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An Immersive Granular Material Visualization System with Haptic Feedback

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Abstract

Recent advances in distributed virtual reality systems and haptic technologies provide new opportunities to augment human sense of touch into highly complex and immersive visual simulations. Haptic feedback further enhances human immersion into virtual environments. We present techniques for portable and distributed rendering of real-time active and passive stereo views of virtual granular material that allow real-time haptic interaction and visualization. The main goal of the research was to develop highly portable and immersive virtual reality system that provides intuitive haptic interactions between a user and the granular material in several VR configurations. We developed real-time haptic enabled virtual sand rendering system for high-end VR systems, a FLEX (CAVE^{TM}-like display from Fakespace Systems), a high resolution tiled-wall display, comparatively low cost portable VR system that consists of two projectors with two horizontally and vertically polarizing filters, and the Disney projection screen. Our system has a distributed rendering architecture where a server computer is dedicated to haptic interaction force rendering and client computers are dedicated to a visual rendering of granular surface. Interconnection between the haptic and visual rendering programs is realized through a stream based TCP/IP communication. We report the architecture and implementation details of this highly portable and haptic force feedback enabled virtual reality system for interactive granular surface manipulation in this paper.

Categories and Subject Descriptors (according to ACM CCS): I.3.3 [Computer Graphics]: Line and Curve Generation

1. INTRODUCTION

Immersive systems and haptic interaction devices are becoming available to wider population of researchers that brings new areas of opportunity as well as new problems to solve. While the visual feedback requires the refresh frequency about 60Hz, the haptic feedback is necessary to be received on a frequency much higher (around 1kHz). The combination of both enhances significantly the immersive experience but poses high demand on the immersive systems with the haptic feedback. A multiprocessor solution is necessary and the communication among its components can present a challenging problem.

We present a solution to such a problem and demonstrate a variety of implementations of the immersive haptic interaction with granular surface – a sand-like material. The haptic feedback is calculated by an underlying physical model and the sand relocation, required for the correct visual feedback, is calculated by a visual model of the sand manipulation. An example of the cheapest solution based on anaglyphs is displayed in Figure 1. The fully stereo image (up) is displayed as a stereo pair of color anaglyphs.

We present the underlying mathematics and the code components that are need to implement solutions for this problem. We also provide a description of how the solution scales across multiple systems ranging from portable VR systems to a FLEX multi-display.

The rest of the paper starts with the introduction and previous work that has been done in the area of haptics, namely Section 2 describes the granular surface manipulation in Computer Graphics and the model used. Section 3 describes the visual and haptic model used to simulate the granular material interaction and visualization. The following Section 4 discusses variety of stereo displaying immersive systems. Implementation of the sand-like manipulation on the

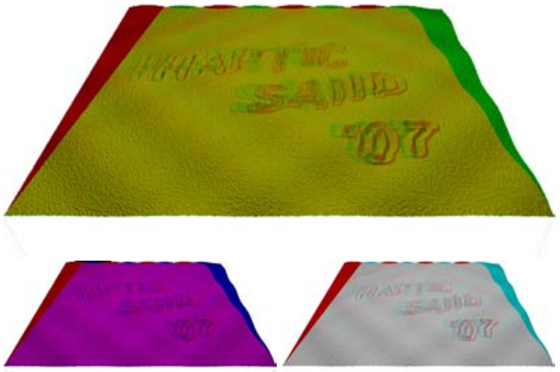

Figure 1: *A stereo image resulting from haptic interaction with the system (top) is displayed as an anaglyph pair of color images*

stereo displays is explained in Section 5 and the following section concludes the paper.

2. PREVIOUS WORK

2.1. Haptics

Haptic interaction, which involves both human tactile and kinesthetic perceptions, provides the user with means to physically interact by feeling touch and thus affect the virtual environment, instead of just passively viewing it [FTBS01]. However, realistic haptic interaction with complex virtual environment comes with high cost of both computation and hardware interfaces [FTBS01, KR05]. Development of commercially available high fidelity haptic devices such as the PHANToM family haptic devices by Sensable Technologies and the Delta and Omega haptic devices by Force Dimension has provided researchers and scientists much needed accuracy, stability, and speed for mainly desktop virtual reality applications. Haptic devices require high controller servo rates in order to achieve stability and provide realistic force feedback to the user.

