

Analyzing and Visualizing Multivariate Volumetric Scalar Data and Their Uncertainties

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Abstract

Data sets from the real world and most scientific simulations are known to be imperfect, often incorporating uncertainty information. Exploration and analysis of such variable data can lead to inaccurate or even incorrect results and inferences. As a powerful tool to communicate the data with users, an effective visualization system should present and inform users of the uncertainty information existing in the data. While some research has been conducted on visualizing uncertainty in spatio-temporal data and univariate data, little work has been reported on multivariate data. In addition, there are two main disadvantages in the existing uncertainty visualization methods for volumetric data. First, they rely heavily on the human perceptual system to recognize the uncertainty information, lacking the capability to depict them quantitatively. Second, they often present large amounts of diverse information in a single display, which may result in visual clutter and occlusion. In this paper, we present our hybrid framework that combines both information visualization techniques and scientific visualization techniques together to allow users to interactively specify features of interest, quantitatively explore and analyze the multivariate volumetric data and their uncertainties as well as localize features in the 3D object space. In comparison with those existing methods, we argue that our method not only allows users to quantitatively visualize the uncertainties within multivariate volumetric data, but also yields a clearer data presentation and facilitates a greater level of visual data analysis. We demonstrate the effectiveness of our framework by reporting a case study from the OpenGGCM (Open Geospace General Circulation Model) simulation in space science application domain.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Picture/Image Generation—Display algorithms

1. Introduction

Data sets from the real world and even most scientific simulations often incorporate uncertainty information. For example, uncertainty can be found in Computational Fluid Dynamics (CFD) data sets, bioinformatics data sets, environmental science or geo-spatial data sets, intelligence and military data sets, commerce databases, etc [DKLP02]. These uncertainties may refer to various quantities associated with data including error, accuracy, variability, noise, or completeness of the data [DAN12]. They often arise throughout the scientific process due to a variety of factors i.e., problems in data acquisition and process, approximation of data interpolation and sampling, variability of instrument's measurement and calculation.

Visualization is a powerful tool to convey data to users and assists them in understanding the phenomenon behind the data. Since uncertainties often exist in data and may significantly affect the validity of decisions made by users, it is very important for visualization researchers to carry out research on graphical representations of the uncertainty information. In this paper, we focus our research on the scalar uncertainty since it is easier to manage and has a large range of application domains.

1.1. Motivation

According to literature review, existing 1D and 2D uncertainty visualization methods [CR00] [GR04] [LV02] [Hen03] cannot be applied directly to visualize the uncertainty in volumetric scalar fields. Also existing volumetric methods have two main drawbacks. First, they rely heav-

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ily on the human perceptual system to recognize the uncertainty information and thus may lead to different interpretation from person to person. They lack the ability to depict the uncertainty quantitatively. Second, they tend to present diverse information including both data attributes and uncertainties in a single display. Such methods may be good when there is a small amount of information. As the amount of information increases, such methods can cause the classic visualization problems i.e., clutter and occlusion. Therefore we argue that it is a better choice to present the data attributes and uncertainties in a framework consisting of multiple linked views for a clearer data presentation. In addition, such a choice may also benefit a greater level of visual data analysis. We can effectively separate the data's feature space from its object space, without representations conflicting with each other. Furthermore, we can combine more interaction techniques together to form a more powerful visualization system.

While most of the work in uncertainty visualization research area has been primarily focused on spatio-temporal or univariate data, little research has been reported on visualizing the uncertainty in multivariate data [XHWR06] [HYX11].

Therefore in this work we explore a hybrid framework that combines both information visualization techniques and scientific visualization techniques to analyze and visualize multivariate volumetric scalar data and their uncertainties. We refer the uncertainty to scalar error in this paper.

1.2. Contributions

The main contributions of this paper are:

- Designed a hybrid framework that allows users to visually explore and analyze the multivariate volumetric data and their errors.
- Developed a new uncertainty visualization method that allows users to select a subset of the numerical domain in 2D plane and then quantitatively visualize the subset's data attributes, errors as well as their relationship using the depth information collected from ray casting.
- Introduced a new case study where the framework has been applied.

