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Conceptualizing Birkhoff's Aesthetic Measure Using Shannon Entropy and Kolmogorov Complexity

Jaume Rigau, Miquel Feixas, and Mateu Sbert

Institut d'Informàtica i Aplicacions, University of Girona, Spain

Abstract

In 1928, George D. Birkhoff introduced the Aesthetic Measure, defined as the ratio between order and complexity, and, in 1965, Max Bense analyzed Birkhoff's measure from an information theory point of view. In this paper, the concepts of order and complexity in an image (in our case, a painting) are analyzed in the light of Shannon entropy and Kolmogorov complexity. We also present a new vision of the creative process: the initial uncertainty, obtained from the Shannon entropy of the repertoire (palette), is transformed into algorithmic information content, defined by the Kolmogorov complexity of the image. From this perspective, the Birkhoff's Aesthetic Measure is presented as the ratio between the algorithmic reduction of uncertainty (order) and the initial uncertainty (complexity). The measures proposed are applied to several works of Mondrian, Pollock, and van Gogh.

Categories and Subject Descriptors (according to ACM CCS): I.4 [Computing Methodologies]: Image Processing and Computer Vision J.5 [Computer Applications]: Arts and Humanities

1. Introduction

From *Birkhoff's aesthetic measure* [Bir33], Moles [Mol68] and Bense [Ben69] developed the *information aesthetics* theory, based on *information theory*. The concepts of *order* and *complexity* were formalized from the notion of *information* provided by Shannon's work [CT91]. Scha and Bod [SB93] stated that in spite of the simplicity of these beauty measures, "if we integrate them with other ideas from perceptual psychology and computational linguistics, they may in fact constitute a starting point for the development of more adequate formal models".

In this paper, we present a new version of Birkhoff's measure based on Zurek's physical entropy [Zur89]. Zurek's work permits us to look at the creative process as an evolutionary process from the initial uncertainty (Shannon entropy) to the final order (Kolmogorov complexity). This approach can be interpreted as a transformation of the initial *probability distribution* of the palette of colors to the *algorithm* which describes the final painting. We also analyze several ratios, obtained from Shannon entropy and Kolmogorov complexity, applied to the global image and different decompositions of it.

We will use here zeroth-order measures such as Shannon entropy of the histogram. This is a first step towards updating classical Birkhoff's measure. A next step could be to use higher-order measures to handle edges, contrast, and spatial frequency, well studied in visual perception [Bla93].

This paper is organized as follows. In section 2, the information theory and Kolmogorov complexity are described. In section 3, origins and related work are reviewed. In sections 4 and 5, global and compositional aesthetic measures are defined and discussed, respectively. Finally, conclusions are presented.

2. Information Theory and Kolmogorov Complexity

In this section, some basic notions of information theory [CT91] and Kolmogorov complexity [LV97] are reviewed.

2.1. Information-Theoretic Measures

Information theory deals with the transmission, storage and processing of information and is used in fields such as physics, computer science, statistics, biology, image processing, learning, etc.

Let $\mathcal{X}$ be a finite set, let X be a random variable taking values x in $\mathcal{X}$ with distribution $p(x) = Pr[X = x]$. Likewise, let Y be a random variable taking values y in $\mathcal{Y}$. The *Shannon entropy* $H(X)$ of a random variable X is defined by

$$H(X) = -\sum_{x\in\mathcal{X}} p(x)\log p(x). \quad (1)$$

The Shannon entropy $H(X)$ measures the average uncertainty of random variable X. If the logarithms are taken in base 2, entropy is expressed in bits. The *conditional entropy* is defined by

$$H(X|Y) = -\sum_{x\in\mathcal{X},y\in\mathcal{Y}} p(x,y)\log p(x|y), \quad (2)$$

where $p(x,y) = Pr[X = x,Y = y]$ is the joint probability and $p(x|y) = Pr[X = x|Y = y]$ is the conditional probability. The conditional entropy $H(X|Y)$ measures the average uncertainty associated with X if we know the outcome of Y. The *mutual information* between X and Y is defined by

$$I(X,Y) = H(X) - H(X|Y) = H(Y) - H(Y|X) \quad (3)$$

and represents the shared information between X and Y.

