Combined Rendering of Polarization and Fluorescence Effects

Alexander Wilkie

Institute of Computer Graphics and Algorithms
Vienna University of Technology

Robert F. Tobler

VRVis Research Center for Virtual Reality and Visualization

Werner Purgathofer

Institute of Computer Graphics and Algorithms
Vienna University of Technology

Abstract. We propose a practicable way to include both polarization and fluorescence effects in a rendering system at the same time. Previous research in this direction only demonstrated support for either one of these phenomena; using both effects simultaneously was so far not possible, mainly because the techniques for the treatment of polarized light were complicated and required rendering systems written specifically for this task.

The key improvement over previous work is that we use a different, more easily handled formalism for the description of polarization state, which also enables us to include fluorescence effects in a natural fashion. Moreover, all of our proposals are straightforward extensions to a conventional spectral rendering system.

1 Introduction

For the purposes of truly predictive photorealistic rendering it is essential that no effect which contributes to the interaction of light with a scene is neglected. Most aspects of object appearance can be accounted for by using just the laws of geometric optics, comparatively simple descriptions of surface reflectivity, tristimulus representations of colour and light, and can nowadays be computed very efficiently through a variety of common rendering algorithms. However, several physical effects, namely fluorescence, diffraction, dispersion and polarization, are still rarely – if at all – supported by contemporary rendering software.

1.1 Polarization

Polarization has received particularly little attention because – while of course being essential for specially contrived setups that e.g. contain polarizing filters – it seemingly does not contribute very prominent effects to the appearance of an average scene. This misconception is in part fostered by the fact that the human eye is normally not capable of distinguishing polarized from unpolarized light[1].

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[1] Contrary to common belief trained observers *can* distinguish polarized from unpolarized light with the naked eye. Named after its discoverer, the effect is known as *Haidinger's brush* and is described by Minnaert in his book about light in outdoor surroundings [7].

One of the main areas where it in fact does make a substantial difference are outdoor scenes; this is due to the usually quite strong polarization of skylight, as one can find documented in G. P. Können's book [6] about polarized light in nature. But since such scenes are currently still problematical for photorealistic renderers for a number of other, more obvious reasons (e.g. scene complexity and related global illumination issues), this has not been given a lot of attention yet. Other known effects which depend on polarization support are certain darkening or discolourization patterns in metal objects and their reflections, and the appearance of faceted transparent objects, such as crystals.

1.2 Fluorescence

Some of the reasons for the small amount of work fluorescence has received are different from those which have made polarization a fringe topic. Firstly, although it causes very prominent effects, these can also be faked comparatively easily through custom shaders at a fraction of the effort involved in actually simulating the real process. Secondly, measurements of fluorescent pigments are very hard to obtain, virtually no publicly accessible data of this kind exists, and designing such spectra by hand is tedious. The third main reason is shared between fluorescence and polarization: they ought to be done using a spectral rendering system, which still rank as comparatively exotic and expensive to use.

2 Previous Work

2.1 Raytracing with Polarization Parameters

Physics has long achieved a very thorough understanding of light polarization; a classic introduction is e.g. to be found in the book of Born and Wolf [1]. In the computer graphics literature there are two publications about it; one by Wolff and Kurlander [16], who demonstrated polarization–aware image synthesis for the first time, and one by Tannenbaum et al. [14], who concentrated on the rendering of anisotropic crystals and extended the techniques used by Wolff et al.

The main goal of these efforts was that of finding an appropriate way to describe, and perform calculations with, polarized light; both groups of authors settled for a notation suggested by standard reference texts from physics literature.

Coherency Matrices. The formalism to describe polarized light used by both Wolff and Tannenbaum is that of *coherency matrices* (CM for short); this technique was introduced by Born and Wolf [1]. As derived in detail by Tannenbaum et al. [14], the coherency state of a monochromatic wave can be expressed in a matrix J of the form

$$J = \begin{pmatrix} J_{xx} & J_{xy} \\ J_{yx} & J_{yy} \end{pmatrix} = \begin{pmatrix} \langle E_x E_x^\star \rangle & \langle E_x E_y^\star \rangle \\ \langle E_y E_x^\star \rangle & \langle E_y E_y^\star \rangle \end{pmatrix}$$

where E_x and E_y are the E–field vectors for the X and Y directions, respectively, and $E_x^\star$ denotes the complex conjugate of E_x. The main diagonal elements – J_{xx} and J_{yy} – are real valued, and the trace $T(J) = J_{xx} + J_{yy}$ of the matrix represents the total light radiation of the wave, while the complex conjugates J_{xy} and J_{yx} represent the correlation of the X and Y components of E. For fully polarized light, these components are fully correlated and $|J|$ vanishes.