The structure of this paper is as follows. Section 2 presents the work that are related to our research in the area of uncertainty visualization. Section 3.1 gives the two reasons about why we utilize MR(Multi-Resolution) modeling. Section 3.2 presents two concrete MR modeling methods to model the appropriate error information for our research. Section 3.3 introduces the model to quantify the error produced from these two methods. In Section 4, we give a great detail of the framework's work flow and its every component. In Section 5, we present a case study from the space science application domain and report the corresponding results to show our framework's effectiveness. Finally in Section 6 we draw

our conclusions and propose future work expected to be conducted.

2. Background and Related Work

Some of earliest research in uncertainty visualization started in the Geographic Information System (GIS) community [Mac92] [WF93]. That work is mainly about representing the error in terrain models. Later on some researchers from computer graphics or scientific visualization community started to pay attention to this research area, and their work mainly involves visualizing the uncertainty in 3D surface [LSPW96] [ATP96].

Uncertainty visualization started to gain momentum when its significance was pointed out by several leading researchers [JS03] [Joh04] [Che05] [JMT*06] [LK07]. According to recent literature [XHWR06] [HYX11] [Pot11], it is now an active research area in the visualization community. A number of researches have explored the visualization of uncertainty for volumetric scalar data [DKLP02] [RLBS03] [LLPY07] [FB09]. These proposed techniques involve from inline DVR, post-processing to hybrid rendering and animation, and they are considered to be effective and straightforward. However, there are two main disadvantages of these techniques. First, they rely on human perceptual system to distinguish and recognize different scales of uncertainty information and thus may lead to different interpretation from person to person. Second, most of the work tends to present both data attributes and uncertainties in a single display, which may result in visual clutter and occlusion, especially when the amount and diversity of information increases.

In recent publications, Potter et al. have presented Ensemble-Vis, a framework consisting of a collection of overview and statistical displays linked through a high level of interactivity, to allow scientists to gain key scientific insight into the distribution of simulation results as well as their uncertainty [PWB*09]. Sanyal et al. also proposed a framework called Noodles, which consists of coordinated views of ribbon, glyph-based uncertainty visualization, spaghetti plots, iso-pressure colormaps and data transect plots to visualize the uncertainty in ensemble data [SZD*10]. In contrast to uncertainty visualization methods that present a diverse collection of information in a single display, they argued that their framework which combines multiple linked views techniques can yield a clearer presentation of the data. Although both techniques mentioned above have indicated that developing the effective framework for uncertainty visualization can be a very promising way, they are only specialized for 2D or 2.5D data.

While most research in uncertainty visualization has been focused on spatio-temporal data and univariate data, little work has been reported on multivariate data. In the information visualization area, Xie et al. described two approaches

to the problem of visually exploring multivariate data with variable quality [XHWR06]. He et al. presented approaches to improve the existing visualizations of parallel coordinates and star glyph and then applied them to visualize multivariate uncertainty [HYX11]. Although both approaches are useful to visualize the uncertainty for multivariate non-spatial-based data, they are not directly applicable for multivariate spatial-based data.

3. Error Information Modeling and Quantification

3.1. Reasons to utilize MR modeling

There are many ways to model the uncertainty incorporated within the volumetric scalar data, depending on the specific application domains explored. Here we will not give a comprehensive review of these modeling approaches since they are beyond the scope of this paper. Instead, we will detail the reasons why we use the MR modeling methods.

3.1.1. Provide simple yet effective error generation approach for generalized uncertainty visualization research

The first reason that we use MR modeling methods is because while relatively simple, they enable us to generate the error data that has the same characteristics as those generated from complex scientific simulations i.e., in [DKLP02]. Here when we say the same characteristics, we do not mean that the MR modeling and the complex scientific simulations can generate the same error results. Instead, we mean that both methods are able to model one or multiple scalar error values that are associated to every grid point of the volumetric data. As a result, the developed uncertainty visualization methods tested by the error data generated from the MR modeling can also be applied to other application domains. MR modeling provides a simple yet effective way for generalized uncertainty visualization research for volumetric scalar data.

3.1.2. Generate the data for data integrity research

The second reason that we use MR modeling is because we want to develop new uncertainty visualization methods to address data integrity issue that is caused by the MR modeling technique itself.