A fundamental result of information theory is the *Shannon source coding theorem*, which deals with the encoding of an object in order to store or transmit it efficiently. The theorem expresses that the minimal length of an optimal code (for instance, a Huffman code) fulfills

$$H(X) \le \bar{\ell} < H(X)+1, \quad (4)$$

where $\bar{\ell}$ is the expected length of the optimal binary code for X.

The *data processing inequality* plays a fundamental role in many information-theoretic applications: if $X \to Y \to Z$ is a Markov chain (i.e., $p(x,y,z) = p(x)p(y|x)p(z|y)$), then

$$I(X,Y) \ge I(X,Z). \quad (5)$$

This inequality demonstrates that no processing of Y, deterministic or random, can increase the information that Y contains about X.

2.2. Kolmogorov Complexity and the Similarity Metric

The *Kolmogorov complexity* $K(x)$ of a string x is the length of the shortest program to compute x on an appropriate universal computer. Essentially, the Kolmogorov complexity of a string is the length of the ultimate compressed version of the string. The conditional complexity $K(x|y)$ of x relative to y is defined as the length of the shortest program to compute x given y as an auxiliary input to the computation. The joint complexity $K(x,y)$ represents the length of the shortest program for the pair (x,y) [LV97]. The Kolmogorov complexity is also called *algorithmic information* and *algorithmic randomness*.

The *information distance* [BGL*98] is defined as the length of the shortest program that computes x from y and y from x. There it was shown that, up to an additive logarithmic term, the information distance is given by

$$E(x,y) = \max\{K(y|x),K(x|y)\}. \quad (6)$$

This measure is a metric. It is interesting to note that long strings that differ by a tiny part are intuitively closer than short strings that differ by the same amount. Hence,the necessity to normalize the information distance arises. Li et al. [LCL*04] defined a normalized version of $E(x,y)$, called the *normalized information distance* or *the similarity metric*:

$$\begin{aligned} NID(x,y) &= \frac{\max\{K(x|y),K(y|x)\}}{\max\{K(x),K(y)\}} \\ &= \frac{K(x,y)-\min\{K(x),K(y)\}}{\max\{K(x),K(y)\}}. \end{aligned} \quad (7)$$

In addition, they shown that *NID* is a metric and takes values in $[0,1]$. It is *universal* in the sense that if two strings are similar according to the particular feature described by a particular normalized admissible distance (not necessarily metric), then they are also similar in the sense of the normalized information metric.

Due to the non-computability of Kolmogorov complexity, a feasible version of NID (7), called *normalized compression distance*, is defined as

$$NCD(x,y) = \frac{C(x,y)-\min\{C(x),C(y)\}}{\max\{C(x),C(y)\}}, \quad (8)$$

where $C(x)$ and $C(y)$ represent the length of compressed string x and y, respectively, and $C(x,y)$ the length of the compressed pair (x,y). Therefore, *NCD* approximates *NID* by using a standard real-world compressor.

2.3. Physical Entropy

Looking at a *system* from an observer's angle, Zurek [Zur89] defined its *physical entropy* as the sum of the missing information (Shannon entropy (1)) and the algorithmic information content (Kolmogorov complexity) of the available data:

$$S_d = H(X_d)+K(d), \quad (9)$$

where d is the observed data of the system, $K(d)$ is the Kolmogorov complexity of d, and $H(X_d)$ is the conditional Shannon entropy or our ignorance about the system given d [Zur89, LV97].

Physical entropy reflects the fact that measurements can increase our knowledge about a system. In the beginning, we have no knowledge about the state of the system, therefore the physical entropy reduces to the Shannon entropy, reflecting our total ignorance. If the system is in a regular state, physical entropy can decrease with the more measurements we make. In this case, we increase our knowledge about the system and we may be able to compress the data efficiently. If the state is not regular, then we cannot achieve compression and the physical entropy remains high [LV97].

3. Origins and Related Work

In 1928, Birkhoff formalized the notion of beauty by the introduction of the *aesthetic measure*, defined as the ratio between *order* and *complexity* [Bir33], where "the complexity is roughly the number of elements that the image consists of and the order is a measure for the number of regularities found in the image" [SB93]. According to Birkhoff, the aesthetic experience is based on three successive phases:

1. A preliminary effort of attention, which is necessary for the act of perception, and that increases proportionally to the *complexity* (C) of the object.
2. The feeling of value or *aesthetic measure* (M) that rewards this effort.
3. The verification that the object is characterized by certain harmony, symmetry or *order* (O), which seems to be necessary for the aesthetic effect.

