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Terrain Models for Mass Movement Erosion

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Abstract
In this paper we present a particle-based method for large scale long time progressive simulation of terrain erosion containing wet granular particles. The wetting process and the propagation through granular material is based on defining the wetness value for each particle representing the amount of water absorbed by granular particles and stored between them, as was originally proposed by Rungjiratananon [RSKN08]. We extend this model by adding a non homogeneous material to simulate differences between different types of soil-like granular material, based on physical constants like stability, plasticity and wetness. With this approach we can create a physical animation of erosion process like mass movement or mass wasting.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Computational Geometry and Object Modeling—Physically based modeling; I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism—Animation

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1. Introduction

The terrain erosion is important process in the nature. Granular materials like sand or soil and their behavior under the impact of erosion is indispensable part of modeling naturally looking terrain. For example, running water forms valleys and ridges, material deposition results in sandstones and wind creates wind ripples. One of the most important factors acting on granular materials is water. Influence of this element changes shape, morphology and properties of material with result in erosion [LS04].

In this paper we present particle-based simulation method of water influence when applied to granular materials resultant in erosion. Although there are many algorithms to simulate such behavior, majority of them is acting on the terrain surface. Our algorithm take into account also water saturated in material. In natural erosion, certain amounts of water are creating wetness and spreading among little gaps between granular particles. As a lot of water is absorbed by lower layers of soil, water factor can influence particles composition with the result in erosion.

We define soil system in this paper as large structure of granular rigid particles with values of stability, wetness and friction. Shape of particles is spherical, as in huge masses of particles, naturally behaving friction, between them, is inefficient [LH93]. For purpose of simulating differences between different types of soil in real world we created layers of soil material, where each layer represents one type of soil in the real world. Layers are bounded by forces acting on them and between them.

Figure 1: *Example of mass movement erosion in real world.*

Similarly to general erosion, mass movement can be also described as three-step erosion process [BF01]. In the first step the regolith or boundary between layers of soil is damaged by influence of gathered wetness between layers and other factors such as physical structure of soil layers. During

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this process, water interacts with terrain by bringing wetness to its structure. The time required to create such failure ranges from few weeks to few years. In the second step, corrupted mass of eroded material is transported by natural factors such as gravity, weight of wet soil and shape of stable, not moving terrain. Last step of erosion involves a deposition of eroded material. At this stage, deposition of material is considered in large scale.

For a simulation of soil material we use Discrete Element Method (DEM), which comprises different techniques suitable for simulating dynamic systems behavior with multiple rigid and separated bodies of various shapes. Continuous changes, computed in contact forces and applied in contact status, turn influence of the subsequent movement of particles [CS79]. Since positions of particles are changed by physical forces designed in the contact states of particles, topology of particle interaction evolves freely. As a result, highly dynamic simulations, such as avalanches and general erosion can be conveniently generated by this meshless approach without sacrificing physical accuracy [BYM05]. Smoothed Particle Hydrodynamics (SPH) [Chl10] is used to simulate water particles.

This paper is organized as follows. In Section 2, we present algorithms, which are most closely related to our work. This is followed by Section 3, where we describe our algorithm for soil simulation system and erosion and Section 4, where input terrain is described. Thereafter, in Section 5, we describe visualization method and optimization of given algorithm. Finally, in Section 6 we present mass movement simulation and results of our algorithm. The paper concludes with conclusions and future work.

2. Related Work

The simulation of terrain erosion was an interesting area of research in Computer Graphics for a long time. Similar to wide variety of erosion phenomena in reality, there are many erosion algorithm presented in Computer Graphics. The existing techniques range from slippage simulation [BYM05,LM93] to water [BTHB06,MKM89] and wind erosion [ON00].

