

Sub-part correspondence using structure and geometry

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Abstract

The paper describes a method for recognizing similar sub-parts of objects described by 3D polygonal meshes. The innovation of our method is the coupling of structure and geometry in the matching process. First of all, the structure of the shape is coded in a graph where each node is associated to a sub-part of the shape. Then, the matching between two shapes is approached using a graph-matching technique relying upon a geometric description of each sub-part.

Categories and Subject Descriptors (according to ACM CCS): I.3.5 [Computer Graphics]: Object Modelling

1. Introduction

Assessing the similarity among 3D shapes is a very complex and challenging research topic. Whilst there are already techniques for rapidly extracting knowledge from massive volumes of texts, there is an increasing demand for tools supporting the automatic search for 3D objects and their sub-parts in digital archives.

There is a growing consensus that shapes are recognized and coded mentally in terms of relevant parts, or features, and their spatial configuration, or structure. Methods approaching the problem from a geometric point of view do not take into account the structure of the shape and generally the similarity distance between two objects depends on their spatial embedding.

The work herein presented is based on results of differential topology, which deals with the description of shape by means of shape properties of one, or more, real-valued functions defined over the shape. Studying these properties, topological descriptions of the shape can be defined, namely the Reeb graphs, which induce a decomposition of the shape into significant regions. Such a decomposition defines a structural description of the shape, which is coupled with an error-correcting subgraph isomorphism to provide a system for shape similarity analysis. Moreover, the proposed framework makes it possible to plug in heuristics for tuning the matching algorithm to the specific application, in partic-

ular the 3D shape sub-parts correspondence and the partial matching.

Aim of this paper is to describe a method for recognizing the sub-parts of two objects the most similar both in geometry and structure and to discuss the feasibility of the proposed approach. Main innovation of the method is the coupling of a structural descriptor, like the Reeb graph proposed in [Bia04], with a geometric descriptor and the inexact graph-matching techniques.

The reminder of the paper is organized as follows. First, a brief overview on existing techniques for shape retrieval and partial matching is given. Then, the description of our approach is proposed; since solving the complete graph matching is computationally expensive, a new heuristic method which speeds up the process is proposed. Finally, results are presented and discussed. Conclusive remarks and suggestions on future work end the paper.

2. Previous work

Concerning 3D shapes, there is a great number of techniques for shape matching. Many methods for 3D object comparison return as output a positive real number which measures how much an object resembles to another one [OFCD01, NK01, CRC*02, VT03, KFR03]. Since no information on the sub-part correspondence of the compared objects is stored, these approaches are not available for partial

matching and mapping of objects. On the contrary, such an information is essential in applications like object modeling, registration and recognition.

The use of spin-images [JH97] for classifying sub-parts of 3D objects has been recently proposed in [HKDH04] and [RCSM03]. Both methods represent the 3D objects as a set of parts, which are compared to obtain an object sub-part mapping. Other approaches like those described in [FGN89, FKS*04] try to match global shapes with a special emphasis on selected sub-parts. For example, in [FGN89] the surface of an object is described by segmenting it into patches; the complete surface description separately represents each patch and their interrelationships. Complex surfaces are segmented into simpler meaningful components (the patches) through shape discontinuities, such as jump boundaries, limbs and creases. Therefore, such a description can be viewed as an attributed graph whose nodes correspond to the surface patches and the edges codify the relations between them.

Finally, the methods proposed in [BSRS04] represent the shape object as binary tree obtained by recursively subdividing the object into two parts. The similarity measure between two objects is obtained by matching the two trees, where the sub-part correspondence is induced by the node mapping provided by the matching algorithm.

All these methods provide only one and arbitrary description of the object shape, while the framework proposed in this paper is based on the consolidated Reeb graph theory that allows the use of different functions to analyze the shape, each one able to identify different relevant sub-parts of the object. In particular, methods based on the spin-images provide a point-to-point correspondence between the object surfaces and do not store any kind of structural and topological information. On the contrary, the structural information represented in [FGN89, BSRS04, FKS*04] either does not guarantee the identification of the most meaningful sub-parts or is not able to modularly incorporate different heuristics capable of adapting the matching to specific application contexts (e.g. global matching, sub-part correspondence, partial matching) or is not fully automatic [FKS*04].

