Appendix A: Residual distribution details

Conservative → primitive variable transformation matrix

$$\mathbf{M} = \frac{\partial \mathbf{q}}{\partial \mathbf{Q}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ u & \rho & 0 & 0 & 0 \\ v & 0 & \rho & 0 & 0 \\ w & 0 & 0 & \rho & 0 \\ \mathbf{V}^2/2 & \rho u & \rho v & \rho w & 1/\gamma_{\text{-}1} \end{bmatrix} \quad (28)$$

with $\mathbf{V}^2 = u^2 + v^2 + w^2$, $\gamma_{\text{-}1} = \gamma - 1$ ($\mathbf{M}^{-1}$ is easily computed analytically).

The primitive variable Euler equations are

$$\mathbf{Q}_t + \mathbf{F_Q} \cdot \nabla \mathbf{Q} = f \quad (29)$$

with the Jacobian components $\mathbf{F_Q} = A\mathbf{e}_x + B\mathbf{e}_y + C\mathbf{e}_z$

$$\mathbf{A} = \begin{bmatrix} u & \rho & 0 & 0 & 0 \\ 0 & u & 0 & 0 & \frac{1}{\rho} \\ 0 & 0 & u & 0 & 0 \\ 0 & 0 & 0 & u & 0 \\ 0 & a & 0 & 0 & u \end{bmatrix} \quad (30)$$

$$\mathbf{B} = \begin{bmatrix} v & 0 & \rho & 0 & 0 \\ 0 & v & 0 & 0 & 0 \\ 0 & 0 & v & 0 & \frac{1}{\rho} \\ 0 & 0 & 0 & v & 0 \\ 0 & 0 & a & 0 & v \end{bmatrix} \quad (31)$$

$$\mathbf{C} = \begin{bmatrix} w & 0 & 0 & \rho & 0 \\ 0 & w & 0 & 0 & 0 \\ 0 & 0 & w & 0 & 0 \\ 0 & 0 & 0 & w & \frac{1}{\rho} \\ 0 & 0 & 0 & a & w \end{bmatrix} \quad (32)$$

with

$$a = \left[\frac{p}{\rho} - \rho\left(\frac{\partial e}{\partial \rho}\right)_p\right]\left(\frac{\partial e}{\partial p}\right)_\rho^{-1} = \gamma p \quad (33)$$

the last equality being valid for a perfect gas.

Roe average. The Roe average for the Euler equations is a weighted average of the vertex velocities (u_R, v_R, w_R) and enthalpy h_R, the quantities that appear in the eigenmodes. The weights are defined by the square root of the vertex density.

$$u_R = \frac{\sum_{i=1}^{5} \sqrt{\rho_i} u_i}{\sum_{i=1}^{5} \sqrt{\rho_i}}, \quad (34)$$

with similar expressions for the other variables.

Inflow/outflow splitting, case 2, 3 inflow matrices.

$$\mathbf{K}_{2i}^{+} = \quad [k_{2i}^{(1)} \quad k_{2i}^{(2)} \quad k_{2i}^{(3)} \quad k_{2i}^{(4)} \quad k_{2i}^{(5)}] \quad (35)$$

$$k_{2i}^{(1)} = \quad [\mathbf{v} \cdot \mathbf{n}_i \quad 0 \quad 0 \quad 0 \quad 0]^T \quad (36)$$

$$\left[\; k_{2i}^{(2)} \quad k_{2i}^{(3)} \quad k_{2i}^{(4)} \;\right] = \quad (37)$$

$$\frac{1}{2}\begin{bmatrix} \rho n_{ix}\lambda_+/c & \rho n_{iy}\lambda_+/c & \rho n_{iz}\lambda_+/c \\ 2V_{ni} - n_{ix}^2\lambda_- & -n_{ix}n_{iy}\lambda_- & -n_{ix}n_{iz}\lambda_- \\ -n_{ix}n_{iy}\lambda_- & 2V_{ni} - n_{iy}^2\lambda_- & -n_{iy}n_{iz}\lambda_- \\ -n_{ix}n_{iz}\lambda_- & -n_{iy}n_{iz}\lambda_- & 2V_{ni} - n_{iz}^2\lambda_- \\ c\rho n_{ix}\lambda_+ & c\rho n_{iy}\lambda_+ & c\rho n_{iz}n_{iy}\lambda_+ \end{bmatrix} \quad (38)$$

$$k_{2i}^{(5)} = \left[\; -\frac{\lambda_-}{2c^2} \quad \frac{n_{ix}\lambda_+}{2\rho c} \quad \frac{n_{iy}\lambda_+}{2\rho c} \quad \frac{n_{iz}\lambda_+}{2\rho c} \quad \frac{\lambda_+}{2} \;\right] \quad (39)$$

$$\mathbf{K}_{3i}^{+} = \frac{\lambda_+}{2}\begin{bmatrix} 0 & \frac{\rho n_{ix}}{c} & \frac{\rho n_{iy}}{c} & \frac{\rho n_{iz}}{c} & \frac{1}{c^2} \\ 0 & n_{ix}^2 & n_{ix}n_{iy} & n_{ix}n_{iz} & \frac{n_{ix}}{c\rho} \\ 0 & n_{ix}n_{iy} & n_{iy}^2 & n_{iy}n_{iz} & \frac{n_{iy}}{c\rho} \\ 0 & n_{ix}n_{iz} & n_{iy}n_{iz} & n_{iz}^2 & \frac{n_{iz}}{c\rho} \\ 0 & c\rho n_{ix} & c\rho n_{iy} & c\rho n_{iz} & 1 \end{bmatrix} \quad (40)$$