For haptic force feedback application to communicate a realistic sense of touch, a haptic force refresh rate must be around 1kHz [Bur96, KR05]. This requirement for haptic rendering leaves a very little room for intensive computations required for force feedback from physically based granular surface simulation and its graphical rendering.

The fundamental challenge for this research lies in integrating physically based high quality haptic rendering into greatly complex and dynamic polygonal surface rendering. A survey of current haptic hardware and software interfaces indicated a lack of generic haptic interfaces for large-scale, high degree Haptic Immersion Virtual Environment (HIVE) [KR05].

2.2. Granular material simulation

Real-time manipulation of granular material has been one of the interesting and challenging topics in computer graphics. Over the years, many terrain modeling techniques that mainly focused on realistic visual representation of a terrain have been developed. However, not much work has been done in a real-time interactive modeling with force and tactile feedback. In this paper, we introduce an idea of manipulating a layer of sand with haptic device that is a direct extension of our previous work [BDAB06]

The seminal paper by [Mus89] is the basis for many of the following works. The granular material is described as a regular height field and is eroded by thermal and hydraulic erosion. The broken parts of the material fall down and pile up. Some amount of material is dissolved by running water and deposited elsewhere. The shape morphology is driven by a set of parameters that defines the material's resistance to the erosion forces.

[SOH99] described a technique for animating soil motion that results from interactions with virtual tools. They can animate material displacement that results from footsteps and trails in the sand and mud.

Wind ripples in sand were shown by [ON00]. The sand particles reallocation is formed by the two principal factors - creep and saltation. This motion is described by a set of equations that is applied to the granular material model represented in a 2D matrix of heights.

[BF01] introduced a layered data structure that allows for erosion modeling and representation of volumetric effects, such as caves, as well as for a surface erosion. They use a compressed data structure that stores layers of material. This volumetric technique has comparable speed of erosion simulation as in the case of height-fields and is provides better performance than voxel-based approaches.

Zhu and Birdson [ZB05] recently presented application of the Navier-Stokes equations to simulation of sand motion. Sand is represented as a cloud of particles and its motion is described by the physically correct equations. Special attention is paid to the surface tracking and rendering.

A full solution of Navier-Stokes equation was used for erosion simulation in [BTHB06]. Running water is calculated as the full 3D physics simulation and the solution is coupled with differential equations describing the erosion and the deposition process. The results are rendered using an ad hoc photon mapper that results in photorealistic animations of receding waterfalls, meander breaks, river bed erosions, etc.

[LM93] described a real-time digging and caving. Their algorithm works with extended regular height-fields and describes the surface changes by means of these tools.

The Virtual Sandbox of [ON03] describes real-time manipulation of sand. The dual sand representation is used. The granular material is described as a height field and as a set of particles. Particles are used for falling sand and the height-field deposits sand.

[Nei05] describe terrain shape morphology that deals with homogenous sand-like structures. This is exposed to rain that erodes the underlying terrain.

As previously mentioned, not much has been done in the haptic feedback for sand simulation and visualization. This paper is a direct extension of our previous work [BDAB06]. The principal difference is the parallelization of the tasks and its visualization by means of different immersive systems.

3. GRANULAR MATERIAL MANIPULATION

The granular material manipulation and the haptic feedback calculation is described in this part. We follow our previously published paper [BDAB06] and refer reader for details there.

Granular material is represented as a regular height field; a 2D matrix of elevation values. The matrix resolution defines the level of detail and significantly influences the system performance and quality. The tools that changes the sand position is a sphere that is manipulated directly by the 3D haptics input device. Once there is a collision of the sphere with the sand layer the underlying physics-based model calculates the corresponding feedback that is sent back to the haptic device. The corresponding amount of sand is removed and, based on the gravity and the internal friction, it slowly falls down.

The sand displacement is calculated as a two pass algorithm. In the first pass some amount of sand is taken out, in the second one it is deposited to the neighboring locations. The corresponding amount is calculated from the penetration depth. As the sphere is dragged in the sand the dragging direction is calculated and the sand is deposited only to the corresponding neighboring locations (see Figure 2).

When the tools leaves the area sand falls down and tends to equilibrate the holes and peaks. This process is slowed down by the inner friction of sand defining so called talus angle of the sand. The deposition process is simulated by successive comparison of the local sand gradient and removing corresponding amounts of sand from the higher locations to the lower ones. Once the talus angle is reached the deposition stops.