From this analysis of the aesthetic experience, Birkhoff suggested that the aesthetic feelings stem from the harmonious interrelations inside the object and that the aesthetic measure is determined by the *order* relations in the aesthetic object. As we will see below, different versions of the aesthetic measure try to capture mainly the order in the object, while the complexity fundamentally plays a normalization role.

On the other hand, it is not realistic to expect that a mathematical theory would be able to explain the complexities of the aesthetic experience [SB93]. Birkhoff recognized the impossibility of comparing objects of different classes and accepted that the aesthetic experience depends on each observer. Hence, he proposed to restrict the group of observers and to only apply the measure to similar objects. Excellent overviews of the history of the aesthetic measures can be found in the reports of Greenfield [Gre05] and Hoenig [Hoe05] which were presented in the first Workshop on Computational Aesthetics. From this point on, this paper will focus on the *informational aesthetics* perspective.

Using information theory, Bense [Ben69] transformed Birkhoff's measure into an informational measure: redundancy divided by statistical information (entropy). According to Bense, in any artistic process of creation, we have a determined *repertoire* of elements (such as a palette of colors, sounds, phonemes, etc.) which is *transmitted* to the final *product*. The creative process is a selective process (i.e, to create is to select). For instance, if the repertoire is given by a palette of colors with a probability distribution, the final product (a painting) is a selection (a realization) of this palette on a canvas. In general, in an artistic process, order is produced from disorder. The distribution of elements of an aesthetic state has a certain *order* and the repertoire shows a certain *complexity*. Bense also distinguished between a global complexity, formed by partial complexities, and a global order, formed by partial orders. His contemporary Moles [Mol68] considered order expressed not only as redundancy but also as the degree of predictability.

Nake [Nak05], one of the pioneers of the *computer or algorithmic art* (i.e., art explicitly generated by an algorithm), considers a painting as a hierarchy of signs, where at each level of the hierarchy the statistical information content could be determined. He conceived the computer as a *Universal Picture Generator* capable of "creating every possible picture out of a combination of available picture elements and colors" [Nak74].

Different authors have introduced several measures with the purpose of quantifying aesthetics. Koshelev et al. [KKY98] consider that the running time $t(p)$ of a program p which generates a given design is a formalization of Birkhoff's complexity C, and a monotonically decreasing function of the length of the program $l(p)$ (i.e., Kolmogorov complexity) represents Birkhoff's order O. Thus, looking for the most attractive design, the aesthetic measure is defined by $M = 2^{-l(p)}/t(p)$. For each possible design, they define its "beauty" as the smallest possible value of the product $t(p)2^{l(p)}$ for all possible programs that generate this design. Machado and Cardoso [MC98] established that an aesthetic visual measure depends on the ratio between *image complexity* and *processing complexity*. Both are estimated using real-world compressors (jpg and fractal, respectively). They consider that images that are simultaneously visually complex and easy to process are the images that have a higher aesthetic value. Svangård and Nordin [SN04] use the universal similarity metric (7) to predict how interesting new images will be to the observer, based on a library of aesthetic images. To compute this measure, they also use a combination of different compressors (8).

4. Global Aesthetic Measures

Next, we present a set of measures to implement, from an informational perspective, the Birkhoff's aesthetic measure applied to an image. We distinguish two kinds of measures: the global ones (Sec. 4) and the compositional ones (Sec. 5).

For a given color image $\mathcal{I}$ of N pixels, we use an sRGB color representation (i.e., a tristimulus color system) and an alphabet $\mathcal{A}$ of 256 symbols (8 bits of information) for channel (i.e., 24 bits per pixel). The luminance Y_{709} (0..255) will be also used as a representative value of a pixel (it is a perceptual function of the importance of the pixel color). The probability distributions of the random variables X_r, X_g, X_b, and X_ℓ are obtained from the normalization of the intensity histogram of the corresponding channel (R, G, B, and luminance, respectively). The maximum entropy or uncertainty for these random variables is $\mathsf{H} = \log|\mathcal{A}| = 8$. Thus, the following properties are fulfilled:

- $0 \leq H(X_r), H(X_g), H(X_b) \leq \mathsf{H}$
- $0 \leq H(X_\ell) \leq \mathsf{H}$
- $0 \leq H_{rgb} \leq \log|\mathcal{A}|^3 = 3\mathsf{H} = \mathsf{H}_{rgb}$,

where $H_{rgb} = H(X_r) + H(X_g) + H(X_b)$ is an upper bound of the joint entropy $H(X_r, X_g, X_b)$.