2.1. Erosion Simulation Methods

One of first representations for terrains started with mathematician, Benoit Mandelbrot [Man82], father of fractal geometry, who introduced using fractals in terrain modeling. One of the first algorithms [MKM89] applied thermal and hydraulic erosion to erode fractal terrains. The terrain generation method includes arbitrary local control of fractal dimension and crossover scale. In the topic of generating differently shaped fractal mountains [PH93], authors used water as main factor in the simulation. Mountains and rivers were generated with combination of midpoint-displacement method and the squig-curve model of a non-branching river. Layered structure and its application in thermal weathering was introduced by Benes et al. [BF01]. The authors represented landscape as two dimensional array, where every element of the array consisted of one dimensional array with information about all underlying layers. Transport of material in the terrain erosion was described in [BTHB06], where erosion model uses also cohesive force between particles. Interactive simulation of erosion using water as main factor for creating changes on the surface of terrain was presented in [vBBK08]. They used multiple material layers and with gravity influence, slippage erosion effect was also simulated. Quite recently terrain erosion simulated with SPH was presented in [KBKv09]. Authors used Smoothed Particle Hydrodynamics to dissolve some amount of material from ground, transported due to water which created deposition of material on a different place. Fast GPU method for computing erosion was used in [MDH07]. Authors created large scaled terrain and realistic erosion effects by rainfall and river flows.

2.2. Granular Materials Simulation Methods

One of the first attempts to simulate granular material was introduced by Cundall [CS79], who described Discrete Element Method for simulating rocks mechanics, based on his earlier works [Cun71,Cun78]. Cundall defined granular material as rigid particles and proposed the contact method for resolving collisions of pair of particles. He used this idea to create 2D simulation of falling rocks. His work became essential for rigid bodies simulation. Because of shape and size of rocks, angular velocity was also computed and angular motion was described and visualised with 2D discs.

Granular material like sand was well described by many articles. Some of them used height field methods for better performance of simulation [LM93,SOH99,ON03] or handle the material as fluid [ZB05]. Although these methods are quite efficient, they are less accurate and difficult to use for more complex simulation system.

Idea of using DEMs for simulation of granular material was revisited by Bell et al. [BYM05], who also described different types of friction and created non spherical particles to demonstrate real friction force in the simulation. He also described different types of friction forces limited by threshold or realistic. For simulation of realistic static friction [LH93] used counter-acting frictional force. They also showed that piles generated by avalanches have finite angle of repose.

Wojtan et al. [WCMT07] simulated sand erosion, but fluids could not percolate into the sand volume. Falappi and Gallati [FG07] coupled granular and fluid phases using SPH. Recently wetness in inner structure of the sand material was introduced with DEM and SPH method. In the first work [RSKN08] authors used DEM for sand simulation and SPH for water simulation. They optimised their work on GPU and achieved detailed results. Aim of their work was primary to create new kind of sand behavior. They succeeded by creating wetness system transporting wetness values through sand material. This work is limited by number of particles in simulation.

In the second work [LD09], material and water were both simulated using unified SPH method. Granular volume was simulated using continuous approach sampled by particles. Authors gained great realism. However, their work include just one type of material, sand.

In the study of [DRM08, JYH06, JYL07, DYCB04] authors introduced approach to simulation of cohesive and non cohesive soil system. To simulate cohesion in the soil system they used DEM and bonding forces between particles. These works introduced more complex forces to numerical 2D simulation and accomplished research in the field of cohesive and non cohesive sands.

3. DEM for Soil

3.1. Discrete Element Method

At first step of our algorithm, we define *Discrete Element Method* for simulating granular material such as soil. Sand is one of soil types, with diameter larger than $0.02mm$. This is first type of material in our simulation. It is well described and simply modeled in particle-based simulations, thus it is starting material in simulation of erosion.

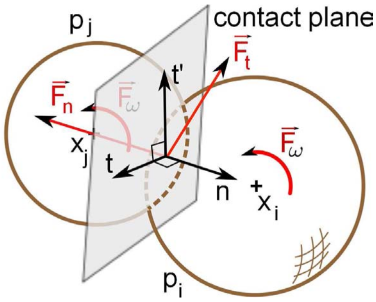


Figure 2: *Contact forces for DEM method.*

Contact forces between two colliding particles p_i and p_j, introduced by *Cundall* [CS79], are constructed acting in contact point between particles. In Figure 2, we can see basic setup scene for contact forces in *DEM*. At first we define overlap value of colliding particles and normal:

$$\xi = max(0, r_i + r_j - ||\overrightarrow{x_i} - \overrightarrow{x_j}||), \tag{1}$$

$$\overrightarrow{N} = \frac{\overrightarrow{x_i} - \overrightarrow{x_j}}{||\overrightarrow{x_i} - \overrightarrow{x_j}||}, \tag{2}$$

where $\overrightarrow{x_i}$ and $\overrightarrow{x_j}$ are positions of particles p_i, p_j and r_i and r_j are radiuses of these particles. Following equations describe normal force and its computation between particles p_i and p_j.

