

# An Improved Friction Model for Cloth Simulation

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## Abstract

*Friction is a complex phenomenon that resists tangential motion between two contacting objects. In computer graphics, existing cloth simulation methods have typically used a Coulomb model for friction, both static and kinetic. However, the Coulomb model is limited since it intends to capture fundamental frictional phenomena. Many observed friction effects such as stiction, Stribeck friction, viscous friction and the stick-slip phenomenon are not modelled by it. This paper describes an improved physically based friction model for simulating such effects, extending the Coulomb model. We show that including these additional effects is relatively straightforward. Results and comparisons with a pure Coulomb model are provided to demonstrate the capability and features of the improved model.*

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling—Physically based modeling

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## 1. Introduction

Friction is an important and complex phenomenon which occurs whenever two objects are in contact in nature. Its development is essential in tribology. Since Leonardo da Vinci and Amonton discovered rules of dry friction, it has been studied in many research fields such as mechanical engineering, material science, fluid and solid object dynamics, and so on. It also plays a significant role in fabric performance, from textile processing to user's tactile comfort. Over time a number of friction models have been developed to capture more observed features. In computer graphics, as an indispensable component of collision handling in physics simulation, friction greatly affects both static and dynamical behaviour. Existing cloth, hair and solid object simulations in graphics typically use a Coulomb model. A more comprehensive friction model is worth investigating.

When two bodies are in contact, dry friction occurs at the contact area as a force resisting the relative tangential motion between the contacting surfaces. Dry friction is separated into static (stiction) and kinetic friction for motionless and sliding contacts respectively, and can be described by:

- Amontons' 1st Law: friction is directly proportional to the applied normal direction.
- Amontons' 2nd Law: friction is independent of obvious contact area.

- Coulomb's Law: kinetic friction is independent of the sliding velocity.

Friction described by the above classic empirical laws is termed Coulomb friction. Because of its simplicity it has been widely used to model friction behaviour. As investigation into friction continued, many phenomena that could not be fully modelled by classic laws were observed, leading to more elaborate models extending the Coulomb model.

One of the most important phenomena not captured by the Coulomb model is that the break-away force (stiction) required to initiate sliding motion from rest is usually larger than kinetic friction [Mor33]. This is observed for almost all contact surfaces. The difference between stiction and kinetic friction causes intermittent motion rather than smooth movement between two objects in contact at low relative speeds. Typically, two contacting objects will not start sliding until the applied force is larger than stiction. Then objects suddenly slide across each other under smaller kinetic friction for a short while until they stick again. This frequent phenomenon is referred to as the stick-slip phenomenon (simply stick-slip) [TP00].

Once the tangential force exceeds the stiction, friction transits from static to kinetic form where another two important phenomena occur, the Stribeck effect and viscosity resulting from the dependency of friction on the relative tan-

gential velocity. The Stribeck effect is the decrease in friction from the stiction peak to lower Coulomb friction when the sliding velocity is small [Str02]. Viscosity indicates that friction increases as the sliding velocity increases [Rey86]. In physics and mechanical engineering, models which are able to capture these phenomena have been developed and successfully used to model friction for years.

These phenomena have been mentioned in discussion of friction of fabrics and fibres [DKV05, Aja92, FSCJ09]. Fabrics exhibit the feature of stiction larger than the kinetic friction resulting from its structural unflattening. According to analysis by Taylor and Pollet [TP00], if the fabric has a certain degree of elasticity, stick-slip occurs at low relative speeds due to the difference between higher stiction and lower kinetic friction. Solid objects such as metals show the Stribeck and viscous friction only under moist or wet conditions, while such velocity-dependent frictional effects are observed under both dry and moist conditions for textile materials, i.e. fibre, yarns and fabrics that deform viscoelastically [Kal88, Aja92, FSCJ09]. These effects have been thoroughly investigated for dry and lubricated textile materials.

In this paper we describe an improved physically based friction model in cloth simulation that extends the Coulomb model to consider these friction phenomena, and report differences in overall cloth behaviour. The model can be easily integrated into most existing cloth systems as a replacement for Coulomb friction, increasing the physical plausibility by accounting for more friction effects. The model is not much more complicated than the Coulomb model and adds little extra computational cost overall. Several experiments and comparisons with cloth simulation using Coulomb friction and the extended models are provided.

