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Stable Interactive Cutting of Deformable Objects

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Abstract

In this paper we present a novel approach for stable interactive cutting of deformable objects in virtual environments. Our method is based on the extended finite elements method, allowing for a modeling of discontinuities without remeshing. As no new elements are created, the impact on performance of the simulation is minimized.

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [COMPUTER GRAPHICS]: Three-Dimensional Graphics and Realism, J.2 [Computer Applications]: PHYSICAL SCIENCES AND ENGINEERING, J.2 [Computer Applications]: LIFE AND MEDICAL SCIENCES, I.6.3 [SIMULATION AND MODELING]: Applications

1. Introduction

Surgical simulation is an important field of application in virtual reality (VR). A virtual surgery trainer can not only help to improve the skills of the surgeons, it also solves ethical issues related to training on animals or humans. Numerous surgical training systems have been developed in the last decade. The main requirement on a surgery simulator is the plausible deformation of the soft tissue in realtime and its interactive manipulation using various surgical instruments. The majority of current surgical simulators use the finite elements method (FEM) for the tissue deformation. Simulation objects are represented using a volumetric mesh of tetrahedral elements. An interactive cutting simulation is an essential feature of a surgery trainer. However, the interactive progressive cutting of a deformable FEM mesh is a challenging problem. The number and quality of the FEM elements have a direct impact on the simulation performance and stability. Although a number of different approaches have been presented recently, the problems have not been solved satisfyingly. [BSMM02] presents a survey of interactive cutting techniques in virtual surgery.

The approach presented here is based on the extended finite elements method (XFEM) as proposed by [BB99]. Instead of actually remeshing the FEM mesh, discontinuous enrichment functions are added. As no new elements are created, the impact on performance of the simulation is minimized. A proper mass lumping technique guarantees the stability of the dynamic simulation.

2. Modeling Discontinuities using XFEM

The dynamic deformation of nodes in an FEM mesh is described by the following formula.

$$\mathbb{M}\ddot{\mathbf{u}} + \mathbb{D}\dot{\mathbf{u}} + \mathbb{K}\mathbf{u} = \mathbf{f}, \tag{1}$$

where $\mathbb{M}$ is the mass matrix, $\mathbb{D}$ is the damping matrix, $\mathbb{K}$ is the global stiffness matrix, $\mathbf{u}$ is the vector of nodal displacements and $\mathbf{f}$ is the external load. The displacement of an arbitrary point can be computed as

$$\mathbf{u}(x) = \sum_{i=1}^{n} N_i(x)\mathbf{u}_i \tag{2}$$

where n is the number of element nodes, N_i are the element shape functions and $\mathbf{u}_i$ are the displacements of the element nodes. For more details on FEM we refer to FEM textbooks, e.g., [Bat82]. When a discontinuity (e.g., a cut or crack) has to be added, the surrounding mesh nodes are enriched by an additional discontinuous function and the corresponding number of nodal degrees of freedom is added.

$$\mathbf{u}(x) = \sum_{i=1}^{n} N_i(x)\mathbf{u}_i + \sum_{j=1}^{m} N_j(x)\psi_j(x)\mathbf{a}_j \tag{3}$$

where m is the number of enriched nodes, N_j are the nodal shape functions of the added nodal degrees of freedom $\mathbf{a}_j$, $\psi(x)$ is the discontinuous enrichment function. [ZB03] proposed an enrichment function in the form $\psi_i(x) = H(x) - H(x_i)$, where $H(x_i)$ is the value of $H(x)$ at the i-th node, which leads to an enormous simplification of an implementation.