While loss of certain trivial information may not affect correct decision making, loss of significant information will greatly affect the validity of decisions. Therefore we need a useful tool to assist users to evaluate the integrity of data. We address this problem by using MR technique to model the errors and then exploring new uncertainty visualization methods to depict them.

3.2. MR modeling

MR modeling is a useful technique to reduce the data size and generate coarse resolution approximations of original

data. Two specific examples of such a technique have been used in our work and they are Haar wavelet transformation and decimation, respectively. We applied 3D version of these two approaches on volumetric scalar data to create its MR hierarchy with different resolution levels. We then used the method as described in Section 3.3 to quantify the introduced errors in the coarse resolution approximations.

3.3. Error quantification model

Given a MR data hierarchy for each attribute of the data, the next step prior to uncertainty visualization is to quantify the local error information between original data and each of its successive low resolution approximations in the hierarchy. We use the Standard Deviation Model (SDM) to measure the local error information, as illustrated in formula (1):

$$e = \sqrt{\frac{1}{n}[(V_1 - \bar{V})^2 + (V_2 - \bar{V})^2 + \dots + (V_n - \bar{V})^2]} \quad (1)$$

Where e represents the local error value that corresponds to every grid point of lower resolution approximations of the original data. It is calculated based on data attributes from the original data and the corresponding data attribute from its selected lower resolution approximation; n is the quantified expression of local measurement. It refers to the number of original data's grid points which are utilized to generate the only one grid point of its lower resolution approximation. V_1, V_2, \dots, V_n represent the n attribute values that are associated to the n grid points in the original data; \bar{V} represents the average attribute value that is calculated from V_1, V_2, \dots, V_n and is associated to the only one grid point in the lower resolution approximation of the original data.

We perform the local error measurement repeatedly on the entire volumetric data for each attribute and record the error value at every grid point in the lower resolution data. Consequently we acquire the multivariate volumetric data with multiple uncertainties associated to them. At this point we can explore the hybrid uncertainty visualization framework.

4. The Hybrid Framework to Analyze and Visualize Multivariate Volumetric Data and Their Errors

In this section, we introduce the framework according to its three main functionalities, as shown in Figure 2 (all the figures of this paper can be found at <http://csvg.blogspot.ie/2012/07/analyzing-and-visualizing-multivariate.html>). We firstly give an overview of the three functionalities and then describe each functionality's components in a separate subsection in more details. In our framework, multiple linked views are used to share camera information, selection and content when appropriate. While each individual view presents the data in either object space or feature space, highlighting certain aspects of the data behavior and providing a clear presentation for the data analysis, combining these views together into a unified framework provides a more powerful method. It brings in

opportunities to get more insights from complex data and is possible to perform a larger scale of data analysis based on more forms of interactions. As a result, our framework provides an effective platform to visualize the errors in multivariate volumetric data.

4.1. Framework overview

Based on the important experience obtained from collaborating with the industrial partners [DMGP05], our framework typically incorporates three main functionalities. The first functionality is indicated by the content inside the red dotted lines as drawn in Figure 2. It allows users to select a Region of Interest (ROI) of a numerical domain from any one object space view and then inspect its corresponding statistical information as well as its multivariate attributes and errors' relationship in the consequent feature space views. The second functionality of this framework is indicated by the content inside the yellow dotted lines as drawn in Figure 2. It allows users to investigate the characteristics and distribution of multivariate attribute data and their errors by specifying a feature in one feature object view and concurrently analyzing other data characteristics or distribution in other feature space views. The third functionality of this framework, known as feature localization, is indicated by the content inside the blue dotted lines as shown in Figure 2. It allows users to look for in the 3D object space where certainty interested features appear. Based on the three functionalities mentioned here, users now are able to interactively explore and analyze the multivariate volumetric data and their errors.

4.2. Functionality 1

4.2.1. DVR views of multivariate volumetric data

This type of views is designed to present both the internal and external structures of the multivariate volumetric data through DVR so that users are able to inspect the data within the 3D environment. Specifically in our research, we take advantage of the volume ray casting-based algorithm to render the data. Due to the complexity incorporated within multivariate volumetric data, presenting them together in a single display may lead to various problems i.e., clutter, occlusion. In order to effectively assign a dedicated object space for every attribute and allow users to inspect it more clearly, we render every attribute of the multivariate data in an individual view. In addition, we enable users to perform a 2D ROI selection on any one of these views. Once a ROI is specified in one view, all of the rest views will be updated automatically, marking out the same ROI in the 3D domain. Consequently it is possible for users to focus their interest only on the specified ROI across all views and distinguish the different characteristics among different data attributes within the ROI.