The remainder of the paper is organised as follows. We start by briefly reviewing previous work directly related to dry friction from engineering research and computer graphics in Section 2. The extended friction model is presented in Section 3, followed by discussion of implementation, results and performance in Sections 4, 6 and 5 respectively. Finally, the conclusion and future work are given in Section 7.

## 2. Related work

We provide a brief introduction to friction in engineering research, then previous work directly related to friction in computer graphics is reviewed.

Early friction models were based on classic laws which only summarise basic aspects of friction. Lately, requirements on more precise estimates of friction behaviour have become the main interest. A number of works [Str02, Kar85, AHDCdW94] improve on the Coulomb model to take into account effects discussed above for a wide range of applications. Their focus is on static models, on which our method is based. For purposes in the microscopic level in mechanical engineering, dynamic models such as Dahl, Bliman-

sorine and LuGre models factor in evolution of friction through time or displacement derivatives, which however is beyond the scope of this work. We refer interested readers to [Alt99, OAdW\*98]. In textile research, investigation of both static and dynamic friction models for fabrics and fibres have been reported in [DKV05, Lah02]. Experimental data by Taloy and Pollet [TP00] show that fabric materials clearly exhibit the stick-slip behaviour.

In computer graphics, the earliest work including friction for cloth is [TPBF87]. Baraff and Witkin [BW98] treat cloth-object friction as constraints. Bridson et al. [BFA02] propose a robust framework that computes friction as impulses for cloth simulation. Based on this framework, Selle et al. [SSIF09] propose a novel cloth-object contact and friction scheme to achieve more accurate collision response. Pabst et al. [PTS09] introduce an anisotropic friction model for contacts between deformable objects (including cloth) and materials with anisotropic surface and inhomogeneous roughness. Chen et al. [CFW13] develop an algorithm to improve cloth-body interactions using friction data captured from real-world. However, to the best of our knowledge, no attempt to model effects of stiction, Stribeck and viscous friction has yet been made for cloth simulation in graphics.

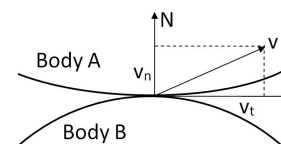
## 3. Friction Model

We assume that at the contact point (Figure 1) the infinitesimal surfaces of two bodies A and B are smooth, so that the relative velocity  $\mathbf{v}$  can be decomposed into a component  $\mathbf{v}_n$  along the normal direction and a tangential component  $\mathbf{v}_t$  opposed to the sliding direction. In the vertex-triangle type of contact, the normal of the triangle face is the contact normal, while in the edge-edge type of contact, the vector connecting the pair of closest points on two edges is treated as the contact normal. In this paper we use the notations as abbreviation for different friction models for convenience:

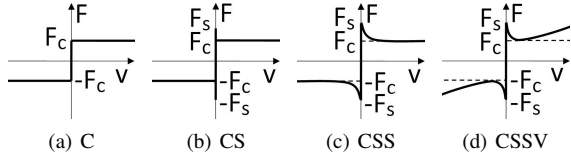
- C: Coulomb
- CS: Coulomb and stiction
- CSS: Coulomb, stiction and Stribeck friction
- CSSV: Coulomb, stiction, Stribeck and viscous friction

### 3.1. Friction Terms

This section describes each friction effect shown in Figure 2 in detail. The simplest model as well as the most common



**Figure 1:** A contact between two bodies. The normal and tangential vectors are defined. The relative velocity is decomposed into the normal and tangential velocity.



**Figure 2:** Friction models

one in graphics is Coulomb friction (Figure 2(a)) as mentioned earlier. As described by three classic laws, the friction force  $\mathbf{F}$  opposes the relative motion and is independent of the relative tangential velocity  $\mathbf{v}_t$ :

$$\begin{aligned} \mathbf{F} &= F_C(-\hat{\mathbf{v}}_t) \\ F_C &= \mu_c |\mathbf{F}_n| \end{aligned} \quad (1)$$

where  $F_C$  is Coulomb friction,  $\mu_c$  the Coulomb coefficient,  $\mathbf{F}_n$  the force in the normal direction of the contact plane and  $\hat{\mathbf{v}}_t$  the unit vector of the relative tangential velocity.