Throughout this paper, the following notions are used:

- *Repertoire*: palette of colors. We assume that is given by the normalized histogram of the luminance values of the image (X_ℓ) or the respective normalized histograms of the RGB values (X_r, X_g, and X_b).
- H_p: entropy of the repertoire or uncertainty of a pixel.
- NH_p: uncertainty or *information content* of an image.
- K: Kolmogorov complexity of an image. The use of this measure implies to take into account an additional constant (see [LV97]).

The measures presented will be applied to the set of paintings shown in Fig. 1. Their sizes are given in Table 1, where we have also indicated the size and rate of compression achieved by two real-world compressors: jpg and png. We select these two compressors as representative examples of lossy and lossless compression, respectively. The jpg compressor will be used with maximum quality.

4.1. Shannon Perspective

As we have seen in Sec. 3, the redundancy was proposed by Bense and Moles to measure the *order* in an aesthetic object. For an image, the *absolute redundancy* $\mathsf{H} - H_p$ expresses the reduction of uncertainty due to the choice of a given repertoire instead of taking a uniform palette. The *relative redundancy* is given by

$$M_H = \frac{\mathsf{H} - H_p}{\mathsf{H}}. \tag{10}$$

From a coding perspective, this measure represents the gain obtained using an optimal code to compress the image (4). It expresses one aspect of the creative process: the reduction of uncertainty due to the choice of a palette.

Table 2 shows M_H for the set of paintings in Fig. 1, where the entropy of a pixel has been computed using the luminance ($H_p \equiv H(X_\ell)$). We can observe, as expected, how a high redundancy is reflected in Mondrian's paintings while low values appear in the Pollock and van Gogh ones. Note that the M_H value for `Pollock-2` stands out due to a more homogeneous repertoire than the other two Pollock's paintings.

4.2. Kolmogorov Perspective

From a Kolmogorov complexity perspective, the *order* in an image can be measured by the normalized difference between the image size obtained using a constant code for each color (i.e., the maximum size of the image) and the Kolmogorov complexity of the image:

$$M_K = \frac{N\mathsf{H}_{rgb} - K}{N\mathsf{H}_{rgb}}. \tag{11}$$

Due to the non-computability of K, real-world compressors are used to estimate it. The complementary of this measure corresponds to the *compression ratio*. It is an adimensional value in [0,1] that expresses the degree of order of the image without any a priori knowledge on the palette (the higher the order of the image, the higher the compression ratio). In practice, this measure could be negative due to the presence of an additive compression constant.

In Table 2, M_K has been calculated for the set of paintings using the jpg and png compressors. Note that the use of M_K alters the ranking obtained by M_H. The compressors take advantage of the degree of order in an image, being detected and used in different ways within the process of compression. A radical case corresponds to `Pollock-2`, that switches from the lowest position for jpg to the fourth one for png. This could be due to the fact that, being jpg a lossy compressor, it can more easily detect regular patterns.

4.3. Zurek Perspective

Using the concept of physical entropy (9), we propose a new aesthetic measure given by the ratio between the *reduction of uncertainty* (due to the compression achieved) and the initial *information content* of the image:

$$M_S = \frac{NH_p - K}{NH_p}. \tag{12}$$

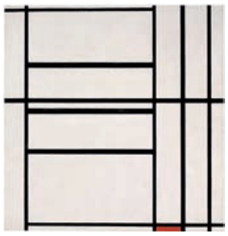
This ratio quantifies the degree of order created from a given palette. It is an adimensional value in [0,1], but in practice it could be negative, similarly to the M_K case.

In Table 2, values M_S have been computed using the jpg and png compressors. We have considered $H_p \equiv H_{rgb}$ for coherence with the compressors that use the knowledge of all the channels. In our experiments, Mondrian's and Pollock's paintings correspond to the highest and lowest values in the png case, respectively. The values near zero in the Pollock's paintings denote their high randomness. In the jpg case, the ordering of some paintings has changed due to the higher capacity of jpg of detecting patterns. For instance, this explains that the value of `vanGogh-3` is lower than the value of `Pollock-1`.

