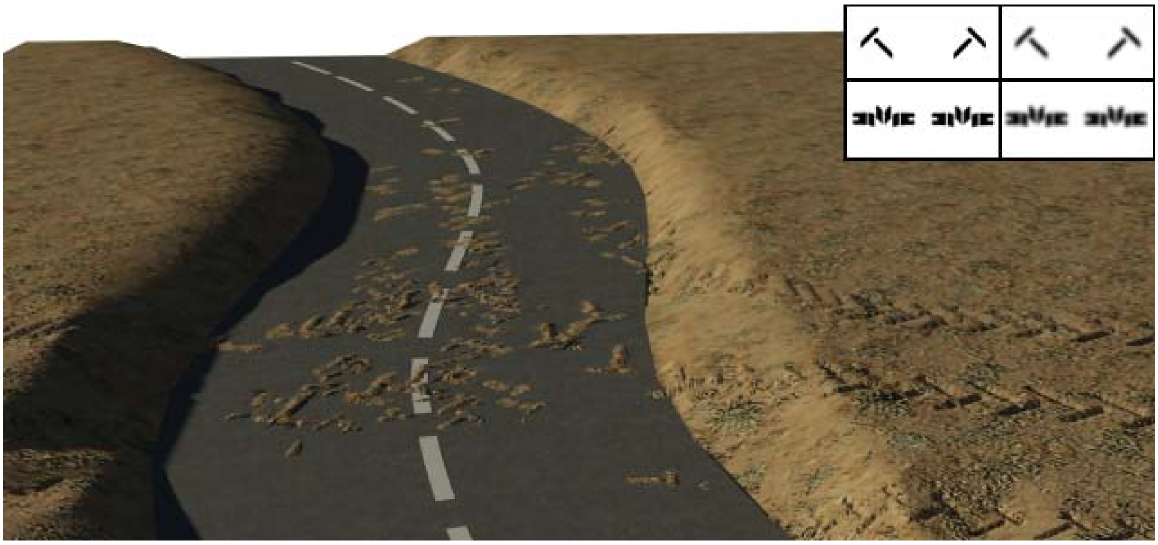

Figure 13: *Material deposit on a road. The material is carried through each vehicle tracks and is then deposed on the road when vehicles leave the muddy ground.*

Figure 14: *Material deposit through a vehicle track.*

5. Conclusion and perspectives

This paper aims to provide an efficient method for modeling tracks on the ground. In natural scenes such tracks represent a vast spectrum of details. Our method is based on stamping and topological operations, and offers a quick and easy way to manage various kinds of marks without using a physical model. We only need to provide a few parameters in order to be able to generate a very large amount of marks on any surface, dry or wet, hard or soft. All these marks are generated without any physical model and all the process is done using phenomenologycal methods.

We propose to accurately shape tracks on the ground using as input the mark shape, the height map and the parameters of the subject. These parameters can be weight, speed, adherence or trajectory. They are used to compute the entire track. Moreover, the borders of each mark can be generated using various ways by combining topological extrusion and external borders elevation. All these data are used to improve the realism of the generated marks. Depending on these parameters, our model is able to generate any kind of mark on any surface without the need of applying a physical simulation to the surface.

We wanted our model to provide the ability to compute tracks with a very large number of marks. In order to have such a feature, we introduced LOD dependent of the camera in order to reduce the number of operations needed to compute the whole ground of a scene. As a result, we are able to generate vast varieties of tracks with a large number of realistic marks on any soil. Moreover, the realism of the marks is preserved no matter if the alterations occur far or close to the camera.

Finally, the material transport method allows to simulate carried material through the whole trajectory. First of all, the material is taken from the ground and is deposed all along the trajectory of the object which alters the terrain. The storage textures represent the surface of the objects used to alter the soil. They permit to keep a coherent management of the material quantity. Therefore, as in real scenes, the material deposit is less important as material quantity decreases in storage textures in the same way as material disappears from the surface of the object carrying it. This material transport method allows us to have a complete method able to handle many aspects of ground deformation due to walkers or vehicles. The generated mesh can be exported along with the procedural tools. In this way, the rendering process can be carried out using any available rendering software.

A possible improvement to our model would consists in handling the material spattering around the marks and through the tracks. When a wheel or a foot sinks into a very wet ground, spattering can occur on the soil with various radiuses. Moreover, we do not currently handle the material behaviour due to cracks and soil evolution. In order to deal with both of these features, we would have to include more physically inspired deformations in our topological model.

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