Stiction describes the friction force at zero relative velocity, and as mentioned earlier stiction is usually higher than Coulomb friction (Figure 2(b)). Hence it is expressed as:

$$\mathbf{F} = \begin{cases} -\mathbf{F}_e & \text{if } \mathbf{v}_t = 0 \text{ and } |\mathbf{F}_e| < |F_S| \\ F_S(-\hat{\mathbf{F}}_e) & \text{if } \mathbf{v}_t = 0 \text{ and } |\mathbf{F}_e| \geq |F_S| \end{cases} \quad (2)$$

where  $\mathbf{F}_e$  is the external tangential force,  $\hat{\mathbf{F}}_e$  the unit vector of  $\mathbf{F}_e$ , and  $F_S$  the stiction threshold whose value is greater than or equal to Coulomb friction:

$$F_S = \mu_s |\mathbf{F}_n| \quad \mu_s \geq \mu_c \quad (3)$$

where  $\mu_s$  is the coefficient of stiction. For fabrics  $\mu_s$  is always larger than  $\mu_c$ . The closer  $\mu_s$  and  $\mu_c$  are, the smoother the material is. For example, compared to silk, the stiction effect is more detectable in jeans. On the contrary the futher  $\mu_s$  and  $\mu_c$  are, the rougher the contacting surfaces are, more easily leading to stick-slip.

In order to model the Stribeck effect (Figure 2(c)) where friction decreases continuously from the stiction threshold to Coulomb friction as the velocity increases:

$$\mathbf{F}_{str}(\mathbf{v}_t) = (F_C + (F_S - F_C)e^{-(|\mathbf{v}_t|/v_S)^{\delta_S}})(-\hat{\mathbf{v}}_t) \quad (4)$$

where  $v_S$  is a positive value standing for the Stribeck velocity, and  $\delta_S \in [0, 1]$  a curve shape factor describing the nonlinear decrease. Under dry conditions friction decreases faster than it does under moist conditions, but we only consider dry cases in this paper. We set  $\delta_S$  to 0.2 (the smaller the value, the more quickly friction decreases). In the Bliman-Sorine friction model [BS95] Stribeck friction is degraded to be the velocity-independent transition from stiction to Coulomb friction. This model has been adopted to compute internal friction in bending behaviour of fabrics [Lah02]. However either way removes the discontinuous decrease.

Under dry conditions, viscous friction in textile materials

results from increasing frictional heating. For moist fabrics and lubricated fibre and yarns, viscous friction is characterized by the fact that a lubricant film exists between contact surfaces and friction is governed by the viscosity of the liquid. For either cases, viscous friction can be obtained by extending Coulomb model to consider the velocity as follows:

$$\mathbf{F} = F_v |\mathbf{v}_t|^{\delta_v} (-\hat{\mathbf{v}}_t) \quad (5)$$

where  $F_v$  is the viscous coefficient, and  $\delta_v$  enables the non-linearity of the dependency when its value is not 1. These two parameters vary a lot depending on types of textile materials in dry conditions, or types of fluid and how moist the condition is, i.e., moist to wet, in moist conditions. According to this equation, the higher the relative tangential velocity the larger friction is yielded (Figure 2(d)).

Therefore a general description of all effects is:

$$\mathbf{F}(\mathbf{v}_t) = \begin{cases} \mathbf{F}_k(\mathbf{v}_t) & \text{if } \mathbf{v}_t \neq 0 \\ -\mathbf{F}_e & \text{if } \mathbf{v}_t = 0 \text{ and } |\mathbf{F}_e| < |F_S| \\ F_S(-\hat{\mathbf{F}}_e) & \text{if } \mathbf{v}_t = 0 \text{ and } |\mathbf{F}_e| \geq |F_S| \end{cases} \quad (6)$$

where,  $\mathbf{F}_k(\mathbf{v}_t)$  is the overall kinetic friction force as the sum of Coulomb, Stribeck and viscous friction:

$$\mathbf{F}_k(\mathbf{v}_t) = (F_C + (F_S - F_C)e^{-(|\mathbf{v}_t|/v_S)^{\delta_S}} + F_v |\mathbf{v}_t|^{\delta_v})(-\hat{\mathbf{v}}_t) \quad (7)$$

There are several contact methods for cloth simulation. The equations described above can be integrated into those systems to model different friction effects. In our work we discuss the use of the extended friction model in two commonly adopted contact methods: the constraint handling approach [BW98] and the velocity filter framework [BFA02]. We use the latter for experiments in this paper.

