Appendix A: Oren-Nayar Diffuse Reflectance Model

The Oren-Nayar reflectance model was designed for rough surfaces. The model is composed of two parts: the direct illumination component and the inter-reflection component. The direct illumination component in the radiance for this model is given by

$$\begin{aligned} & \rho_d^1(\theta_r, \theta_i, \phi_r - \phi_i; \sigma_d, K_d) \\ = & \frac{K_d}{\pi}\Big[C_1(\sigma_d) + \cos(\phi_r - \phi_i) C_2(\alpha, \beta, \phi_r - \phi_i, \sigma_d) \tan\beta \\ & + (1 - |\cos(\phi_r - \phi_i)|) C_3(\alpha, \beta, \sigma_d) \tan(\frac{\alpha + \beta}{2})\Big] \qquad (1) \end{aligned}$$

where,

$$\alpha = max(\theta_r, \theta_i) \qquad (2)$$

$$\beta = min(\theta_r, \theta_i) \qquad (3)$$

$$C_1 = 1 - 0.5\frac{\sigma_d^2}{\sigma_d^2 + 0.33} \qquad (4)$$

$$C_2 = \begin{cases} 0.45\frac{\sigma_d^2}{\sigma_d^2+0.09}\sin\alpha & \cos(\phi_r - \phi_i) \geq 0 \\ 0.45\frac{\sigma_d^2}{\sigma_d^2+0.09}(\sin\alpha - (\frac{2\beta}{\pi})^3) & \text{otherwise} \end{cases} \qquad (5)$$

$$C_3 = 0.125(\frac{\sigma_d^2}{\sigma_d^2 + 0.09})(\frac{4\alpha\beta}{\pi^2})^2 \qquad (6)$$

The inter-reflection component is given by

$$\begin{aligned} & \rho_d^2(\theta_r, \theta_i, \phi_r - \phi_i; \sigma_d, K_d) \\ = & 0.17\frac{K_d^2}{\pi}\frac{\sigma_d^2}{\sigma_d^2 + 0.13}\Big[1 - \cos(\phi_r - \phi_i)(\frac{2\beta}{\pi})^2\Big] \qquad (7) \end{aligned}$$

These two components combine to give the total diffuse surface radiance.

$$\begin{aligned} & \rho_d(\theta_r, \theta_i, \phi_r - \phi_i; \sigma_d, K_d) \qquad (8) \\ = & \rho_d^1(\theta_r, \theta_i, \phi_r - \phi_i; \sigma_d, K_d) + \rho_d^2(\theta_r, \theta_i, \phi_r - \phi_i; \sigma_d, K_d) \end{aligned}$$

Appendix B: Torrance-Sparrow Specular Reflectance Model

The Torrance-Sparrow specular model is expressed by facet normal distribution, geometrical attenuation and fresnel reflection terms as

$$\rho_s = \frac{D \cdot G \cdot F}{4\cos\theta_i \cos\theta_r} \qquad (9)$$

where D describes the distribution of facet normals over the surface and G is a geometrical attenuation factor.

$$D = e^{-(\theta_h/\sigma_s)^2} \qquad (10)$$

$$\begin{aligned} G = & max(0, min(1, \frac{2\cos\theta_i\cos\theta_h}{\cos\theta_i\cos\theta_h + \sin\theta_i\sin\theta_h\cos(\phi_i - \phi_h)}, \\ & \frac{2\cos\theta_r\cos\theta_h}{\cos\theta_r\cos\theta_h + \sin\theta_r\sin\theta_h\cos(\phi_r - \phi_h)})) \qquad (11) \end{aligned}$$

F is the fresnel reflection term and depends on the refractive index n of the material. We have set the Fresnel term to 1 for convenience of measurement and fitting.

Appendix C: Top-lit Dust Reflectance Model

The dust reflectance ρ_{dust} is from the top lit brightness function in Blinn's dust reflectance function which is given by

$$\rho_{dust}(\theta_r, \theta_i, \phi_r - \phi_i; g, w_{r,g,b}) = w_{r,g,b}\Phi(\gamma)\frac{\cos\theta_i}{(\cos\theta_i + \cos\theta_r)} \tag{12}$$

where γ is computed as the angle between the light and viewing ray. Φ is the popular Henyey-Greenstein phase function which describes the dependence of scattering on deviation angle γ.

$$\Phi(\gamma, g) = \frac{1 - g^2}{(1 + g^2 - 2g\cos\gamma)^{3/2}} \tag{13}$$

This is the equation of an ellipse in polar coordinates, centered at one focus. The parameter g is the eccentricity of the ellipse and is a property of the material. When g equals 0, scattering is isotropic. When g is greater than 0, it is predominantly forward scattering.