4.2.2. DVR views of error data

This type of views has the same functionalities as the one describe in Section 4.2.1, apart from that it is used to render multivariate volumetric data's error information. We also render the error information in regards to every data attribute in an individual view and enable users to perform the 2D ROI specification on any one of them. Moreover, we put every error view side by side to its corresponding data attribute view so that users are able to make a visual comparison between them. It provides an effective mechanism to guide users to identify their ROI.

4.2.3. Histograms of attribute data

We use histograms (shown in Figure 4(a3)) to visually and quantitatively represent the statistical information of the multivariate data that is specified within a ROI by users in the 3D object space. For every attribute data, there is only one histogram in regards to it. The horizontal axis of the histogram represents the attribute variations, while its vertical axis represents the number of sampling points (obtained from volume ray casting-based rendering) that fall into a particular interval.

4.2.4. Histograms of error data

Similar to Section 4.2.3, we use histograms to visually and quantitatively represent the statistical information for error data. However, instead of utilizing the horizontal axis to represent attribute variations, we use it to represent the error variations. We utilize the vertical axis to represent the number of sampling points of error information.

4.2.5. Parallel coordinates

In the information visualization community, parallel coordinates is a common and useful tool. It is always utilized to explore the multivariate data as well as their relationship. We also include a parallel coordinates view into our framework. We arrange the parallel coordinates in such a way that every error data is next to and on top of its associated attribute data. This is considered to be convenient and efficient for users to compare them in pairs. Based on the clear information displayed by the parallel coordinates, users can now study the relationship between multivariate attributes and their errors.

4.2.6. Linking between views by ROI selection using depth information

While each of the above mentioned components has certainty specific functionality, linking them together make up a more useful system that allow users to perform quantitative exploration and analysis tasks based on their ROI. A typical work flow of functionality 1 is explained as follows. Users start their tasks by inspecting the feature of multivariate volumetric data and their errors in the DVR views (here designing effective transfer functions for these views is necessary in order to guide users to recognize their interest).

They can specify their ROI by dragging a rectangle on any one of these views through a mouse interaction. Once a ROI is selected, four things happen. First, the rest of the views that have not been used for the ROI selection are updated, focusing on the same ROI with a green rectangle indicated. Second, a group of histograms are displayed and each one corresponds to an attribute, describing the attribute's statistical information within the specified ROI. Third, another group of histograms are displayed and each one corresponds to an attribute's error information, summarizing its statistical information within the ROI. Last, a parallel coordinates is plotted and it is used to reveal the relationship between the multivariate data and their associated errors within the ROI. One important thing that needs to be highlighted here is that the data we utilized to plot the histograms and parallel coordinates actually include the depth information of the volumetric data, collected from the ray casting process. They are not simply the information obtained from the surface of the volumetric data. We provide a mechanism to allow users to plot the ROI in 2D plane but extract its corresponding 3D information (2D + depth). Consequently we provide an effective mechanism for quantitative visualization of multivariate attribute data and their errors.

4.3. Functionality 2

4.3.1. Interactive scatter plots for data feature specification

Scatter plot is a useful visualization tool to quantitatively depict values of two variables for a given data set in the Cartesian Coordinates. It is used throughout the statistical analysis. In our research, we construct a scatter plot by plotting an attribute data against its associated error data. Arranging it in such a way has two benefits. First, we effectively assign a separate feature object for every pair of attribute data and its associated error data to facilitate users' observation. Second, we allow users to specify a feature through mouse interaction on it.

We can consider scatter plots as condition filters on which users can perform various operations to formulate their complex selection. In Section 4.3.1.1 and 4.3.1.2 we introduce two operations, named AND and OR, respectively.