For constraint based models like [BW98], stiction and kinetic friction effects can be obtained by enforcing a complete constraint that eliminates relative motion and a partial constraint that only allows sliding on the plane proportional to the contact normal force, respectively. In the sticking case, the number of degrees of freedom for relative acceleration is set to 0, while in the sliding case the contact normal indicates the prohibited direction. The change in relative velocity caused by contact is included in the linear system for explicit control. The change in relative velocity is computed as  $-\mathbf{v}_n + \mathbf{F}(\mathbf{v}_t)dt/m$ , where  $m$  is the mass of the colliding primitive pair.

In order to replace the friction component in [BFA02] where friction effects are achieved by using impulses, friction needs to be converted into the impulse  $I$ , which can be simply computed as

$$I = \frac{1}{2} |\mathbf{F}(\mathbf{v}_t)dt/m| \quad (8)$$

However in [BFA02] the impulse is directly obtained from the change in relative velocity in the normal direction  $d\mathbf{v}_n$

	incline plane	skirt	stick-slip	card	sphere
$\mu_c^{obj}$	.2	.2	.1	.1	.2
$\mu_c^{self}$	.1	.1	.1	.1	.1
$\mu_s^{obj}$	.3	.4	.4	.2	.35
$\mu_s^{self}$	.15	.2	.1	.2	.15
$V_S$	.4	.4	.4	.4	.4
$\delta_S$	.2	.2	.2	.2	.2
$F_v$	N/A	N/A	N/A	.00006	.00006
$\delta_v$	N/A	N/A	N/A	1	1

**Table 1:** Values of parameters used in each experiment. Superscripts *obj* and *self* indicate cloth-object and cloth self contacts respectively.

avoiding manipulating forces:  $I = \frac{1}{2} \left| \min(\mu |d\mathbf{v}_n| / |\mathbf{v}_t|, 1) \mathbf{v}_t \right|$ . Hence Equation 6 and 8 are replaced with:

$$\xi = \frac{|d\mathbf{v}_n| (\mu_c + (\mu_s - \mu_c) e^{-(|\mathbf{v}_t|/v_s)^{\delta_s}}) / |\mathbf{v}_t| + (F_v |\mathbf{v}_t|^{(\delta_v-1)}) dt / m}{I = \begin{cases} \frac{1}{2} |\mathbf{v}_t| & \mu |d\mathbf{v}_n| / |\mathbf{v}_t| \geq 1 \\ \frac{1}{2} |\xi \mathbf{v}_t| & \mu |d\mathbf{v}_n| / |\mathbf{v}_t| < 1 \end{cases} \quad (9)$$

By applying the impulse  $I$  into impulse weighting equations in [BFA02], the relative tangential velocity will be cancelled out due to stiction or slowed down due to kinetic friction.

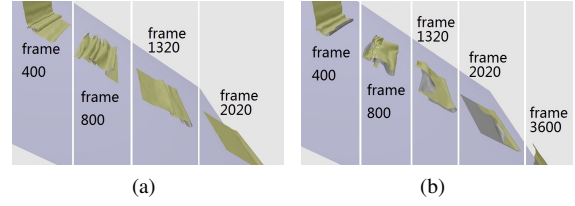
These two contact mechanisms discussed above are explicit, assuming the normal force and relative velocity are known. It is worth considering the extended friction model as part of implicit solvers, which we suggest as future work.