Coherency Matrix Modifiers. Besides being able to describe a ray of light, it is also necessary to process the interaction of light with a medium or surface. Such *filtering operations* (or *filters* for short) on polarized light described by a coherency matrix can be performed by using *coherency matrix modifiers*, or CMM. Tannenbaum et al. brought this approach, which was originally introduced by Parrent et al. [8], to the computer graphics world for use in their rendering system.

CMMs have the form of a complex–valued 2×2 matrix. If all participating elements have the same reference coordinate system, such a matrix M_p can be applied to a given coherence matrix J in the sequence of $J_p = \mathcal{M}_p J \mathcal{M}_p^\dagger$, where $\mathcal{M}_p^\dagger$ signifies the conjugate transpose of $\mathcal{M}_p$. If the modifier and the coherency matrix are not in the same coordinate system, an appropriate transformation – as discussed in section 2.3 of the full version of Tannenbaum et al. [14] (to be found on the proceedings CD–ROM) – has to be applied first.

2.2 Reflection Models which take Polarization into Account

Apart from the modified Cook–Torrance model used by Wolff et al., only one other surface model proposed so far, namely that of He et al. [5], attempts to consider polarization effects. As could be inferred from the results section of this paper, the high complexity of their surface model apparently led the authors to only implement a simpler, non–polarizing version in practice, and to contain themselves with just providing the theoretical derivation of the polarization–aware model in the text.

2.3 Rendering of Fluorescence Effects

So far Glassner has apparently been the only graphics researcher who investigated the rendering of fluorescence phenomena [3]. The main focus of his work was centered around the proper formulation of the rendering equation in the presence of phosphorescence and fluorescence, and he provided striking results generated with a modified version of the public domain raytracer *rayshade*. Sadly, this work was not followed up, nor was the modified version of rayshade made public. Also, we are not aware of any work that aims at considering the inclusion of fluorescence effects in sophisticated reflectance models.

3 A Combined Renderer

We first introduce an alternative notation that can be used for polarization support, and then show how this formalism can easily be combined with fluorescence support.

3.1 Alternative Polarization Support

A description for polarized radiation which due to its simpler mathematical characteristicsis is better suited for use in raytracing–based rendering systems is that of *Stokes parameters*. This description, while equivalent to coherency matrices, has the advantage of using only real–valued terms to describe all polarization states of optical radiation, and has an – also noncomplex – corresponding description of ray weights in the form of Müller matrices [11].

It has to be kept in mind that – similar to coherency matrices – both Stokes parameters and Müller matrices are meaningful only when considered within their own local reference frame; the main effect of this is that in a rendering system not only light, but

also filters are *oriented* and have to store an appropriate reference in some way. However, for the sake brevity this spatial dependency is omitted in our following discussion except in the section about matrix realignment.

Stokes Parameters.   Apart from coherency matrices, the polarization state of an electromagnetic wave of a given frequency can also be described in several other ways. Three real–valued parameters are required to describe a general polarization ellipse, but the *Stokes vector* notation defined by

$$
\begin{aligned}
E_{n,0} &= \kappa(V_x^2 + V_y^2) && [W \cdot m^{-2}] \\
E_{n,1} &= \kappa(V_x^2 - V_y^2) \\
E_{n,2} &= \kappa(2V_x^2 \cdot V_y^2 \cdot \cos\gamma) \\
E_{n,3} &= \kappa(2V_x^2 \cdot V_y^2 \cdot \sin\gamma)
\end{aligned}
\tag{1}
$$

(in the notation used by Shumaker [11]) has proven itself in the optical measurements community, and has the key advantage that the first component of this 4–vector is the unpolarized intensity of the light wave in question (i.e. the same quantity that a non-polarizing renderer uses). Components 2 and 3 describe the preference of the wave towards linear polarization at zero and 45 degrees, respectively, while the fourth encodes preference for right–circular polarization. While the first component is obviously always positive, the values for the three latter parameters are bounded by $[-E_{n,0}, E_{n,0}]$; e.g. for an intensity $E_{n,0} = 2$, a value of $E_{n,3} = -2$ would indicate light which is totally left circularly polarized.

