AA Patterns

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Abstract

AA Patterns is the name of a recently developed algorithmic art technique that employs a simple linear 2D transform to generate ornamental patterns of pixels. The patterns are controlled by a single real parameter between 1 and 2. By appropriately anatomizing this parameter it is shown that this single parameter actually allows much control over the patterns. Some potential uses are pointed out, too.

1. Introduction

Flaws and bugs in algorithms occasionally result in artifacts which are undesirable most of the times, but which sometimes exhibit patterns that look really interesting, like the ones discovered by Kaplan [Kap05]. An example artifact occurs in forward scanning to transform a bitmap using:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \left\lfloor \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \right\rfloor \quad (1)$$

Forward scanning means going through all image pixels (x, y) and copying color to target view-port pixels (X, Y). Such scanning order could leave holes (unpainted pixels) in the target.

The author accidentally discovered that the holes make aesthetic and interesting patterns, like the ones shown in Figure 1, when the transform has the form:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \left\lfloor \frac{1}{2} \begin{bmatrix} \alpha & -1 \\ 1 & \alpha \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \right\rfloor ; \ \alpha \in (1, 2) \quad (2)$$

Long investigations have been conducted to understand and analyze these patterns, which were called AA Patterns. The study led to a paper [Ahmss] which focused mainly on practical aspects, like generation and coloring algorithms. In this paper we spot lights on some properties of AA Patterns, in a way that could suggest some uses and/or allow some control over the patterns.

2. Properties

The interesting thing about AA Patterns is that although they are controlled by a single parameter, this parameter actually embodies many sub-parameters. A rational parameter

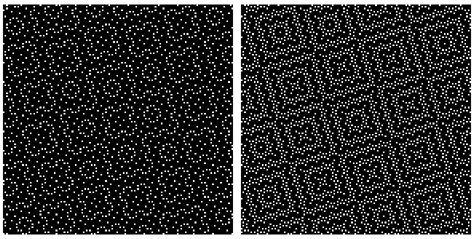

Figure 1: *Sample AA Patterns. Holes can group in threads (left) or clusters (right).*

$$\alpha = p/q \quad (3)$$

can be broken into the following integer parameters:

$$t = p - q \quad (4)$$
$$m = q/t \quad (5)$$
$$r = q\%t \quad (6)$$

In the following subsections we list some properties of AA Patterns and relate them to these parameters.

2.1. Plotting Plane

Besides the apparently tilted XY plots it has been shown in [Ahmss] that the patterns can be plotted in the xy plane instead, effectively straightening the patterns, as shown in Figure 2.

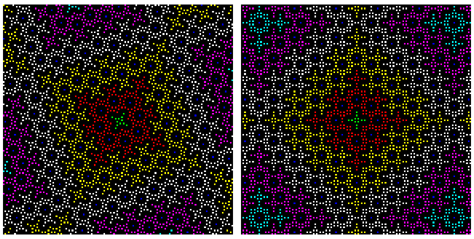

Figure 2: *$AA(108/61)$ plotted in XY plane (left) and xy plane (right).*

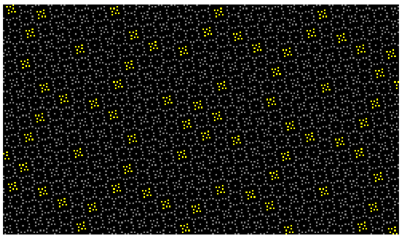

Figure 3: *$AA(1365/794)$ with level 2 highlighted to show how every level is also has an ornamental distribution.*

2.2. Periodicity

With a rational parameter p/q the resulting pattern is periodic. The fundamental domain of XY plots is $(p^2+q^2) \times (p^2+q^2)$ pixels, and contains $t^2 \times (p^2+q^2)$ holes. That of an xy plot is $2q \times 2q$ pixels, and contains $t \times t$ holes. Thus, there are two options to control the number of holes in a given area. On the other hand, an irrational parameter generates an aperiodic pattern.

2.3. Threads and Clusters

As shown in Figure 1, there are two types of AA Patterns: threads and clusters. When $t < 2r$ holes are grouped in threads, otherwise they group in clusters. Note that m is not relevant here.

2.4. Levels

Groups of holes, whether threads or clusters, exhibit hierarchical structures, or levels. Levels are best described by looking at Figure 2, where different levels are painted in different colors.

There are clues that different levels are controlled by entries of continued fraction expansion of α, which suggests potential uses of AA Patterns in visualization. One interesting thing about levels is that clusters of every level are also distributed ornamentally, as shown in Figure 3.

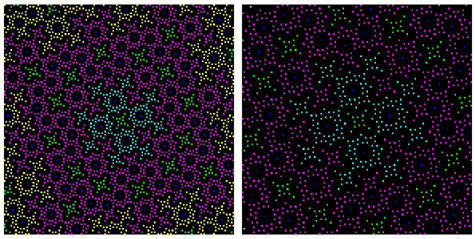

Figure 4: *Dispersion of patterns with $m = 1$ (left) and $m = 2$ (right); t and r held the same.*

2.5. Dispersion

Given a hole at (X, Y), it was shown in [Ahmss] that there are specific locations where nearby holes might be found, and these locations depend solely on m.

Thus, t and r decide the layout of the pattern, while m describes how disperse it is. The effect of m is shown in Figure 4.

2.6. Self-Similarity

Repeating cycles of the continued fraction of α are reflected in the patterns in a form of self-similarity, where higher levels resemble smaller ones, but add more details.

Self-similarity is a characteristic of fractals [PS88], and having this property here suggests a relationship between AA Patterns and fractals, but that relationship is not clear yet.

On the other hand, AA Patterns do not involve iterations which are common in fractals. The importance of this is point is that if the exact relationship of AA Patterns with fractals is discovered, it might help in developing iteration-less fractals generation techniques.

References

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[Kap05] Kaplan C. S.: Aliasing artifacts and accidental algorithmic art. *In Renaissance Banff: Bridges 2005: Mathematical Connections in Art, Music and Science* (2005), 349–356.

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