The plots shown in Fig. 2 express, for three paintings, the *evolution* of the physical entropy as we make more and more measurements. To do this, the content of each painting is progressively discovered (by columns from left to right), reducing the missing information (Shannon entropy) and compressing the discovered one (Kolmogorov complexity). Observe the higher compression obtained with jpg (lossy) compressor compared with png (lossless) one. As we expected, Mondrian's paintings show a greater order than van Gogh's and Pollock's ones.

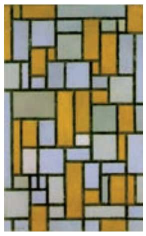
5. Compositional Aesthetic Measures

In this section, we analyze the behavior of two new implementations of Birkhoff's measure using different decompositions of an image.



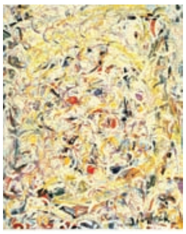
(*a.1*) Composition with Red
Piet Mondrian, 1938-1939

(*a.2*) Composition with Red, Blue,
Black, Yellow, and Gray
Piet Mondrian, 1921

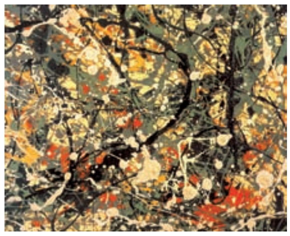


(*a.3*) Composition with Gray and
Light Brown
Piet Mondrian, 1918

(*b.1*) Number 1
Jackson Pollock, 1950



(*b.2*) Shimmering Substance
Jackson Pollock, 1946



(*b.3*) Number 8 (detail)
Jackson Pollock, 1949

(*c.1*) The Starry Night
Vincent van Gogh, 1889

(*c.2*) Olive Trees with the Alpilles
in the Background
Vincent van Gogh, 1889

(*c.3*) Wheat Field Under
Threatening Skies
Vincent van Gogh, 1890

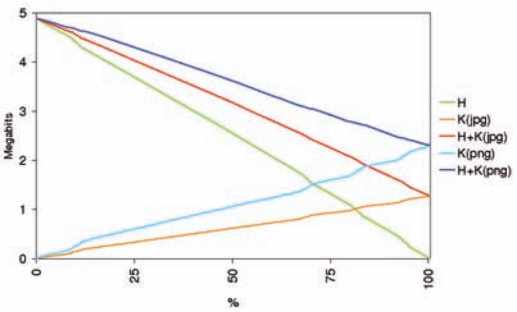
Figure 1: *Set of paintings.*

Image				jpg		png	
Fig.	*Painting*	*Pixels*	*Size*	*Size*	*Ratio*	*Size*	*Rate*
1.(*a.1*)	`Mondrian-1`	316888	951862	160557	0.169	290073	0.305
1.(*a.2*)	`Mondrian-2`	139050	417654	41539	0.100	123193	0.295
1.(*a.3*)	`Mondrian-3`	817740	2453274	855074	0.349	1830941	0.746
1.(*b.1*)	`Pollock-1`	766976	2300982	1049561	0.456	2255842	0.980
1.(*b.2*)	`Pollock-2`	869010	2609178	1377137	0.528	2355263	0.903
1.(*b.3*)	`Pollock-3`	899300	2699654	1286395	0.477	2596686	0.962
1.(*c.1*)	`vanGogh-1`	831416	2495126	919913	0.369	2370415	0.950
1.(*c.2*)	`vanGogh-2`	836991	2511850	862274	0.343	2320454	0.924
1.(*c.3*)	`vanGogh-3`	856449	2570034	1203527	0.468	2402468	0.935

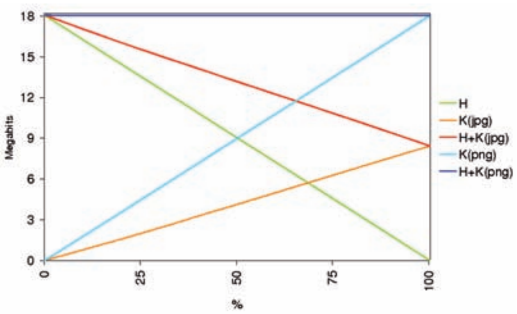
Table 1: *Data for paintings in Fig. 1. The sizes (bytes) for their original and compressed files (jpg and png) are shown. The respective compression ratios achieved are also indicated.*