The haptic feedback is calculated from the depth of the immersion of the tools into the sand and from the neighboring vertices of the height field that are directly influenced.

The repulse force is a resultant of the tension of sand that depends on the depth. This force has the opposite direction to the normal vector to the surface, close to the surface, and becomes vertical with increasing depth. It is expressed as

$$\vec{F_r} = -k_s e^{k_c d} - 1 \qquad (1)$$

where k_s, k_c are the material dependent constants and d is the depth of the layer of sand. This force has always vertical, or nearly vertical direction.

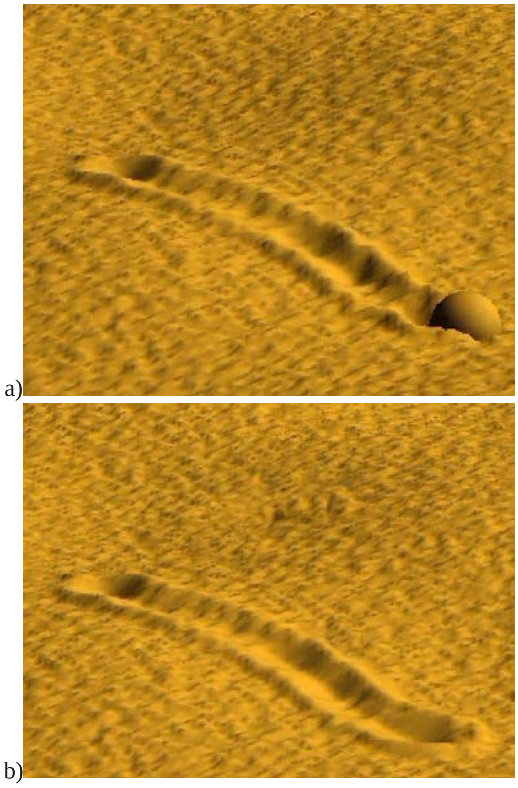

Figure 2: *The stroke in the sand (a) is faded out by erosion and deposition in (b)*

The second force is the inner *friction* of the sand that has only the vertical component. It is also a function of depth but depends on the velocity of the tools:

$$\vec{F_f} = -k\vec{F}|\vec{F_r}|. \qquad (2)$$

The force $\vec{F}$ depends on the friction area, the velocity $\vec{v}$, and the size of the repulse force $\vec{F_r}$. The friction area is constant for the sphere so $\vec{F} = k_a\vec{v}$.

The three constants depend on the relative sizes of the objects in the scene. Setting the total sand area size to $\langle -1, 1\rangle^2$ and the sphere radius to 10^{-2}, the $k_s = 10^3$, $k_c = 1/2$ and $k_f = 10$.

4. STEREOSCOPIC VIEWING

There are many visual cues that contribute to the human perception of depth. The primary depth cues (also called binocular depth cues) are binocular disparity, convergence, and accommodation. Secondary depth cues (also called monocular depth cues) are linear perspective, relative size, texture, shading and shadow, relative motion, reference frames, and aerial perspective [HWSB99]. However, stereopsis is considered the most dominant depth cue for providing the viewer 3D information of the objects in the scene.

In general, a number of techniques such as anaglyph (see Figure 1), active stereo and passive polarized stereo can be used to create stereoscopic view of an image that provides perception of depth in virtual reality applications. Anaglyph stereo can come at cheap but losses significant to severe amount of color information due to two color filtering glasses. On the other hand, active stereo can generate a high quality stereo image using expensive shutter glasses, left/right eye image synchronizer, and CAVE systems. Although these high-end VR systems are capable of creating a highly immersive virtual environments for multiple users, they are extremely expensive and many cases have no or limited portability.

A relatively low cost active stereo can be achieved by fish tank stereo VR system that can be created using a single screen multiplexing CRT display and graphics card with high refresh rates of at least 120Hz [Cli04]. But, this configuration is limited to a single user and suffers from poor user immersion to the environment. However, reasonably high quality stereo viewing can be achieved with passive polarized stereo at considerably low cost.

4.1. Passive polarized stereo rendering

Passive polarized stereo provides high-quality stereo image and is suitable for multiple users. This configuration requires a pair of LCD or DLP projectors with polarizing filters, polarization preserving projection screen (silver screen), and polarizing glasses for a viewer. To create stereo view of a scene, images for both left and right need to be rendered separately. In computer graphics, this can be achieved by rendering the scene from left and right eye positions.