$$\overrightarrow{F_n} = \overrightarrow{F_s} + \overrightarrow{F_d}, \tag{3}$$

$$\overrightarrow{F_s} = k_s \xi \frac{\overrightarrow{N}}{||\overrightarrow{N}||}, \tag{4}$$

$$\overrightarrow{F_d} = k_d \overrightarrow{N}, \tag{5}$$

where $\overrightarrow{F_s}$, $\overrightarrow{F_d}$ is spring and damping force and k_s, k_d is spring and damping coefficient. Coefficient k_s is determined and k_d is selected same or smaller. After positive overlap, normal force is applied to accelerations of particles p_i and p_j. In summation through all accelerations of particle p_i, we compute its velocity using Newton's second motion law and determine its new position after one step in time Δt.

For simulating natural granular material we are using friction applied in tangent direction. Friction or tangential force causes negative contribution to summation in generating particle's acceleration. We implemented basic types of friction forces described by Bell et al. [BYM05]. We also implemented counter-acting friction force to simulate natural friction [LH93]. Unfortunately, to update this force, we need to hold list of old and new neighbors in system, which is very inefficient. Moreover, this force has minimal effect to more stable friction between particles. With this reasons, we decided to use following friction force

$$\overrightarrow{F_t} = -min(\mu f_n, k_t ||\overrightarrow{V_t}||) \frac{\overrightarrow{V_t}}{||\overrightarrow{V_t}||}, \tag{6}$$

where $\overrightarrow{V_t}$ is tangential velocity, which is tangent to the contact plane and perpendicular to the normal direction. The tangential velocity is defined using the relative velocity of the particles p_i and p_j at the contact point. Coefficient of friction k_t is limited by Coulomb law of friction, where $\mu$is friction coefficient and f_n is value of applied normal force $\overrightarrow{F_n}$. In Figure 3 we can see simulated sand with *DEM* in our algorithm.

3.2. Strength of Material

Of course, sand-like material is generally too simple for erosion of soil causing mass movement. We need material which will be acting like stable structure with range of strength simulating natural soil. Forces applied in *DEM* during sand simulation are limited. For simulating dry soil we

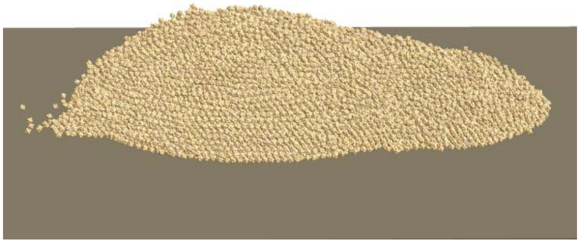


Figure 3: *Sand simulated with our algorithm. Dry feature of granular material.*

need to introduce another force to preserve strength of dry material constructed from rigid particles.

For that purpose we implemented bonds to our algorithm. Bond is relation between two colliding particles in case when they are in relax state or overlapping [JYL07]. This force is applied to basic definition of *DEM* in the meaning that *normal bond* is applied to *normal force* $\overrightarrow{F_n}$ etc. In Figure 4 we can see setup example of two disks representing two particles p_i and p_j.

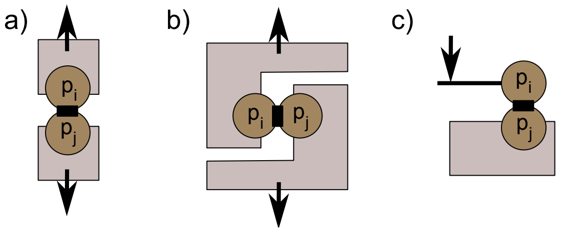


Figure 4: *Bonded motion of pair of particles. Black squares are representing bonds. Arrows are pushing particles away from each other in (a) normal, (b) tangential and (c) angular direction. Bonds interlock particles and keep them together.*

In example, for normal force, if $\overrightarrow{F_n}$ in (3) is resolving collision of particles by pushing them away from each other, than bonding force in normal direction (7) acts conversely. We can define normal bonding force as follows

$$\overrightarrow{B_n} = (R_n/\xi) * \overrightarrow{N}, \tag{7}$$

where R_n is normal bonding coefficient and ξ is overlap value between particles. Tangential and angular motion are locked in similar way using tangential and angular bonding coefficients R_t and R_ω. After considering this force, we need to update all equations in *DEM* contact model by subtraction of these bonding forces. Bonds are limited by spring and tangential forces acting in *DEM*. It means that failure of system is not allowed. In our algorithm, we are using diameter value of particles.