Image	*Shannon*					jpg		png	
Code	$H(X_\ell)$	$H(X_r)$	$H(X_g)$	$H(X_b)$	M_H	M_K	M_S	M_K	M_S
`Mondrian-1`	5.069	5.154	5.072	5.194	0.366	0.831	0.737	0.695	0.525
`Mondrian-2`	6.461	6.330	6.514	6.554	0.192	0.900	0.877	0.705	0.635
`Mondrian-3`	7.328	7.129	7.343	6.421	0.084	0.651	0.600	0.254	0.143
`Pollock-1`	7.874	7.891	7.869	7.801	0.016	0.544	0.535	0.020	0.001
`Pollock-2`	7.091	6.115	7.133	7.528	0.114	0.472	0.390	0.097	-0.044
`Pollock-3`	7.830	7.305	7.866	7.597	0.021	0.523	0.497	0.038	-0.015
`vanGogh-1`	7.858	7.628	7.896	7.712	0.018	0.631	0.619	0.050	0.018
`vanGogh-2`	7.787	7.878	7.766	7.671	0.027	0.657	0.647	0.076	0.049
`vanGogh-3`	7.634	7.901	7.670	7.044	0.046	0.532	0.503	0.065	0.008

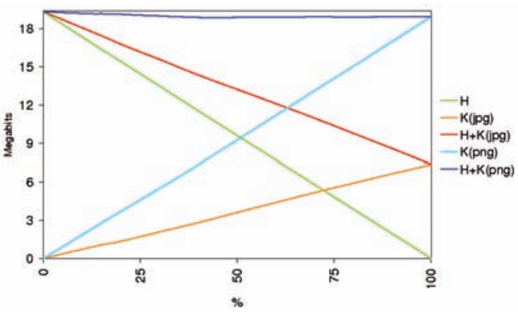
Table 2: *Aesthetic measures (M_H, M_K, and M_S) for the paintings in Fig. 1 with their respective entropies.*



(*a*) `Mondrian-1`



(*b*) `Pollock-1`



(*c*) `vanGogh-1`

Figure 2: *Evolution of the physical entropy (missing information + Kolmogorov complexity) for three paintings shown in Fig. 1.*

5.1. Shannon Perspective

From a Shannon perspective, we present here an implementation of the aesthetic measure based on the degree of order captured by a determined decomposition of the image.

To analyze the order of an image, we use a partitioning algorithm based on mutual information [RFS04]. Given an image with N pixels and an intensity histogram with n_i pixels in bin i, we define a discrete information channel where input X represents the bins of the histogram, with probability distribution $\{p_i\} = \{\frac{n_i}{N}\}$, and output Y the pixel-to-pixel image partition, with uniform distribution $\{q_j\} = \{\frac{1}{N}\}$. The conditional probability $\{p_{j|i}\}$ of the channel is the transition probability from bin i of the histogram to pixel j of the image. This *information channel* is represented by

$$\{p_i\} \quad X \stackrel{\{p_{j|i}\}}{\longrightarrow} Y \quad \{q_j\} \tag{13}$$

To partition the image, a greedy strategy which splits the image in quasi-homogeneous regions is adopted. The procedure takes the full image as the unique initial partition and progressively subdivides it, in a binary space partition (BSP) or quad-tree, chosen according to the maximum mutual information gain for each partitioning step. The algorithm generates a partitioning tree $T(\mathcal{I})$ for a given ratio of mutual information gain or a predefined number of regions (i.e, leaves of the tree, $L(T(\mathcal{I}))$).

This process can also be visualized from

$$H(X) = I(X,\widehat{Y}) + H(X|\widehat{Y}) = I(X,\widehat{Y}) + \sum_{i \in L(T(\mathcal{I}))} p_i H_i(X_i), \tag{14}$$

where p_i and $H_i(X_i)$ correspond, respectively, to the area fraction of region i and the entropy of its normalized histogram. The acquisition of information increases $I(X,\widehat{Y})$ (5) and decreases $H(X|\widehat{Y})$, producing a reduction of uncertainty due to the equalization of the regions. Observe that the maximum mutual information that can be achieved is $H(X)$.