All stereoscopic viewing systems have to independently present a left and right eye image to the corresponding eye. In passive polarized stereo, this is achieved using linear or circular polarizations techniques. A light wave is polarized in a single direction for the linear polarization and polarized in rotating multiple directions for the circular polarization. The matching polarizing glasses for left and right eyes must be used to see the stereo image generated by those polarizing filters. In our design, we used a simple but an effective linear polarization technique.

4.2. Rendering left and right eye images

The simplest way of rendering left and right eye images is to use two virtual cameras with symmetric camera frusta that are pointed at the same focal point in 3D. Unfortunately, this approach introduces erroneous stereo view with vertical parallax. The vertical parallax can be eliminated by setting two camera views parallel aiming at separate focal points. However, this approach introduces another type of erroneous stereo view with horizontal parallax. The correct stereo viewing requires two parallel camera views with asymmetric camera frusta [OSW*05].

We modified open source CHAI 3D library [CBB*03] for computer haptics, visualization and interactive real-time simulation to render the left and right eye images into a single display buffer using asymmetric camera frusta for each camera. Since dual head graphics cards can be set to render two separate screens from a single display buffer, left and right eye images can be drawn into a single buffer side by side to be show on two separate projectors. In our case, dual head ATI MOBILITY RADEON 9600/9700 graphics card and Casio XJ-450 DLP projector with 2800 ANSI lumens and native resolution of 1024x768 pixels were used.

5. IMPLEMENTATION

Our TCP/IP stream based distributed haptics and graphics rendering architecture for granular surface is based on Portable Haptic Display (PHD) concept introduced in [DCDB06]. The basic idea of PHD is to separate high frequency haptic rendering loop (1kHz) from low frequency graphics rendering loop (60Hz) and distribute those computations to client and server applications with shared virtual environment (see Figure 3).

The server application is dedicated to granular surface interaction force rendering at about 1kHz and the client application is dedicated to rendering stereo view of the granular sand surface and its fast erosion in three different virtual environment settings: FLEX, tiled-wall, and portable VR display. To keep the network communication overhead low between client and server we only synchronize haptic device probe position between haptic and graphics rendering applications.

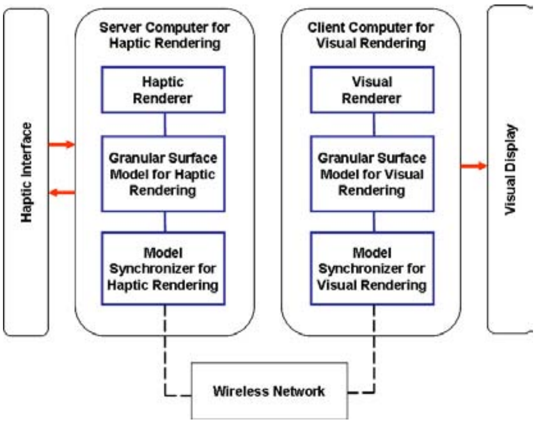

Figure 3: *Hardware and software architectures of the Portable Haptic Display*

The same Windows based haptic sand rendering server application was developed based on CHAI 3D [CBB*03], an open source C++ libraries for computer haptics, visualization and interactive real-time simulation. The CHAI 3D library provides generic interface for several commonly used commercially available haptic devices such as Phantom, Omega, and Delta haptics devices. This device independence provided by CHAI 3D significantly increases portability of our distributed haptic sand rendering system.

The server runs three threads for the following fundamental processes:

- Contact point based haptic force feedback computation and material displacement,
- continues erosion following material displacement, and
- device position synchronization with a client node.

The first two processes share the same sand surface model (height-field data) to synchronize surface change. Our granular material involves $1/4M$ vertices and we are drawing about 125000 quadrilaterals in each graphics rendering cycle. The entire model must be upgraded and displayed in real time and the haptic feedback must be generated.

Dedicated threads in haptic rendering server and graphics rendering client applications were created for TCP/IP socket stream communication to deliver fast network connection between the two. In the following sections, we provide detailed description of client sand rendering application implementations for each virtual environment.