The use of structural descriptions for shape similarity has been firstly addressed in [HSKK01], where the Reeb graph is proposed in a multi-resolution fashion to build a graph and perform shape similarity by means of graph matching techniques. Similar criteria have been successively used in [BRS03] and further enriched by [TS05], where for each slice the geometric attributes considered are the volume, a statistic measure of the extent and the orientation of the triangles, an estimation of the Koenderink shape index and a statistic of the texture. Other geometric descriptors have been proposed for associating to the nodes of a skeletal graph the description of the related model sub-parts. In particular, methods based on the medial axis [Blu67] like [SSGD03, ZSM*05, IJL*05] code in a vector the *relevance*

of the skeletal edges incident in a node (e.g. edge length, diameters and average circumference of the skeleton loops) or use geometric descriptors, like the mean curvature histogram [ZSM*05].

3. Sub-Part Correspondence

In our idea, the partial matching problem may be grouped in three main issues:

1. recognizing similar sub-parts in objects that are both structurally and geometrically similar (that is, having similar overall shape);
2. recognizing analogous sub-parts in objects having different overall shape;
3. distinguishing if an object shape is itself a sub-part of another.

In particular, we observe that the third problem is a particular case of the second one. In the first case similar sub-parts of the two objects should be automatically recognized and mapped. The method proposed in this paper is able to approach all facets of the sub-part shape correspondence problem with major emphasis on the first one (due to the properties of the structural descriptor we have adopted).

The second kind of correspondence deals with objects having different overall shape but similar sub-parts. The partial correspondence should recognize similar sub-parts of the two objects and produce the correspondent mapping.

3.1. Shape descriptors

Our implementation of the matching process involves both structure and geometry of the two objects. At the moment, the structural information of the object is captured by the Reeb graph computed with respect to a position invariant function [BMM*03], see figure 1.a). The functions used to validate the matching framework proposed in this paper are the distance from the center of mass of the object and the integral geodesic distance in [HSKK01]. As shown in [Bia04], the shape characterization and the Reeb graph construction naturally induce a decomposition of the shape in topologically significant regions, see figure 1.(a). The region decomposition obtained from our contouring approach does not admit slices with internal holes and each border component of the surface patches is shared by only two distinct patches. Since each border component is completely shared by two patches, cutting and pasting operations along such a contour may be performed independently from other contours. Moreover, this segmentation produces a directed graph [BMM*03], in which each node corresponds to an object patch and each edge connects two nodes, see figure 1.(b).

characterization, see figure 1.c). The value of f and a geometric descriptor are associated to each node in the simplified ERG. In our approach we move from a local description

of the surface slice to a more general representation of the model sub-parts, based on the assumption that the larger the model portion associated to a node is, the more relevant the node should be.

Since our graph is directed, each node identifies a sub-graph and the geometric attribute associated to the node is obtained from the surface related to its subgraph, see figure 1.d). For leaf nodes, whose sub-graph is empty, we consider only the slice of the shape that correspond to them. Once sub-parts have been associated to each node, we use the spherical harmonic analysis of the sub-part to describe its geometry. Spherical harmonic analysis has been defined in [KFR03] and this descriptor is rotation invariant. In [KFR03] the scale invariance is obtained by uniformly scaling a model in a cube whose edge length is two; therefore, in our case each sub-part is separately scaled.