4.3.1.1. AND operation AND is one of the most basic logic operations in computer programming and digital circuit. It is very useful to judge whether all expressions meet conditions simultaneously. In [Gas04], Gasser borrowed the AND idea from the logic operation and take advantage of it to implement the complex selection for his visualization tool. In this research we also incorporate it into our framework for two reasons. First, we want to study that if certainty features of a given multivariate data are met simultaneously, what will the other features look like? Second, we want to know that where the shared feature (based on a previous se-

lection step) appear in the 3D object space? We use Figure 1(a) to illustrate the concept of AND operation.

4.3.1.2. OR operation Similar to the AND operation, we use Figure 1(b) to illustrate the concept of OR operation. The reason to incorporate it into our framework is because we want to know that if certainty features of a given multivariate data are selected, where all these features appear in the 3D object space?

4.3.2. Linking between scatter plots

In contrast to the OR operation under which every scatter plot is used to simply display the data information and specify a feature, the AND operation has an extra functionality: users are allowed to specify a feature of the multivariate data on any one of these scatter plots and concurrently analyzing the other features of the multivariate data in the other scatter plots. This functionality is achieved by the following steps. First, users specify a feature on one scatter plot by dragging a rectangle using their mouse. Second, based on users' specification, we search the entire data to find out the grid points where the feature matches user's specification. Third, we display the other features that reside on those grid points in the other scatter plots (each feature corresponds to one scatter plot). At this point users are able to analyze the other features of the data based on their previous feature specification. Fourth, users can repeat the above mentioned three steps in order on the other scatter plots which have not been used for the feature specification. As a result we provide users with an effective guidance mechanism to refine the ROI.

4.4. Functionality 3

4.4.1. Feature localization view

Our feature localization view is used to display the final positions of features that have been specified by users in the previous steps, as described in Section 4.3. It is where the 3D object is rendered by the volume ray casting-based algorithm. Dependent on this view users can clearly observe where their interested features are located.

4.4.2. Linking between scatter plots and feature localization view

The linking between the scatter plots and the feature localization view is achieved by the feature specification done in the feature spaces. And a synchronize effect between every scatter plot and the feature localization view is achieved by the mouse interaction so that users can know which scatter plot/feature corresponds to the current image shown in the feature localization view. In addition, for the AND operation mentioned in Section 4.3.1.1, we also apply the same color on both scatter plot and the feature localization view to indicate the synchronization.

5. Case Study

A case study is described here to show the effectiveness of our framework.

5.1. Data set from OpenGGCM simulation

The data set we used for our research comes from the OpenGGCM simulation performed at Space Science Research Center in the University of New Hampshire. It is used to study the phenomenon of solar wind and its interaction with the earth's magnetosphere [ADO]. The data is three dimensional plus time, and is sampled on a stretched Cartesian grid [Rae95]. Many attributes are available from the output of the simulation, including pressure, density, resistivity, bulk plasma velocity and magnetic field. Since in this paper we focus our research on multivariate volumetric scalar data, we only select three scalar attributes (density, pressure and resistivity) from the entire attribute space to use. Every file size of the three scalar attributes (sampled at time step 900 seconds) is 1.05GB and each format is 32-bit floating point values with little endian.

5.2. Objectives

An initial framework prototype has been implemented using C++ language with two graphical packages: GLUT and OpenGL. We use the prototype to show the effectiveness of our hybrid framework. Due to the restriction of maximum paper length, here we only present some typical questions that may be asked by users:

1. If the MR data's certain features have been specified in certain feature spaces, what the other features will look like in the rest of feature spaces?
2. Where will the shared feature appear in the object space after we have specified certain features in the feature spaces?
3. For the same positions (in the object space) between two different low resolution approximations, what quantitative conclusions can we get?
4. For the different positions (in the object space) of the same low resolution approximation, what quantitative conclusion can we get?

5.3. Experimental results and discussion

Since the main purpose of our experiments is to demonstrate the effectiveness of our framework prototype, rather than how to handling large-scale data, we reduce the size of the three attributes's original data (each one with dimensions of 1024 x 512 x 512) twice with the MR techniques and utilize them as the initial data (each one with dimensions of 256 x 128 x 128) in order to obtain a quicker test results. Figure 3(a) illustrates these initial data, with each picture corresponding to an attribute. For the sake of clarity, in the rest of this section we only use the haar wavelet transformation as a specific

example to describe our experiments. However, the decimation method can be equally applied as well.