In summation from above equations we can see, that bonds create forces, which hold particles together in normal, tangential and angular direction, if they are overlapping. In the same time rigidness of granular particle in simulation is preserved.

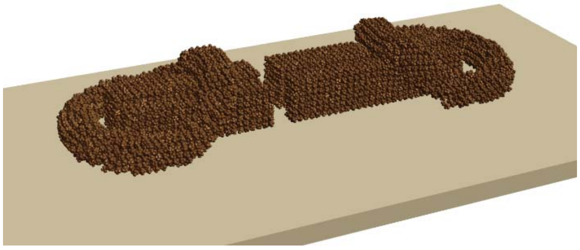


Figure 5: *Dry feature of soil-like material with defined normal, tangential and angular bonds.*

In case of sand, rotation of particles can be neglected, because particles can be easily separated during time of simulation. In case of such complex and complicated process as terrain erosion we must also consider rotations of bigger masses of soil constructed with certain amounts of particles. Thus in our algorithm rotation can not be ignored. To include angular motion of particle we compute angular acceleration of colliding particles. Then in integration of step of time Δt we determine particle's angular velocity. Rotation is applied to particle's body using rotation matrix, computed using quaternions. Then in next time step relative velocity $\overrightarrow{v_{p_i}}$ of particle is simply updated to consider also angular velocity

$$\overrightarrow{v_{p_i}} = \overrightarrow{v_{p_i}} + \omega_{p_i} \times (\overrightarrow{x} - \overrightarrow{p_i}), \tag{8}$$

where ω_{p_i} is angular velocity of particle p_i, $\overrightarrow{x}$ is position of contact and $\overrightarrow{p_i}$ is position of particle p_i.

3.3. Wetness

Water is main factor to cause mass movement. It is water absorbed in inner soil structure, which causes bigger weight of wetted soil and then after some time, failure state of system, when mass movement starts. With this condition we implemented wetting system to our granular soil-like material.

This part of algorithm is based on wetness system presented by Rungjiratananon et al. [RSKN08]. In our algorithm there are also three states of particles, dry, wet and overwet as we can see in Figure 6. Wetness is then percentage expression of each state, applied to particle. This percentage value represents water saporated in little gaps between particles.

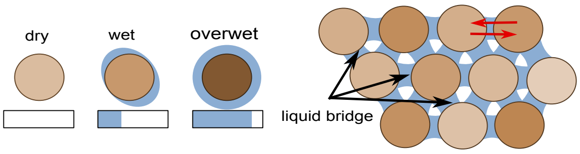


Figure 6: *States of wetness in particles (left). Liquid bridges (right).*

Figure 7: *Example of wetness system with different materials using our algorithm.*

Amounts of water between two particles represented by wetness, create attractive acting force, which can simulate cohesion of material. This force is acting between particles in case they are moving away from each other

$$\overrightarrow{F_i^{attract}} = max\left\{0, w_f - \frac{w_i + w_j}{2}\right\}(v_j - v_i), \qquad (9)$$

where w_f is fluidization coefficient. As we can see, wetness system has impact on *DEM* contact model. Contact forces are updated similar to approach in Rungjiratananon's article [RSKN08]. With this feature, system becomes more plastic during loading wetness to its structure. Wetness between particles of material is propagated through material and controlled by coefficient of propagation k_p. In the layer with bigger strength, propagation coefficient is smaller. Wetness is propagated to all contacts N_i of particle p_i as follows

$$w_i^{t+\Delta t} = w_i^t + k_p \frac{\Delta w_i^t}{N_i} \Delta t, \qquad (10)$$

$$\Delta w_i^t = w_i^t + w_t, \qquad (11)$$

where Δw_i^t is excessive wetness of particle p_i. As propagation speed of wetness is different in different layers of soil, excessive wetness is most visible on boundaries between layers of different material. These regions in materials are very hazardous because their behavior is water-like and they are creating chance for bigger mass of material to start a mass movement.