#### 4. Implementation

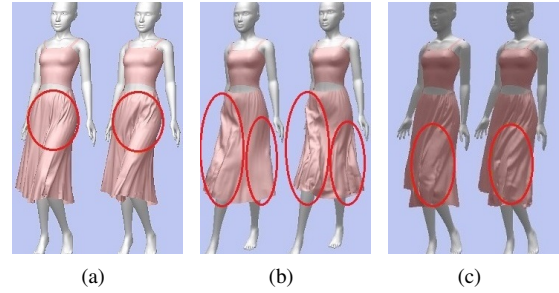
We adopt a cloth model similar to [BW98] to model tensile forces. Bending force computation is based on [PKST08]. Following [BW98], the integrator for evolving tensile and damping forces is an implicit backward Euler scheme. We handle collision detection by using a GPU-based virtual triangle subdivision scheme with a uniform spatial partition [WLZ12]. For collision response the widely adopted framework of Bridson et al. [BFA02] is used, but where the friction model is replaced by the extended model discussed here. A strain rate limiting procedure similar to the scheme in [BFA02] is used to remove bad strain rates.

#### 5. Results

We investigate friction models using several scenarios, and point out differences and advantages by comparing with the C model. Friction is applied for both cloth-object contacts and self contacts. Since we are interested in differences in behaviour between different models, our aim is not to investigate and evaluate deeply how parameters of the extended model influence the output. Values of parameters used in



**Figure 3:** A piece of cloth falls on an incline plane. Results of (a) the C model and (b) the CSS model at several frames.

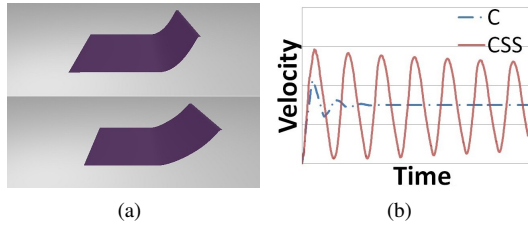


**Figure 4:** A walking character wearing a skirt. Results of the C (left) and CSS model (right) at several frames.

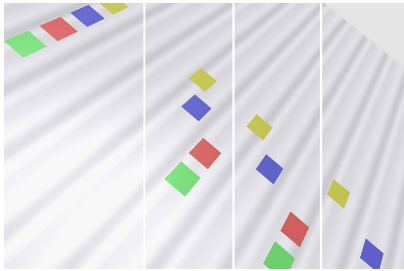
each experiment are given in table 1. For overall dynamic behaviour we refer readers to accompanying animations.

We first test the CSS model. Figure 3 shows frames of a piece of cloth falling on an inclined plane. Figure 3(a) displays results of simulation using the C model, while Figure 3(b) shows friction effects generated by the CSS model. Because of the higher stiction and the Stribeck effect, the cloth in simulation with the CSS model is more likely to experience static friction, and higher kinetic friction when the tangential velocity is small, and thus tends to fold more and slides slower than the cloth in the simulation using the C model. We have also applied these two friction models to a scene where a female wearing a skirt walks (Figure 4). In the CSS case, more and finer wrinkles are formed and hold longer, especially in the skirt as highlighted in the figure.

Since stiction larger than kinetic friction is the main cause of stick-slip, we built a scenario to test this mechanism (Figure 5(a)). A hang cloth has an edge attached. The lower part of the cloth touches the ground with friction. The attached edge moves at a constant velocity so that the cloth is pulled by the internal force. In the CSS model case where the kinetic friction coefficient is much less than the stiction coefficient, the cloth moves intermittently. The cloth stays stuck before the tangential force reaches stiction, then it suddenly starts sliding with a large acceleration due to the much smaller kinetic friction. In the slip-phase, the internal force decreases as the stretched cloth tends to recover to its un-



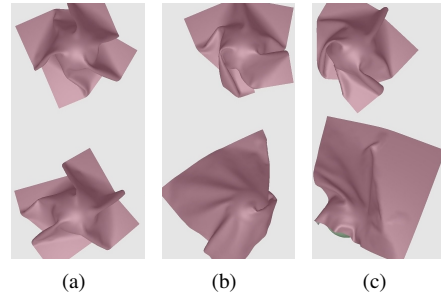
**Figure 5:** (a) A scene to test stick-slip: results of the C (top) and CSS model (bottom). (b) Average displacements of the lower part cloth in simulations using the C and CSS model.



**Figure 6:** A card slides on an inclined plane. From left to right results of C model (green), CS model (red), CSS model (blue) and CSSV model (yellow) at several frames.

stretched state, leading to the cloth stuck again. As expected the entire process is repeated periodically. This stick-slip behaviour is depicted by the red solid curve in Figure 5(b), where average velocity of the lower part cloth are shown. On the contrary, the cloth in the C model case moves much smoother (the blue dashed curve).

