AA Patterns

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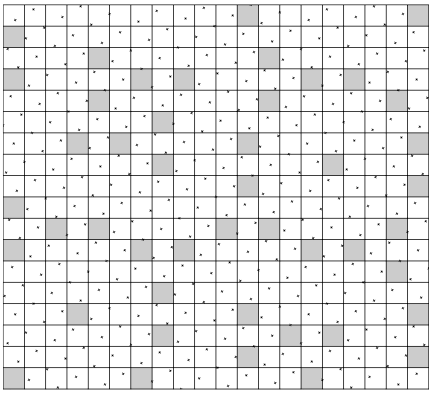
Concept

Overlay a 2D grid over another using the transform:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \left\lfloor \frac{1}{2} \begin{bmatrix} \alpha & -1 \\ 1 & \alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \right\rfloor; \quad 1 < \alpha < 2$$

and scanning over all (x,y) points. Owing to quantization, some (X,Y) points will be skipped. We call these skipped points 'holes', and we call mapped-to (X,Y) points 'solids'. AA(α) is the is the set of holes, as in this figure adapted from [1].



Although AA Patterns have got a single parameter, they offer much control over the patterns by tuning some sub-parameters embedded in α.

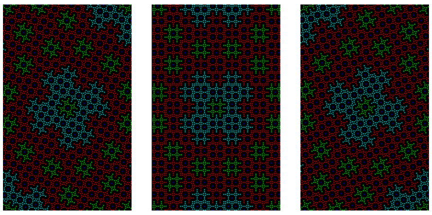
So far a rational parameter could be anatomized as follows:

$$\begin{aligned} \alpha &= p/q \\ t &= p - q \\ r &= q \% t \\ m &= q/t \end{aligned}$$

In what follows we show the effect of these parameters.

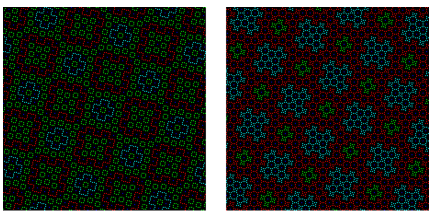
Orientation

The original patterns are apparently tilted in the direction of the overlaid grid. By flipping the patterns horizontally or vertically we find a new orientation. It was also found [1] that there is a trick to plot the patterns in the overlaid grid (x,y), effectively straightening the patterns. This gives us three orientation options for AA Patterns: XY, xy, or –XY, as illustrated in this figure:



Threads and Clusters

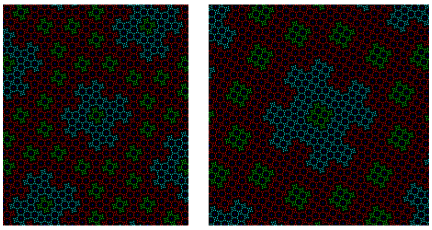
Holes in AA Patterns tend to group either in threads or clusters, depending on whether $t < 2r$ or $t > 2r$, respectively. The difference is apparent in the figure. Generally, for a given t, if some r gives clusters, then $r' = t\text{-}r$ gives similarly structured threads.



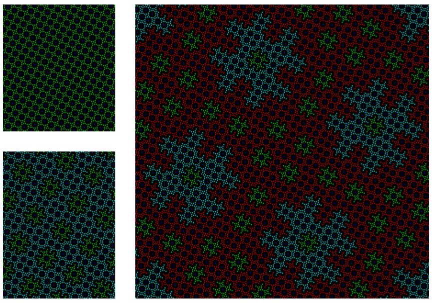
Levels

Holes in AA Patterns group into structured levels, each level might contain all or some of the lower levels inside.