5.1. Sand in FLEX VR system

Our FLEX virtual reality system consists of cluster of four Linux HP PCs with genlocked graphics cards one of which is a head node and other three nodes are slave nodes. Each node has Dual Core AMD Opteron Processor with 2.6GHz speed and 1MB of cache, 2GB RAM, and NVIDIA Quadro FX4500 graphics card with 512MB memory. All four nodes are involved in rendering and are connected with 1 Gigabit Ethernet connection. We developed the sand rendering client application for Linux cluster using the VRJuggler [BJH*01], open source community-oriented virtual reality application development framework to integrate haptic force feedback enabled granular surface manipulation with a highly immersive active stereo rendering in FLEX (see the Figure 3) and high resolution polarized passive stereo rendering in tiled-wall. The only sand rendering client running on master node of the cluster communicates with the haptic rendering server that connected to the haptic device through TCP/IP stream socket connection from its dedicated network communication thread to receive haptic device proxy point position and synchronizes the received data with other slave nodes. This data synchronization is based on common shared data structure that is implemented using VRJuggler [BJH*01] application data manager in clustered environment. The sand rendering clients running on the slave nodes have only possessed read access to the shared data while the client running on the master node had both read and write access to the shared data (see Figure 4).

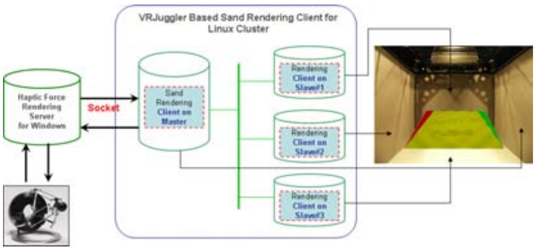

Figure 4: *Haptic drives the application and its output is displayed in the immersive system*

To match fast erosion that is happening on the server high priority dedicated erosion thread was created in all clients running on all nodes in the cluster (including master node). Once device proxy position synchronizes, an intensive sand erosion process happens on each node in parallel.

5.2. Sand in High Resolution Tiled-wall Display system

Our high resolution tiled wall system consists of twelve (3x4) 1024x768 resolution displays. For each display two projectors are utilized to render left and right eye images respectively. A cluster of 13 nodes (12 rendering nodes and a master node) that has 2.4GHz speed and 1MB of cache, 2Gb RAM, and NVIDIA Quadro FX4500 graphics card with 512MB memory is driving the tiled wall. The exact same principle of cluster rendering that used for sand in FLEX was used for the sand simulation in tiled wall environment. In this environment our sand client is running on all 13 nodes and only the client running on the master node communicated with the server running on Dell precision M90 notebook computer with 2.00GHz Intel Core 2 Duo Processor and NVIDIA Quadro FX 1500M with 256MB memory through TCP/IP socket and synchronizing haptic device position with other 12 nodes. Hence the architecture of haptic sand simulation is the same as the sand simulation architecture in FLEX except it has more client nodes involved.

5.3. Sand on Portable VR display

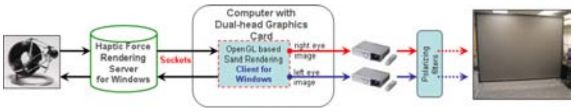

Figure 5: *Haptic drives the application and its output is displayed in the simple system*

The client granular surface rendering application creates a full dual screen window of size 2048x768 pixels (two times the normal screen) to draw both left and right eye images into a single rendering cycle (see Figure 5). The main idea is to draw left and right images to a single back buffer for two continuous displays using two separate viewports with asymmetric frustum that one is pointing to the first half and the other is pointing to the second half of the full dual screen window buffer, and then to swap the back buffer with the front buffer in a single rendering cycle. Since both left and right eye images are rendered in a single buffer and in a single cycle it does not require a high frequency refresh rate for stereoscopic viewing.

The OpenGL `glFrustum()` function is altered to create of asymmetric frusta for left and right eye perspective projections. This can be done by altering near clipping plane of the left and right eye projection frusta as follows (see also Figure 6):

$$glFrustum(l+d, r+d, bot, top, near, far)$$

The displacement d will be positive for the left eye projection to move the near clipping plane to the right and will be negative for the right eye projection to move the near clipping plane to the left.