3.4. Summation of Algorithm

In Figure 8 we can see diagram of accumulation forces. *SPH*, *DEM*, *Bonds* and *Wetness* are here different methods used for computing forces between colliding particles.

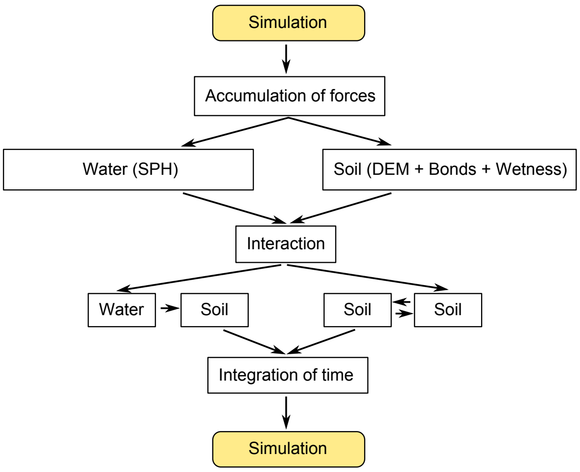


Figure 8: *Accumulation of forces between particles.*

4. Mass Movement

In Figure 9, we can see illustration of mass movement erosion, where surface of rupture is region of soil particles with most excessive wetness. This region is forced to behave like mud due to forces in overwetted particles. Wetness between layers creates slide. Then slump block is volume of soil above surface of rupture. This volume is transported during mass movement erosion and it's basic result from this kind of erosion. There are many other products of mass movement. Most closely related to change of shape of terrain is production of scarps. Mass movement can be very fast, or very depending on gradient, shape of terrain, amount of wetness, weight of wetted particles and also construction of initial layers.

Simulation starts with setup input scene constructed from particles of soil and water. Model of terrain is created in *Blender* and saved in *.obj* format. With our application *VolumetoParticles*, we scan input model and create representation of particles using 3D scanline with defined positions of particles of soil with density and contact radiuses. Layers of terrain are presented with different objects in model. With this approach we can easily create synthetic input scene representing terrain as we can see in Figure 10.

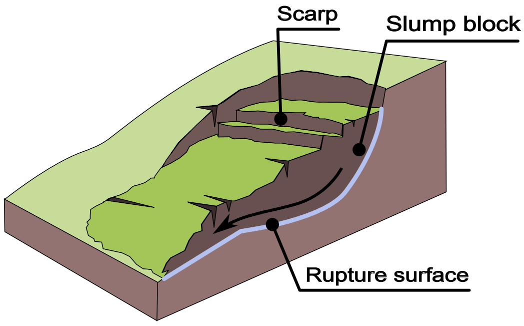


Figure 9: *Mass movement illustration.*

As initialization to our algorithm we setup starting scene. Then wetting of terrain can start. Particles of water simulate rain and bring required wetness to system of soil. Wetness is propagating through particles of soil to lower layers of material.

Figure 10: *Input terrain. Layers are orderd from top to bottom in the meaning of water propagation speed.*

Each layer has different properties of strength, density and speed of wetness propagation. With combination of different layers in scene we can create non homogeneous material. Even each particle can have different initial properties and also predefined wetness. With increasing wetness within particles, weight is also increasing. After certain time of simulation wetness gathered on boundaries of layers is creating a slide action. In this process forces between couple of particles are corrupted and they can slide over each other. This action is essential in our simulation of mass movement.

5. Visualization and Optimization

For visualization of results of our work we are using *OpenGL*. Without lose of resolution in simulation it is not possible to simulate this type of erosion in real time using *DEM* method. As differential equations solver, we use basic *Euler* algorithm.

Because of optimization we joined particles of water and soil to one programmable structure. They have different properties and they are simulated with different methods. With this feature we can compute contacts between particles more effectively. With assumption of using particles of water just to bring required wetness to soil system, we do not need to create surface of water and visualization. Physical correctness of particles of water is preserved.