We consider that the resulting tree captures the structure and hierarchy of the image, and the mutual information

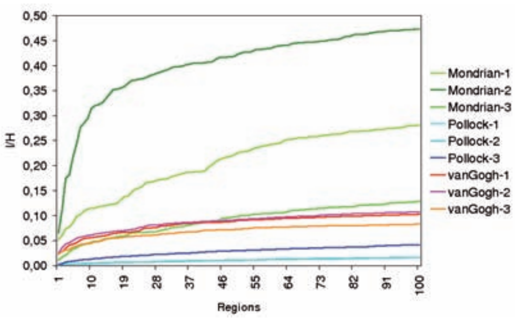


Figure 3: *Evolution of ratio M_I for the set of paintings in Fig. 1.*

gained in this decomposition process quantifies the degree of *order* from an informational perspective. In other words, the mutual information of this channel measures the capacity of an image to be ordered or the feasibility of decomposing it by an observer.

Similarly to Bense's communication channel between the repertoire and the final product, the channel (13) can be seen as the information (or communication) channel that expresses the *selection* of colors on a canvas. Hence, given an initial entropy or uncertainty of the image, the evolution of the ratio $I(X,\widehat{Y})/H(X)$, represents this selection process:

$$M_I(r) = \frac{I(X_\ell,\widehat{Y}_r)}{H(X_\ell)}, \tag{15}$$

where r is the resolution level (i.e., number of regions desired or, equivalently, number of leaves in the tree), $X \equiv X_\ell$, and $\widehat{Y}_r$ is the random variable defined from the area distribution. It is an adimensional ratio in [0,1].

In Fig. 3 we show the evolution of M_I for the set of paintings. Observe that the capacity of extracting order from each painting coincides with the behavior expected by an observer. Note the grouping of the three different painting styles (plots of `Pollock-1` and `Pollock-2` are overlapped). In Fig. 4, the resulting partitions of three paintings are shown for $r = 10$.

5.2. Kolmogorov Perspective

The degree of order of an image is now analyzed using a similarity measure between its different parts. To do this, given a predefined decomposition of the image, we compute the average of the normalized information distance (7) between each pair of subimages:

$$M_D(r) = 1 - \mathrm{avg}_{1\le i<j\le r}\{NID(i,j)\}, \tag{16}$$

where r is the number of regions or subimages provided by the decomposition, and $NID(i,j)$ the distance between the subimages $\mathcal{I}_i$ and $\mathcal{I}_j$. This value ranges from 0 to 1 and expresses the order inside the image.

(*a*) `Mondrian-2` (*b*) `Pollock-2`

(*c*) `vanGogh-2`

Figure 4: *Decomposition obtained for $r = 10$ using the partitioning algorithm based on the maximization of mutual information. The corresponding values M_I for each image are 0.314, 0.005, and 0.061, respectively.*

Image	jpg	png
Code	M_D	M_D
`Mondrian-1`	0.208	0.104
`Mondrian-2`	0.299	0.099
`Mondrian-3`	0.148	0.038
`Pollock-1`	0.129	0.028
`Pollock-2`	0.123	0.026
`Pollock-3`	0.128	0.023
`vanGogh-1`	0.140	0.021
`vanGogh-2`	0.142	0.020
`vanGogh-3`	0.123	0.022

Table 3: *The values M_D for the set of paintings in Fig. 1.*

In Table 3, the values M_D for the set of paintings are calculated using a 3×3 regular grid and $NCD(i,j)$ (8) as an approximation of $NID(i,j)$. Note that, like the previous compositional measure (15), the paintings are classified according to the author (specially in the png case).

6. Conclusions

In this paper, the concepts of order and complexity in an image have been analyzed using the notions of Shannon entropy and Kolmogorov complexity. Different ratios based on these measures have been studied for different decompositions of the image. A new version of Birkhoff's aesthetic measure has also been presented using Zurek's physical entropy. This new measure allowed us to introduce a simple formalization of the creative process based on the concepts of uncertainty reduction and information compression.

The analysis presented can be extended in two different lines. On the one hand, the zeroth order measures used in this paper can be extended to higher order measures such as entropy rate and excess entropy of an image [BFBS06, FC03]. These measures can be used to quantify, respectively, the irreducible randomness and the degree of structure of an image. On the other hand, following Zurek's work, the artistic process can be analyzed from the viewpoint of a Maxwell's demon-type artist or observer [Zur89].

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