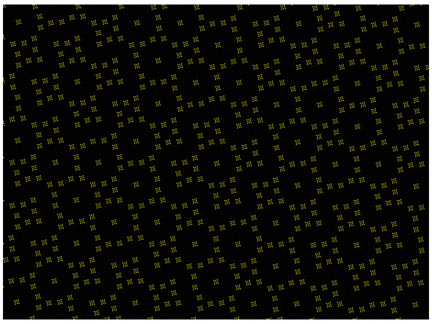
There are strong clues that levels are tightly related to entries of continued fraction representation of α. This figure shows the impact of changing α from [1 1 2 1 2 1 3] to [1 1 2 1 4 1 3].



By truncating the continued fraction at a certain convergent, the pattern is restricted to the corresponding level, as shown in this figure which shows patterns for α = [1 1 3], [1 1 3 4], and [1 1 3 4 4].



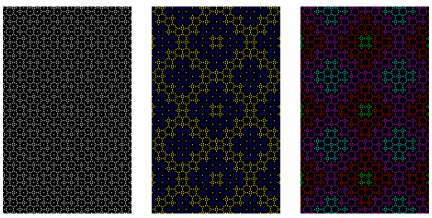
An interesting thing about levels is that every individual level is also laid out ornamentally, in a similar style to individual pixels, as illustrated in this figure:



Resolution

AA Patterns are inherently pixel-based, which means that they do not need high spatial resolution to display well. Hence they are readily usable for floor tiling, for example.

Originally AA Patterns are monochrome, but coloring reveals details and give the patterns a richer look. Typical coloring uses a different color for each level, which means only a few colors to paint the patterns. Alternatively, it is also possible to color the holes with only 2 alternating colors in an even-odd style.

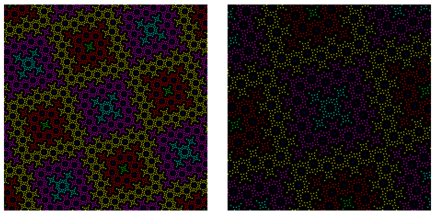


Symmetries

(Almost) All clusters and threads of AA Patterns exhibit D4 symmetries, as can be seen in the figures.

Dispersion

The sub-parameter m controls how disperse the pattern is: how far apart the holes are. It has no much effect otherwise.

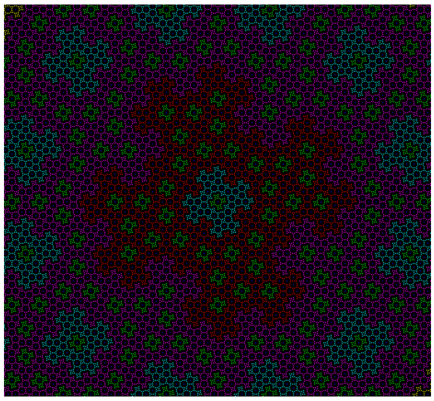


Periodicity

All AA Patterns with rational parameters are periodic, and (theoretically) all patterns with an irrational parameter are aperiodic.

Self-Similarity

Cycles in continued fractions of α are reflected as a sort of self-similarity in the resulting pattern; that is, higher levels resemble smaller ones, but add more details. This suggests a strong relationship between AA Patterns and fractals, but that relationship is still unresolved.



Conclusion

This is a brief introduction to AA Patterns. The generation logic of AA Patterns is very simple, yet it offer a rich variety, and many aspects can be controlled easily.

One of the important directions for future research about AA Patterns is to explore how these patterns relate to fractals. Understanding that relationship can really help in developing computationally-cheap fractals techniques.

Another area of interest is the relationship between levels and continued fractions, which suggests a use in visualization.

Besides these applications, and besides aesthetics, AA Patterns offer a rich educational material. Indeed, the author found it worthwhile developing various generation and coloring algorithms.

Further Readings

You are invited to interactively play with AA Patterns at http://aapatterns.abdallagafar.com. You can test various parameters, and see how different levels interact to make the pattern.

For technical study of AA Patterns, and for generation and coloring algorithms, you are advised to read the article:

[1] Abdalla G. M. Ahmed: "Pixel Patterns from Quantization Artifacts of Forward Affine Mapping", to appear in the forthcoming issue of the "Journal of Graphics, GPU, and Game Tools."