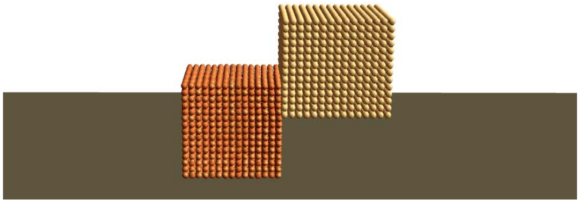
scene	particles	1 core	OpenMP	time step
sand	130k	0.2 fps	0.6 fps	0.001
wetness	20k	7 fps	13 fps	0.0005
input	50k	4 fps	5 fps	0.0002
hill	30k	2 fps	3 fps	0.0001
layers	60k	1.2 fps	3.4 fps	0.0002

Table 1: *Comparison of frames per second on different terrains without and with OpenMP.*

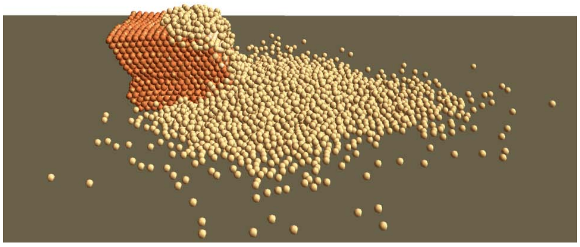
For optimization of performance we used *OpenMP* for simulation on more threads of *CPU*. As hardware for simulation we used Intel i7 950 CPU with 8 cores. In Table 1, we can see performance of our algorithm and comparison with OpenMP optimization.

6. Results

We simulated random input terrains to test our algorithm. Figure 11 shows difference between sand and soil in simulation and interaction.



(a) Before simulation.



(b) After simulation.

Figure 11: *Comparison of soil and sand interaction. Sand material (right) is falling towards soil material (left).*

In Figure 12 we can see wetting simulation performed on terrain with layers. In this terrain, there are 6 layers with different speed of wetness propagation. After close measurement, layer with red color is most resistant to wetness. In result, the layer, which is directly above most resistant one is layer with biggest amount of wetness between particles.

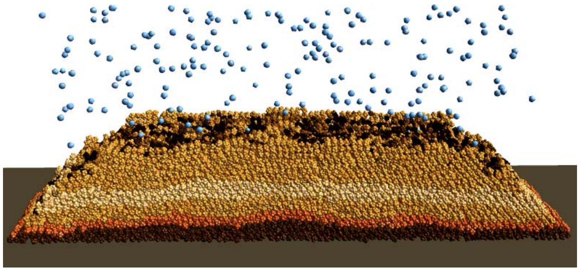


Figure 12: *Wetting system on set of layers.*

Figure 13 shows the mass movement simulation. Sample hill was constructed from three soil-like layers. Inspired by previous result, bottom layer is here the most resistant to wetness. As result red layer is capturing all incoming wetness and creates surface of rupture with result in movement.

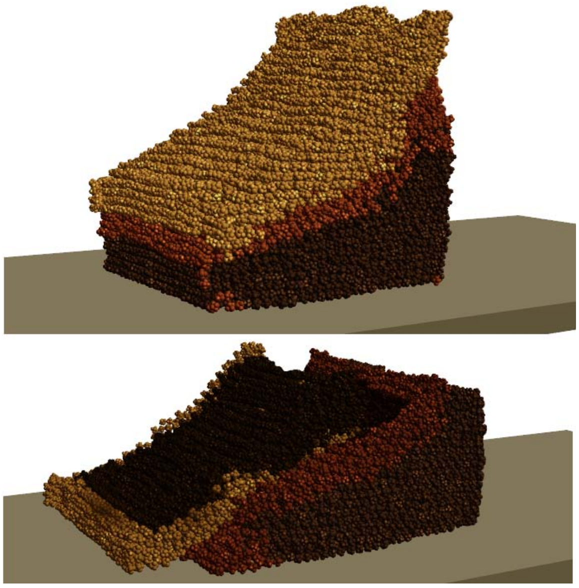


Figure 13: *Mass movement on sample hill with 3 layers.*

7. Conclusions and Future Work

In conclusion, we created particle based system for simulating soil particles and mass movement erosion. Non homogeneous material, layered data structure of soil, additional wetness, wetting and over wetting of material in our algorithm provide ideas for future work. With different layers of soil and interaction with water we are able to simulate formation of underlying structures such as caves and underlying water. With definition of high stability and strength of dry and also wet soil, it is possible to simulate drying of particles of soil. Although, described methods are not suitable for real time simulations, real time simulation of this processes is also our future goal.

8. Acknowledgment

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