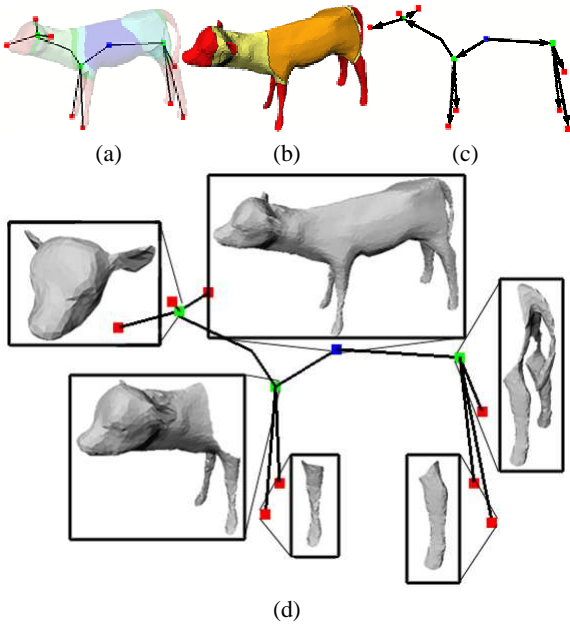


Figure 1: A Reeb graph of a calf a), the surface segmentation associated b) and the oriented graph c). In d) the surface portions associated to some nodes are highlighted; these regions contain all patches associated to the subgraph nodes.

3.2. Graph matching

The sub-part shape correspondence between two objects is obtained by matching the directed attributed graphs. Inexact graph matching has been topic of research since many years and several techniques are available [MB98, CFSV04]: recently, the framework proposed in [MSF05] formalizes the enumeration of all common subgraphs of two graphs in a way that makes it straightforward usable for plugging heuristics in it and, according to the specific case, achieves different approximations of the optimal solution. The matching

algorithm proposed in this paper is a specialization of that described in [MSF05] for partial graph matching applications.

According to the notation proposed in [MSF05] we name a graph \mathcal{G} a *common subgraph* of \mathcal{G}_1 and \mathcal{G}_2 if there exists a subgraph isomorphism from \mathcal{G}_1 to \mathcal{G} and from \mathcal{G}_2 to \mathcal{G} . A *maximal common subgraph* is a common subgraph that can not be extended to another subgraph by the addition of nodes or edges. A *maximum common subgraph* of \mathcal{G}_1 and \mathcal{G}_2 is a common subgraph \mathcal{G} such that there exists no other common subgraph having more nodes than \mathcal{G} . The maximum common subgraph is not necessarily unique.

The output of the matching process should be the largest maximal common subgraph that minimizes the geometrical and the structural differences. The proposed matching algorithm can be synthetically described by the two following steps:

1. Select a mappings M among the nodes of the two graphs \mathcal{G}_1 and \mathcal{G}_2 . The mapping M is a set of node pairs (v_1, v_2) , where v_1 is a node of \mathcal{G}_1 and v_2 is a node of \mathcal{G}_2 ;
2. compute the common subgraph between \mathcal{G}_1 and \mathcal{G}_2 by expanding the mapping M .

The step 2. expands the initial mapping M as much as possible while respecting the definition of common subgraph.

As discussed in [Mar05], running the algorithm on M yields an approximation of the maximum common subgraph. Heuristic techniques can be used to select the best initial mappings and the subgraph expansion rules, in order to better approximate the optimal solution.

Depending on the attributes and on the topology of the graph, some nodes are more relevant than others. Since the considered input graph \mathcal{G} is directed, each node v identifies a subgraph \mathcal{G}' induced by the set of nodes having v as ancestor plus v itself [BMM*03]. The notion of node relevance is used to select the initial mapping M and it is captured by the subgraph associated to the node: a large subgraph corresponds to a large amount of structural and geometric information and then to a more relevant articulated object sub-part. Another useful heuristic has been constructed associating to the pair of nodes (v_1, v_2) the information about how much the common subgraph would expand with the addition of that pair. To this end, a distance function $d(v_1, v_2)$ between two nodes v_1 and v_2 that involves node and edge attributes and the approximation of the structure of the subgraphs related to v_1 and v_2 has been defined in [MSF05]. Then, the similarity between the subgraph structures is defined as:

$$St_S = \frac{\overline{in} + \overline{out} + \overline{sub_n} + \overline{sub_in} + \overline{sub_out}}{5},$$

where

$$\overline{X} = \begin{cases} 0 & \text{if } \max(X(v_1), X(v_2)) = 0 \\ \frac{|X(v_1) - X(v_2)|}{\max(X(v_1), X(v_2))} & \text{otherwise} \end{cases}$$

in and *out* represent the indegree and the outdegree of the two nodes, *sub_n* the number of the subgraph nodes, *sub_in* and *sub_out* the indegree and outdegree sum of the subgraph nodes. Finally $Sz_S = \overline{sub_s}$, where *sub_s* is the sum of the edge attributes of the subgraph.