We apply two successive 3D haar wavelet transformation on the initial data of every attribute to obtain its MR hierarchy with 3 different resolutions, as shown in Figure 3(a), (b) and (c). Figure 3(b) illustrates the data after one pass of haar wavelet transformation and its dimensions are 128 x 64 x 64. In contrast to Figure 3(a), it is clear from Figure 3(b) that some data information has been lost i.e., at the bottom of the volumetric data some density attribute information and pressure attribute information are missing. But the overall impression of the renderings is fine. Figure 3(c) illustrates the data after two passes of haar wavelet transformation and its dimensions are 64 x 32 x 32. Compared to Figure 3(a), it is clear that the data in Figure 3(c) has lost a substantial of information i.e., the internal structures for both density attribute and pressure attribute have gone. Besides, the pictures in Figure 3(c) tend to be more blocky and blurring, compared to Figure 3(b). It indicates that the resolution of the data is coarser than the one in Figure 3(b).

Figure 5 shows the result corresponding to our objectives 1. The data we used here is after two passes of haar wavelet transformation and its dimensions are 64 x 32 x 32. The original features for density and its errors, pressure and its errors as well as resistivity and its errors are illustrated in Figure 5(a1), (a2) and (a3). Figure 5(b1), (b2) and (b3) present their features after we have specified a ROI in the density scatter plot. From Figure 5(b1) we can see clearly that the ROI meets the following two conditions: $0 \leq \text{density} \leq 9.9$ and $0 \leq \text{Dens. Error} \leq 24.0$. All values that fall into this range are highlighted in red color. Figure 5(b2) presents the corresponding feature for pressure attribute and its errors of which relevant density attribute and errors (shared the same grid points with them) have met the ROI conditions. In contrast to Figure 5(a2), it is clear that the dots in Figure 5(b2) have been reduced dramatically. It indicates that there are many grid points where the density and its errors do not meet the ROI conditions. Figure 5(b3) illustrates the corresponding feature for resistivity attribute and its error information of which relevant density attribute and errors meet the ROI conditions. Compared to the feature in Figure 5(a3), it is clear that the one in Figure 5(b3) follows a similar pattern. It implies that the resistivity attribute and its errors information on those grid points where the associated density and its errors meet the conditions is relatively complete. Figure 5(c1), (c2) and (c3) presents the corresponding features (highlight in green color) after we specified another ROI ($0 \leq \text{pressure} \leq 890.0$ and $285.25 \leq \text{Pres. Error} \leq 855.75$) in the pressure scatter plot. It is clear that the dots in Figure 5(c3) are less than the one in Figure 5(b3). It implies that there are less grid points where the density and pressure attributes as well as their errors meet both specified ROIs conditions.

Figure 6 (corresponds to objectives 2.) illustrates the

shared feature (meets both ROIs) in the feature localization view based on our previous feature specifications. It is clear from this figure that where these shared feature appears in the 3D space.

Figure 7(a) and (b) illustrate the results in regards to our objectives 3.. They are two successive low resolution approximations after one pass (128 x 64 x 64) and two passes (64 x 32 x 32) haar wavelet transformation from the initial data. We specify an identical ROI for both cases in order to compare the same subset of the numerical domain. Its starting coordinates on the 2D screen is (115, 79). And its width and height is 3 x 3 pixels. Although the same, the position of the ROI shown in Figure 7(b) looks differently from the one shown in Figure 7(a). This is due to the effect of haar wavelet transformation. For keeping the paper concise while proving the quantitative exploration and analysis characteristics of our framework, here we only discuss the density attribute and its errors from both cases. We believe users can draw their own conclusions for the rest of the attribute/errors based on this framework prototype.