The – at least in comparison to coherency matrices – much more comprehensible relationship between the elements of a Stokes vector and the state of the wavetrain it describes is, amongst other things, very beneficial during the debugging stage of a polarizing renderer, since it is much easier to construct verifiable test cases.

Müller Matrices.   Müller matrices (MM for short) are the data structure used to describe a filtering operation by materials that are capable of altering the polarization state of incident light represented by a Stokes vector. The general modifier for a 4–vector is a 4×4–matrix, and the structure of the Stokes vectors implies that the elements of such a matrix correspond to certain physical filter properties. As with Stokes vectors, the better comprehensibilty of these real–valued data structures is of considerable benefit during filter specification and testing.

The degenerate case of MM is that of a nonpolarizing filter; this could equally well be described by a simple reflection spectrum. For such a filter the corresponding MM is the identity matrix. A more practical example is the MM of an ideal linear polarizer T_{lin}, where the polarization axis is tilted by an angle of ϕ against the reference coordinate system of the optical path under consideration, and which has the form of

$$
T_{lin}(\phi) = \frac{1}{2}
\begin{bmatrix}
1 & \cos 2\phi & \sin 2\phi & 0 \\
\cos 2\phi & \cos^2 2\phi & \sin 2\phi \cdot \cos 2\phi & 0 \\
\sin 2\phi & \sin 2\phi \cdot \cos 2\phi & \sin^2 2\phi & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}.
$$

For the purposes of physically correct rendering it is important to know the MM which is caused by evaluation of the Fresnel terms; for this we use the notation from [16].

For a given wavelength and intersection geometry (i.e. specified index of refraction and angle of incidence), the resulting terms $F_\perp$, $F_\parallel$, $\delta_\perp$ and $\delta_\parallel$ have to be used as

$$T_{Fresnel} = \begin{bmatrix} A & B & 0 & 0 \\ B & A & 0 & 0 \\ 0 & 0 & C & -S \\ 0 & 0 & S & C \end{bmatrix},$$

where $A = (F_\perp + F_\parallel)/2$, $B = (F_\perp - F_\parallel)/2$, $C = \cos(\delta_\perp - \delta_\parallel)$ and $S = \sin(\delta_\perp - \delta_\parallel)$; $\delta_\perp - \delta_\parallel$ is the total retardance the incident wavetrain is subjected to.

Filter Rotation. In order to correctly concatenate a filter chain, as effectively first described by Hall et al. in [4] (which basically amounts to matrix multiplications of the MMs in the chain), we have to be able to re–align a MM to a new reference system, which amounts to rotating it along the direction of propagation to match the other operands.

Contrary to first intuition, directional realignment operations are *not* necessary along the path of a concatenated filter chain; the retardance component of a surface interaction is responsible for the alterations that result from changes in wavetrain direction.

Since in the case of polarized light a rotation by an angle of ϕ can only affect the second and third components of a Stokes vector (i.e. those components that describe the linear component of the polarization state), the appropriate rotation matrix $M(\phi)$ has the form of

$$M(\phi) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 2\phi & \sin 2\phi & 0 \\ 0 & -\sin 2\phi & \cos 2\phi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{2}$$

Matrices of the same form are also used to rotate Müller matrices. In order to obtain the rotated version of a MM $T(0)$, $M(\phi)$ has to be applied in a way similar to that shown for CMMs, namely $T(\phi) = M(-\phi) \cdot T(0) \cdot M(\phi)$.

Apart from being useful to re–align a filter through rotation, the matrix M given in equation (2) is also the MM of an ideal *circular retarder*. Linearly polarized light entering a material of this type will emerge with its plane of polarization rotated by an angle of ϕ; certain materials, such as crystal quartz or dextrose, exhibit this property, which is also referred to as *optical activity*.

3.2   Rendering of Fluorescence Effects

Since fluorescence is a material property, its description only affects the filter data structure. Specifically, for a system which uses n samples to represent spectra, filter values of fluorescent substances have to be a re–radiation matrix (RRM) of $n \times n$ elements.

The fact that fluorescence only ever causes light to be re–radiated at *lower* wavelengths than those at which it is absorbed allows us to only consider the lower half of this matrix. In practice, we use the same reflection spectra as for normal materials, only augmented with a data structure that holds the area below the main diagonal. All filtering operations were adapted to handle the presence of this crosstalk component as a sticky property; even if only one filter in a concatenation chain has an off–diagonal component, the overall result has to be also fluorescent.