The graph matching algorithm follows the framework proposed in [MSF05]. The relevance of a node is computed with respect to the average size of all subgraphs induced, and the set of the initial nodes mapping (CANDIDATES) is obtained by combining all relevant graph nodes. The node pairs belonging to CANDIDATES are ordered with respect to *d*; then, the candidate node pair with the smallest value of *d* is extracted. New node pairs are added to CANDIDATES by combining all the nodes outcoming from the initial pair. In particular we observe that the initial mapping among relevant nodes makes the algorithm robust with respect to structural noise allowing the construction of a not necessarily connected common subgraph enabling the recognition of similar sub-parts even if the overall objects shape/structure is dissimilar.

3.3. Computational complexity

The computational cost of the algorithm is given by the sum of the costs of two main steps: the extraction of the structural algorithm and the graph matching phase. The first step of the algorithm (that includes the extraction of the ERG structure and the coding of the models sub-parts using the spherical harmonic descriptor in [KFR03]) may be stored in an out-of-core pre-processing phase. As shown in [Bia04] the complexity of the ERG extraction process is $O(\max(m+n, n \log(n)))$ with *m* the number of vertices inserted in the triangulation during the slicing phase and *n* the number of vertices in the original mesh. The storage of the graph nodes using the spherical harmonic descriptor requires $O(b^4)$ operations using a volumetric regular grid having $O(b^3)$ cells, see [Kaz06] for details.

The computation of the common subgraph between two graphs, with *n* and *m* nodes respectively, is polynomial with respect to $k = \max(n, m)$. In fact, identifying the relevant nodes is linear with respect to the number of nodes of the graph $O(k)$, while the mapping *M* is obtained by combining all relevant graph nodes $O(s * k^2)$, where *s* is the computational cost of the comparison of two geometric attributes associated to the graph nodes. The construction of the ordered set CANDIDATES takes $O(k * \log * k)$ while the extraction of the smallest value of *d* is constant; new elements of CANDIDATES are obtained by combining all the nodes outcoming from the pair of nodes $O(s * k^2)$ and this operation is repeated at most once for each node belonging to the common subgraph. By all these observations the computational complexity of the graph matching algorithm is $O(s * k^3)$.

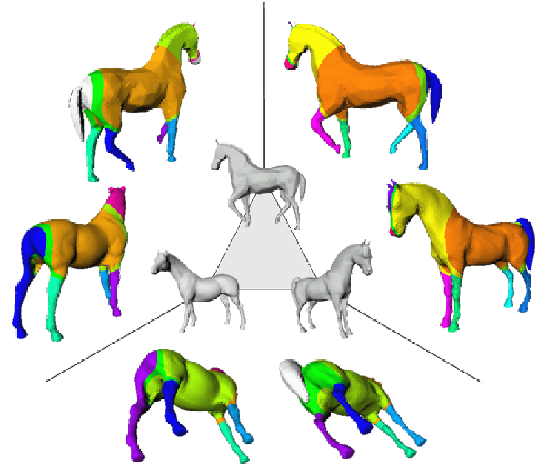


Figure 2: Sub-part correspondence among three animals: these models have similar structure and geometry.

4. Examples and discussion

In this section some experimental results are provided and discussed. Sub-part correspondence represented in the figures of this section is obtained by giving the same color to similar sub-parts.