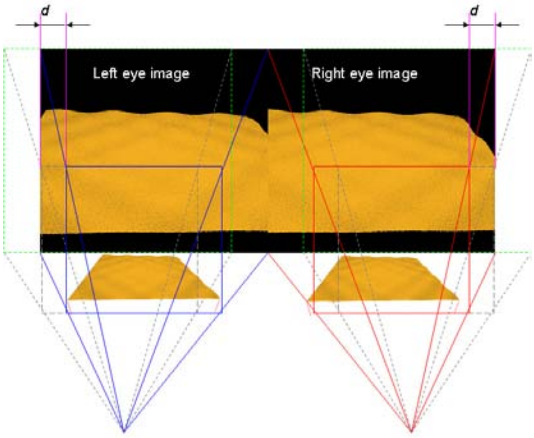

Figure 6: *Passive stereo rendering for dual head graphics system*

5.4. Performance measurements

Special test message that has the same length as the actual device position message sent from the server to the client at random rate to accurately measure TCP/IP packet round trip time (RTT) in our client-server TCP/IP communication. Using processor high resolution counter the server starts timer when the test message is sent and stops timer when the client acknowledges to that test message. In the following table, we showed average packet travel time between server and client (half of the RTT) in both wired and wireless network.

The measurements indicate that four node FLEX cluster performs significantly better than tiled-wall and portable VR system in wired LAN connection. However, in wireless LAN connection the FLEX cluster performs almost the same as 13 node tiled-wall cluster. This is reasonable since synchronization of increasing number of nodes take more time compared to synchronization less number of nodes. The performance of the portable VR system that involved peer-to-peer (P2P) connection between two notebook computers was significantly worse than the two clustered systems. Overall, the TCP/IP stream based client-server application performed poorly in wireless connections (both wireless LAN and wireless P2P) due to repeated packet loss common to any wireless networking.

	FLEX		Tiled-wall		Portable VR	
[ms]	μ	σ	μ	σ	μ	σ
Wired	9.7	5.6	13.6	9.8	16.2	9.8
Wireless	16.0	12.3	16.2	14.0	24.9	14.8

6. CONCLUSIONS AND FUTURE WORK

According to our distributed architecture of integrating high quality force feedback into immersive computationally intensive sand simulation running on clustered nodes through TCP/IP communication is doable but limited to the number of nodes involved in the computation and their individual capacities. The main downside of this architecture is nonparallel computations of cluster nodes due to the network synchronization latency bottleneck among the nodes. In our future work, we plan on improving the immersive haptic sand simulation in several different ways. First, we would like to move some part of the intensive and parallel computations such as erosion into GPU to give more CPU cycles to high frequency haptic force computation. Second, we would like to use high bandwidth InfiniBand based cluster for rendering and develop MPI based synchronization among cluster nodes.

In this paper, we introduced a low cost, portable VR system configuration for interactive granular surface manipulation with haptic force feedback. We have introduced an innovative approach that provides users with access to a new realm of interaction that was difficult to achieve before. The paper discussed one approach that addresses the fundamental challenge in integrating physically based high quality haptic rendering into greatly complex and dynamic polygonal surface rendering. Furthermore, we discussed algorithms and TCP/IP based communication components for a variety of systems ranging from a portable VR system to a FLEX multi-display. The approach and modality of interactions presented in this paper provide a new branch of applications that allow users to exploit such interactions at smaller scales such as the nanoscale. One of the key problems that is currently being addressed in the realm of nanoscale science, engineering, and technology is the "touch-and-feel" of the nano landscape. While Atomic Force Microscopes and Scanning Tunneling Microscopes provide glimpses of the landscape that are exciting, the realm of haptic interactions at that scale level are not obvious. Next steps for the work presented here could lead to algorithms that allow researchers to perceive phenomena that occurs at the nanoscale. One of the key problems with perception of nanoscale is the time lengths at which the interactions occur. This work provides avenues for users to slow down the time scales to proportions that can be perceived. The inclusion of haptics has major implications for research, as well as for learning. We have provided nanoscale science, engineering, and technology as a possible application domain. Clearly, there are other domains that could benefit from this approach.

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References

[BDAB06] Beneš B., Dorjgotov E., Arns L., Bertoline G.: Granular material interactive manipulation: Touching sand with haptic feedback. In *Proceedings of the 14-th International Conference in Central Europe on Computer Graphics, Visualization and Computer Vision 2006* (2006), pp. 295–304.