3.3 Combining Fluorescence and Polarization

Similar to the previous section, this is a problem which only concerns the filtering operations. Specifically, the question is in which way nonpolarizing $n \times n$ (or n plus sub–diagonal crosstalk of $(n \times (n-1))/2$) re–radiation matrix filters and $n \times (4 \times 4)$ reflectance spectra with Müller matrices as samples can be properly mixed.

Practical Considerations. Fortunately, the solution is straightforward. The key observation here is that light which is re–emitted by fluorescent molecules can be considered to be unpolarized for our purposes, since this kind of light interaction with pigments usually has no directional character. This means that, while the full combination of the two properties would require the rather unwieldy construct of a data structure with $n \times n \times 4 \times 4$ entries (a MM for each RRM entry), we can get by with using a much smaller entity.

Our combined filter data structure uses the same nonpolarizing crosstalk component as the plain fluorescence–aware filter described in the previous section, and just replaces the main diagonal reflection spectrum with its polarizing counterpart.

The only area where this change causes a considerable increase in the complexity of a rendering system are the filter manipulation methods, which now have to account for four possible states of each operand (fluorescent yes/no, polarizing yes/no) in each procedure.

4 Results

We implemented the proposed dual polarization and fluorescence support in the public domain rendering software under development at our institute, the Advanced Rendering Toolkit (ART for short).

Filters and Light. The first task was to introduce the distinction between filters and light – as described in the previous sections – throughout the raytracing code of ART; the previously used polymorphous colour data type had to be appropriately replaced by filter and light structures. As an unasked–for fringe benefit of this quite substantial task it transpired that the entire rendering code became much clearer semantically through the introduction of this distinction; this might serve as an encouragement for others who face the same task.

Fluorescence. The filters were then extended so that they also became capable of encoding fluorescence information as described in section 3.2. The main work during this step was the alteration of the filter manipulation routines and development of specialized storage classes for fluorescent reflection data.

Polarization. In order to be able to make meaningful performance comparisons, and also to keep the option of using a faster renderer without polarization capabilities for the majority of users who do not need the feature, polarization support was implemented as a compile–time option in ART. For the non–polarizeable renderer the procedures for polarization support expand to NOPs, so no overhead is incurred, and for the polarization–aware renderer they are expanded as inline funtions.

Apart from additional large changes to the light and filter manipulation code – which had already been fleshed out and pushed into one module during the first step – the

major changes in this stage involved the introduction of the capability to store and manipulate the orientation of light and filter data structures during rendering processes. Three sample images obtained with this hybrid renderer are shown in figure 1 in the colour plate section.

Optimization. The only computational optimization with respect to polarization that has been implemented in ART so far is that each instance of the light and filter data structures is tagged as to whether it actually describes polarized light or a polarizing filter. This makes it possible to use faster routines if both operands in a calculation involving lights and/or filters are not polarized (specifically, multiplication of the samples instead of matrix operations). If one of the operands is polarized or polarizing, then so is the result of the computation: polarization is a sticky property of light and filters.

This simple optimization leads to a polarization–aware renderer which is on average only $10 - 30$ percent slower than its plain counterpart for scenes which do not contain any polarizing surfaces, lightsources or materials. Since practically all scenes contain at least a certain percentage of objects and lightsources that do not exhibit any polarized or polarizing property, this shortcut actually improves the performance of the polarization–aware renderer for all but extreme scenes.

For scenes with large amounts of polarizing objects – like e.g. the example scene shown in figure 1 – the slowdown of the polarizeable version compared the plain renderer is naturally higher and depends strongly on the scene. The example in figure 1 exhibits a fairly typical slowdown of around 500 percent (30 vs. 150 seconds rendering time); in some rare cases we have observed even larger performance drops. This drastic increase in rendering time is the price one has to pay for increased physical accuracy, although we estimate that some additional performance could be gained from aggressive global optimization of the numeric code in our rendering system.

5   Conclusion

We presented a practical way to implement combined polarization and fluorescence support in the context of a modern rendering system. The key improvements over previous approaches are

- The use of Stokes vectors to describe both unpolarized an polarized light with a single, intuitive formalism.
- The use of Müller matrices to describe the polarizing effect of materials and surfaces in an understandable way that is complementary to Stokes vectors.
- The combination of these formalisms with fluorescence information into a single rendering system.