In figure 2 is shown the partial correspondence obtained by comparing similar models, in our case three horses. Although the overall shape of the animals is the same, the models differ over some details: for example, the structure of the head, the tail and the posture. Our partial matching correctly recognizes the correspondence among the bodies and the front/rear sub-parts of the models. If some shape features have no correspondence in the other models (like the tail) they are not mapped at all. Nevertheless it could happen that features like the legs may be switched. This is caused by a lack of structural information into the leg description, in these cases the sub-parts mapping is completely demanded to the geometric descriptor that produces this output.

Other experimental results of the application of our matching method to models having same overall shape but different spatial embedding are shown in figure 3. The geodesic distance distribution on a human model does not change if the legs and the arms are stretched rather than curled up. Therefore, since our structural descriptor is independent of different poses of the same object, our method can recognize human features like hands, head, legs and body in arbitrary positions, see figure 3.

An example of partial correspondence among models having different overall appearance is given in figure 4. In this case the models differ both in structure and geometry but they have similar sub-parts: even if the two big protrusions are not similar from a geometric point of view they have similar structure. The coupling of geometry and structure pur-



Figure 3: Correspondence of shape features on a human model in different poses.

sued by our algorithm correctly map these sub-parts. On the contrary, the central parts of the models are different both in structure and geometry, thus they are not recognized as similar sub-parts.



Figure 4: The correspondence between the sub-parts of the two models is highlighted by the color.

In figure 5 an example where a whole model is a sub-part of another one is shown: the graph of the cow head is a sub-graph of the cow graph. The matching algorithm computes the common subgraph reasoning on the graph structure and on the geometric attributes, with the result that the mouth, the ears and the horns are correctly mapped. In this case, the algorithm expands as much as possible the initial mapping among the relevant nodes so that the front part of the cow head is correctly mapped, but its neck is mapped with a part of the body. This is intrinsic to the sub-parts correspondence problem. The cow head is described as a global object and the Reeb graph has a different edges orientation with respect of the head of the whole model. This fact may produce unexpected node mappings where the structural descriptor carries a small amount of information, that is the neck of the cow head.

5. Concluding remarks and future work

In summary, in this paper we have presented a new method for measuring similarity and recognizing sub-part correspondence between 3D shapes. Main research contribution is the new similarity matching mechanism to compare 3D shapes coupling geometry and topology. Since this method

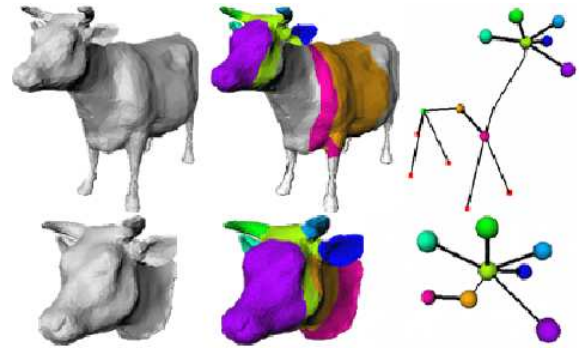


Figure 5: Recognition of the head of a cow with respect to the whole animal model.

computes an approximation of the maximal common subgraph of two structural shape descriptors, it is particularly suitable for sub-part shape correspondence. In addition, it is flexible, because it can be applied to any skeletal structure with the same properties of our topological graph (attributed, directed and acyclic), and tuneable, as it can be used in a multistep query approach, to progressively refine the set of geometrically similar candidates. Even if the flexibility of the mapping function makes adaptable to various application contexts, other shape descriptors like the *shape graph* proposed in [MPS*03] may be considered.

The method proposed in this paper produces an object segmentation without any user interaction while the approaches in [RCSM03] and [FKS*04] do. Furthermore, differently from [HKDH04] that always splits a shape into three parts, our segmentation depends on the shape complexity of the object and decomposes it into a set of significant sub-parts. The method proposed in [FGN89] produces an automatic and structural subdivision of the object surface but it works only on simple surfaces where shape discontinuities are present and easily recognized. As shown in figures 2 and 3, coupling structure and geometry like in our approach is advantageous for comparing models having similar overall shape and structure but different posture.

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