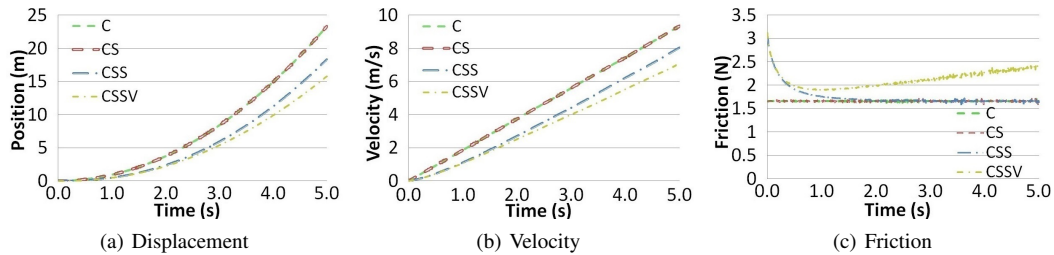
Now we test all friction models. We build a test of four cards sliding on an inclined plane (Figure 6). Cards start from rest and accelerate. During the whole process, cards stay in contact with the plane. All four friction models are used for four cards respectively. In all four cases,  $\mu_s$  is set to  $2\mu_c$ . When the value of  $\mu_c$  is above .3 no cards slide. If  $\mu_c$



**Figure 8:** A cloth drapes over an accelerating rotating sphere. Results of the C model (top) and the CSSV model (bottom) at frame (a) 400, (b) 2400 and (c) 2980.

is set to between .15 and .3 only the card in simulation using the C model slides. By setting  $\mu_c$  below .15, cards in all simulations move as the force towards the sliding direction is greater than the stiction, providing an initial sliding velocity. The value of  $\mu_c$  in the experiment shown in Figure 6 is .1. Observed results match the expectation. Since the stiction cannot keep the card stuck, unsurprisingly the cloth in simulation using the C model has the same velocity with the one in the CS model simulation, as they undergo the same kinetic friction and therefore accelerate at an identical rate. However, the CSS and CSSV models yield higher kinetic friction at the beginning when the velocity is small, hence cards using these two models move slower. As their velocities increase, friction in the CSS model simulation gradually decreases to the Coulomb level, while friction by the CSSV model rises due to viscosity. Hence, the card using the CSSV model slides slowest. Plots of friction, velocity and displacement over time for this experiment are shown in Figure 7.

Another scenario (Figure 8) in which a piece of cloth drapes over an accelerating rotating sphere is designed to show the effect of viscous friction in a more complicated case. As the angular speed of the sphere increases from zero, the cloth in the CSSV model case (bottom) spins faster and faster at an increasing acceleration due to increasing friction,



**Figure 7:** The displacement, velocity and friction over time for a card sliding down on an incline plane using four different friction models are shown. Configurations of this test are identical to the cylinder experiment.

	no friction	C	CS	CSS	CSSV
time (s)	0.71	0.86	0.90	1.16	1.19
friction ratio	N/A	17%	21%	39%	40%

**Table 2:** For a simulation of 5 seconds ( $\Delta t = 1\text{ms}$ ), running time of collision response using different models.

while the cloth in simulation using the C model (top) spins at a constant acceleration caused by a constant friction force, as the friction is independent of velocity. Therefore, the cloth undergoing viscous friction is cast off earlier.

## 6. Performance

In our system computation time is dominated by collision detection. Collision response takes a small amount of total time. Hence the time for the friction part is even smaller. We use the scene (8.1K triangles) shown in Figure 3 to examine the performance. The time for collision response using different friction models are given in Table 2. The CSSV model introduces a 40% increase in collision response computation time which takes less than 8% of total time.

## 7. Conclusion and Future Work

The extended friction model allows additional friction effects of stiction (therefore Stick-slip), Stribeck and viscosity easily incorporated into commonly adopted systems. Results of several experiments and comparisons are presented to demonstrate the capability of modelling friction effects and differences in cloth behaviour. The focus of our work here is cloth simulation. Clearly, animating rigid and deformable objects with these additional friction effects would be beneficial. Considering the extended friction model as part of the implicit solve remains an area for future work as well.

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