The stiffness matrix of the enriched element has the form

$$\tilde{\mathbb{K}}_e = \begin{bmatrix} \mathbb{K}^{uu} & \mathbb{K}^{ua} \\ \mathbb{K}^{au} & \mathbb{K}^{aa} \end{bmatrix} \tag{4}$$

where $\mathbb{K}^{uu}$ is the original element stiffness matrix, whereas $\mathbb{K}^{ua}$, $\mathbb{K}^{au}$ and $\mathbb{K}^{aa}$ correspond to the added degrees of freedom. In the following, we assume linear tetrahedron with constant strain. The enrichment function $\psi_i(x)$ takes on constant values over the domain above (V_1) and below (V_2) the crack plane and only changes value from one to another. Depending on possible combinations of ψ_i and ψ_j, $\mathbb{K}^{ua}_{ij}$ and $\mathbb{K}^{aa}_{ij}$ takes on the following values.

$$\mathbb{K}^{ua}_{ij} = \begin{cases} 2\mathbb{K}^{uu}_{ij}\dfrac{\mathrm{V}_1}{\mathrm{V}} & \text{if } \psi_j = -1 \\ -2\mathbb{K}^{uu}_{ij}\dfrac{\mathrm{V}_2}{\mathrm{V}} & \text{if } \psi_j = 1 \end{cases} \tag{5}$$

$$\mathbb{K}^{aa}_{ij} = \begin{cases} 4\mathbb{K}^{uu}_{ij}\dfrac{\mathrm{V}_1}{\mathrm{V}} & \text{if } \psi_i = \psi_j = -1 \\ 4\mathbb{K}^{uu}_{ij}\dfrac{\mathrm{V}_2}{\mathrm{V}} & \text{if } \psi_i = \psi_j = 1 \\ 0 & \text{if } \psi_i \neq \psi_j \end{cases} \tag{6}$$

Similarly to the enriched stiffness matrix, the enriched mass matrix has the form

$$\tilde{\mathbb{M}}_e = \begin{bmatrix} \mathbb{M}^{uu} & \mathbb{M}^{ua} \\ \mathbb{M}^{au} & \mathbb{M}^{aa} \end{bmatrix} \tag{7}$$

where $\mathbb{M}^{uu}$ is the original element mass matrix, whereas $\mathbb{M}^{ua}$, $\mathbb{M}^{au}$ and $\mathbb{M}^{aa}$ correspond to the added degrees of freedom. See [ZCXB05] for more details on the calculation of the submatrices. In order to solve equation 1 numerically, the inversion of the mass matrix is needed in order to compute the nodal accelerations. Therefore, the mass matrices are diagonalized using a technique called mass lumping. [MRCB06] propose a mass lumping for the enriched part of the matrix. The submatrices $\mathbb{M}^{ua}_{ij}$ and $\mathbb{M}^{au}_{ij}$ are zero and $\mathbb{M}^{aa}_{ij}$ is defined as

$$\bar{\mathbb{M}}^{aa}_{ii} = \frac{m}{n}\frac{1}{V^e}\int_{V^e}(\psi_i(x))^2\,dV \tag{8}$$

For the case depicted here, equation 8 can be rearranged to

$$\bar{\mathbb{M}}^{aa}_{ii} = \begin{cases} 4\bar{\mathbb{M}}^{uu}_{ii}\dfrac{\mathrm{V}_1}{\mathrm{V}} & \text{if } \psi_i = -1 \\ 4\bar{\mathbb{M}}^{uu}_{ii}\dfrac{\mathrm{V}_2}{\mathrm{V}} & \text{if } \psi_i = 1 \end{cases} \tag{9}$$

3. Summary

Figure 1 illustrates a complete dissection of an object. The cut is created non-progressively using a cutting plane. Although the enriched elements are shared by both parts of the object, the parts fall down and deform independently on each other.

The XFEM presented here in the context of an interactive graphical application can effectively model discontinuities within an FEM mesh without creating new mesh elements and thus minimizing the impact on the performance of the simulation.

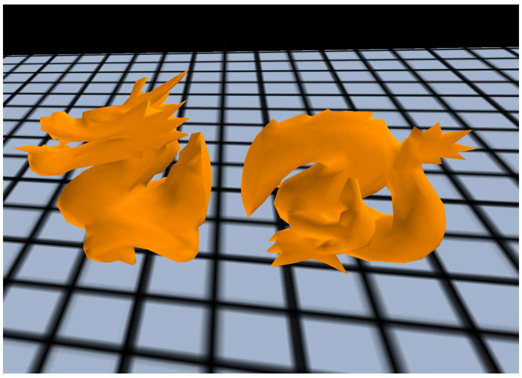

Figure 1: *A complete dissection of a deformable object using the XFEM approach.*

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