[BF01] Beneš B., Forsbach R.: Layered Data Structure for Visual Simulation of Terrain Erosion. In *SCCG '01: Proceedings of the 17th Spring conference on Computer graphics* (2001), vol. 25(4), IEEE Computer Society, pp. 80–86.

[BJH*01] Bierbaum A., Just C., Hartling P., Meinert K., Baker A., Cruz-Neira C.: Vr juggler: A virtual platform for virtual reality application development. In *VR '01: Proceedings of the Virtual Reality 2001 Conference (VR'01)* (Washington, DC, USA, 2001), IEEE Computer Society, p. 89.

[BTHB06] Beneš B., Těšínský V., Hornyš J., Bhatia S. K.: Hydraulic erosion: Research articles. *Computer Animation and Virtual Worlds 17*, 2 (2006), 99–108.

[Bur96] Burdea G. C.: *Force and touch feedback for virtual reality*. John Wiley & Sons, Inc., New York, NY, USA, 1996.

[CBB*03] Conti F., Barbagli F., Balaniuk R., Halg M., Lu C., Morris D.: The CHAI libraries. In *Proceedings of Eurohaptics 2003* (Dublin, IE, July 6–9 2003), pp. 496–500.

[Cli04] Cliburn D. C.: Virtual reality for small colleges. *J. Comput. Small Coll. 19*, 4 (2004), 28–38.

[DCDB06] Dorjgotov E., Choi S., Dunlop S. R., Bertoline G.: Portable haptic display for large immersive virtual environments. In *HAPTICS '06: Proceedings of the Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems (HAPTICS'06)* (Washington, DC, USA, 2006), IEEE Computer Society, p. 48.

[FTBS01] Frank A. O., Twombly I. A., Barth T. J., Smith J. D.: Finite element methods for real-time haptic feedback of soft-tissue models in virtual reality simulators. In *VR '01: Proceedings of the Virtual Reality 2001 Conference (VR'01)* (Washington, DC, USA, 2001), IEEE Computer Society, p. 257.

[HWSB99] Hubona G. S., Wheeler P. N., Shirah G. W., Brandt M.: The relative contributions of stereo, lighting, and background scenes in promoting 3d depth visualization. *ACM Trans. Comput.-Hum. Interact. 6*, 3 (1999), 214–242.

[KR05] Kunzler U., Runde C.: Kinesthetic haptics integration into large-scale virtual environments. In *WHC '05: Proceedings of the First Joint Eurohaptics Conference and Symposium on Haptic Interfaces for Virtual Environment and Teleoperator Systems* (Washington, DC, USA, 2005), IEEE Computer Society, pp. 551–556.

[LM93] Li X., Moshell M.: Modeling Soil: Realtime Dynamic Models for Soil Slippage and Manipulation. In *Proceedings of SIGGRAPH'93* (1993), vol. 27(4) of *Annual Conference Series*, pp. 361–368.

[Mus89] Musgrave, F.K. and Kolb, C.E. and Mace, R.S: The Synthesis and Rendering of Eroded Fractal Terrains. In *Proceedings of Siggraph'89* (1989), vol. 23(3) of *Annual Conference Series*, pp. 44–50.

[Nei05] Neidhold, B. and Wacker, M. and Deussen, O.: Interactive physically based Fluid and Erosion Simulation. In *Proceedings of Eurographics Workshop on Natural Phenomena* (2005), vol. 1, pp. 25–32.

[ON00] Onoue K., Nishita T.: A Method for Modeling and Rendering Dunes with Wind-ripples. In *Proceedings of Pacific Graphics'00* (2000), pp. 427–428.

[ON03] Onoue K., Nishita T.: Virtual sandbox. In *Proceedings of Pacific Graphics'03* (2003), IEEE Computer Society, pp. 252–260.

[OSW*05] OpenGL ARB, Shreiner D., Woo M., Neider J., Davis T.: *OpenGL(R) Programming Guide: The Official Guide to Learning OpenGL(R), Version 2 (5th Edition)*. Addison-Wesley Publishing Company, 2005.

[SOH99] Sumner R. W., O'Brien J. F., Hodgins J. K.: Animating Sand, Mud, and Snow. *Computer Graphics Forum 18*, 1 (1999), 17–26.

[ZB05] Zhu Y., Bridson R.: Animating sand as a fluid. *ACM Trans. Graph. 24*, 3 (2005), 965–972.