Our future research will concern itself with the efficiency of spectral rendering in general, and in particular the choice of the most suitable spectral representation for photorealistic rendering; this is a question for which a good answer that is generally valid has yet to be given. In the course of these investigations we plan to also investigate if both polarization and fluorescence can be represented even more efficiently, and investigate whether and how the usually large similarity in polarization state amongst samples in a given light spectrum can be safely exploited to reduce storage and computation requirements.

Acknowledgements

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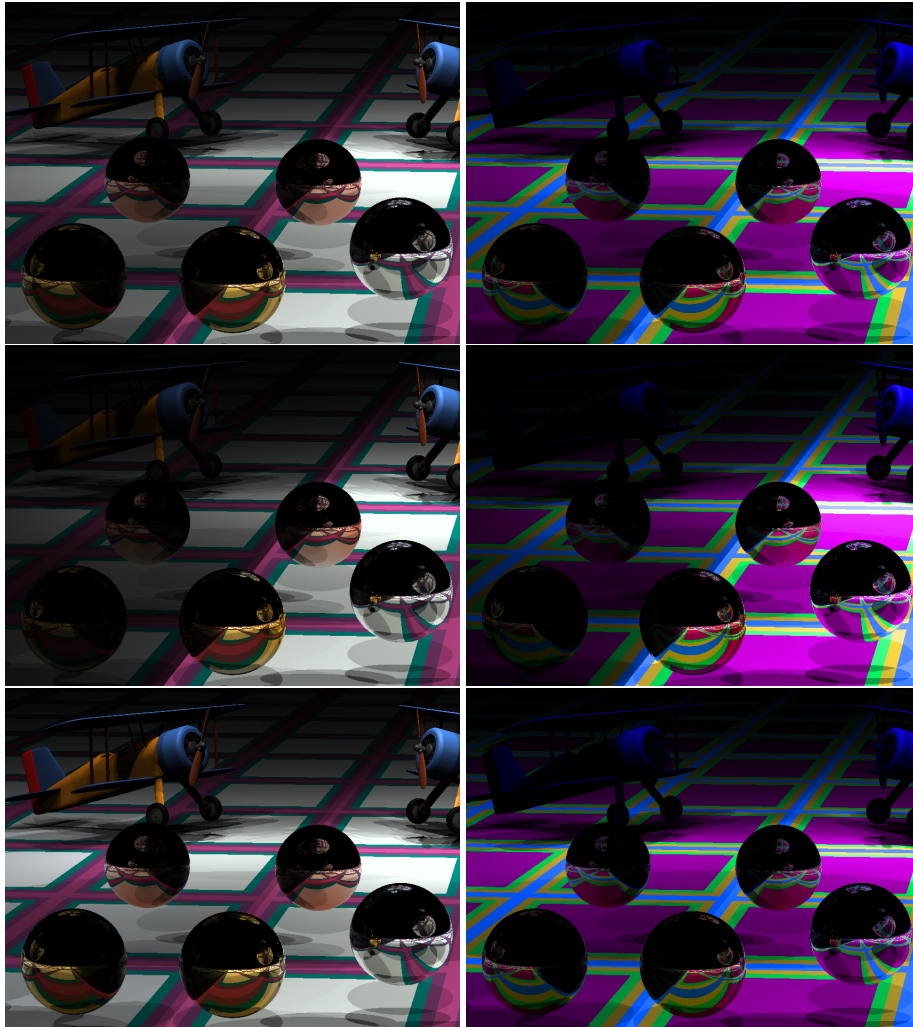
Fig. 1. Example renderings of polarization effects combined with fluorescent objects. The left and right columns show similar setups under two different illuminations - at the left D65, and at the right UV blacklight. The scene shows several metal spheres (gold, copper, silver) and a nonfluorescent object (the biplane model) which float over a diffuse floor with fluorescent properties, and which are reflected in a large block of a dielectric material (glass). The reflection is viewed well below Brewster's angle in order to increase its intensity; because of this, the polarizing filters which are placed in front of the camera in the two lower images (horizontal polarizer in the middle, vertical at the bottom) do not affect the entire reflected energy. For comparison purposes, the topmost two images have a 50 percent neutral grey filter instead of a polarizer placed in front of the camera. These images can also be viewed in higher resolution at http://www.artoolkit.org/Gallery/Fluorescence/.