By observing and comparing the four histograms that are related to the density attribute and its errors from Figure 7(a) and (b), we can get the following information: (1) after one more haar wavelet transformation from the data shown in Figure 7(a), it is clear from Figure 7(b) that the distribution range of the density attribute is reduced by 1.65, while the distribution range of its errors is increased from 16.0 to 56.0; (2) although in both figures the most density sampling points fall into the same interval: [6.6, 8.25], the number of them are different. The number from Figure 7(a) is 821.0, while the number from Figure 7(b) is 446.0; (3) in both figures, the most error sampling points also fall into the same interval: [0.0, 4.0]. The number shown in Figure 7(a) is 994.0, while the number shown in Figure 7(b) is 419.0; (4) the least density sampling points fall into different intervals in Figure 7(a) and (b). While the least density sampling points fall into interval [11.55, 13.2] in Figure 7(a), it falls into interval [24.75, 26.4] in Figure 7(b); (5) the least error sampling points fall into different intervals in both figures. In Figure 7(a) the interval is [12.0, 16.0]. In Figure 7(b) the intervals are [12.0, 16.0], [16.0, 20.0], [40.0, 44.0] and [48.0, 52.0], respectively; (6) other intervals vs. the number of sampling information are also easy to obtain from the histograms. In term of the relationship between the density and its errors, from the parallel coordinates in Figure 7(a) we can see that while the distribution range of density is broad from [6.6, 26.4] (consistent with the histograms), most errors are kept in the low values (less than 16.0). However, with on more haar wavelet transformation, we can see clearly from the parallel coordinates in Figure 7(b) that higher error values increase.

Figure 8 illustrates the results in regards to our objectives 4.. The dimensions of data we used here for the test are 128 x 64 x 64 (after one pass of haar wavelet transforma-

tion). The ROIs we selected for both figures have the same dimensions, with 2 pixels width and one pixel height. For keeping the paper short while proving the quantitative capability of our framework, we take the pressure attribute within the two ROIs as an example to analyze their difference. By observing and comparing the four histograms in Figure 8(a) and (b), we can obtain the following information: (1) in contrast to the pressure histogram shown in Figure 8(a), the one shown in Figure 8(b) has a wider range, which is from [0.0, 1223.75]; (2) the distribution range of pressure errors shown in Figure 8(b) (from 0.0 to 855.75) is slightly wider than the one shown in Figure 8(a) (from 0.0 to 285.25); (3) in both figures, the most pressure sampling points fall into the same interval: [0.0, 111.25]. While the number in Figure 8(a) is 345.0, the number in Figure 8(b) is 260.0; (4) the most pressure error sampling points in both figures fall into the same interval: [0.0, 285.25]. While the number in Figure 8(a) is 351.0, the number in Figure 8(b) is 383.0; (5) the least pressure sampling points in Figure 8(a) fall into the interval [111.25, 222.5], with a number that is less than 17.25; the least pressure sampling points in Figure 8(b) fall into the intervals [222.5, 333.75], [333.75, 445.0], [445.0, 556.25] and [667.5, 778.75], with a number less than 13.00; (6) there is only one number of pressure error sampling points in Figure 8(a), falling into the interval [0.0, 285.25]; the least pressure error sampling points in Figure 8(b) fall into the interval [570.5, 855.75], with a number less than 10.15. In terms of the relationship between the pressure attribute and its associated errors, it is clear from the parallel coordinates in Figure 8(a) that there is a strong relationship between the low-value range of the pressure attribute and the low-value range of its errors. Compared to it, it is clear from the parallel coordinates in Figure 8(b) that the values of the pressure errors have been slightly increased, and the values of pressure itself have been greatly increased. In addition, we can notice there is a cluster which follows a strong relationship between the middle-value range of the pressure attribute and the low-value range of its errors.

6. Conclusion and Future Work

In this article we have presented a framework to multivariate volumetric data and their uncertainties visualization using a federation of both information visualization and scientific visualization representations that, when used in combination, provide a powerful tool for interactive feature specification and quantitative exploration and analysis. We detailed every component of this framework and introduced the MR modeling approaches utilized to generate the error information. Compared to the traditional uncertainty visualization methods that heavily rely on the human perceptual system and often present diverse information into a single display, our framework is capable to quantitatively depict the complex uncertainty information while keeping a clearer data presentation. We also developed a framework prototype and ap-

plied it to the OpenGGCM simulation from the space science application domain to show its effectiveness.

We see three principle directions for our future research. First, the relationship displayed by the parallel coordinates component tend to be difficult to recognize when we selected a wider range of ROI with more sampling data. An improved version of the parallel coordinates with a good clustering method is required in order to display a clearer relationship. Second, a comprehensive user study is expected to perform in order to reveal the advantages and disadvantages of our framework. Third, we will explore more uncertainty visualization methods to visualize the uncertainty in multivariate volumetric data.

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