Human Activity Modeling on Shape Manifold

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Abstract

In this paper we propose a stochastic modeling of human activity on shape manifold. From a video sequence, human activity are extracted as a sequence of shape. Such sequence is considered as one realization of a random process on shape manifold. Then Different activity is modeled by manifold valued random process with different distribution. To solve the stochastic modeling on manifold, we first map the process on the shape manifold to a Euclidean process. Then the process is modeled by linear models such as stationary incremental process and a piecewise stationary incremental process. The mapping from manifold valued process to Euclidean process is known as stochastic development. The idea is to parallelly transport the tangent of curve on manifold to a single tangent space. The advantage of such technique is the one to one correspondence between the process in flat space and the one on manifold. The proposed algorithm is tested on two activity data base [RS01] [BGSB05]. The result demonstrate the high accuracy of our modeling in characterizing different activities.

Categories and Subject Descriptors (according to ACM CCS): I.2.10 [Vision and Scene Understanding]: Motion— Shape Modeling and recovery of physical attributes

1. Introduction

Human activity recognition is of interest in a wide range of applications, spanning areas such as security surveillance, person identification and content-based image retrieval. In addition to security applications, the explosively increasing daily usage of video cameras has and continues to motivate a great interest in motion analysis and understanding in video for diverse applications. Recent progress on human activity analysis from video data has been well documented in [PRSV08] [AC99].

As well remarked in [EL04], the observability of human activity is restricted by the body's configuration. This implies that for a specific representation, the space of human activity is not necessarily linear. Of the various possible nonlinear representations of human activity [VCC04], we choose to view any given activity of interest as a shape sequence [BGSB05] [VRCC05].

Different shape representations lead to different shape manifolds. In Kendall's shape theory [Ken84], a shape is considered to be a set of land marks on the boundary of an object. The Kendall pre-shape space is a quotient space (where translation and scaling were quotiented out) and geometrically as a hyper sphere. An AR/ARMA model of hu-

man activities was proposed in [VRCC05] by projecting the shape sequences onto the tangent space of Kendall's pre-shape space. To overcome the problem of systematically picking consistent landmarks of shapes, we consider a shape as a simple and closed planar curve. Such a shape formulation was first proposed in [SL02] and further developed in [KSWM04] with a numerically efficient computation of a geodesic metric between two shapes. With a similar goal of statistical classification of shape sequences as in [VRCC05], our goal here is to first develop a stochastic model of a curve evolution (representing a shape sequence trajectory on a manifold) on a non-linear space by regressing the problem onto a linear space. In [VRCC05] a shape on a preshape sphere is projected onto a tangent space at the mean shape. Adopting such a tangent approximation is, however, valid in only a sufficiently small neighborhood. The invertibility of such a projection on pre-shape sphere only holds when the shape sequence does not cross the "north or south poles" of the hypersphere. Generally on a smooth manifold, the condition of such an orthogonal projection is restricted to a local area of the manifold. In contrast, our proposed regression is intrinsically constructed by a curve development [BC94] as a 1-1 mapping an evolution curve on any smooth manifold to a curve in a flat space. Intuitively, our proposed method,

seeks to exploit all the tangent spaces along the curve on the manifold in order to construct the mapping to flat space. The curve's tangents at different tangent spaces of the manifold are parallelly transported to a single tangent space. By integration, we proceed to obtain the called curve development in a tangent space.

We exploit the afore-described approach to develop in this paper, an intrinsic stochastic model with a goal to classify activities. Assuming a proper human silhouette segmentation[†] of each frame in a video sequence of interest, a specific activity may be summarized by a sequence of individual closed curves/shapes in a form of an evolution curve on the underlying shape manifold. An reasonable modeling for a activity, for instance like "running", are expected to describe the different data samples of "running". In this paper, the set of the different representative curves of "running" are viewed as the realizations of the "running process", which more precisely is a random process on the shape manifold. As a result, any activity process of interest, may hence be modeled as a manifold valued random process.

Using the development in [Hsu01], any random process on a finite dimensional manifold may be considered as the solution of some stochastic differential equation $SDE(X_0, V, Z)$ defined on the manifold. In other words, the space of human activities in the same class, is modeled by a solution space of $SDE(X_0, V, Z)$. It follows that for a certain class of activities, the vector field V and the driving process Z characterize the dynamic information on the manifold.

A direct analysis of a manifold valued SDE is, however very challenging. Such a problem is solved in [Hsu01] by first mapping a random process on a manifold to a Euclidean space, where the stochastic analysis is then carried out. Such mapping from manifold to flat space consists of two steps: a horizontal lift is first completed, followed by a curve development (the definition of these differential geometry concepts will be provided in section 2). The advantage of such a framework is in achieving a 1-1 correspondence of the regression from the manifold to the Euclidean space. By constructing a flat connection on the shape manifold, we show that the solution space of $SDE(X_0, V, Z)$ is in 1-1 correspondence to that of $SDE(X_0, U, W)$ in the linear frame bundle based on the shape manifold (see details below). It is much more convenient to work with $SDE(X_0, U, W)$, because the vector field U is a known unique to X_t. The resulting equation is only characterized by the Euclidean valued driving process W, which is called a stochastic curve development.

In the balance of this paper, we first provide a brief (but sufficient for this development) introduction to manifold geometry and to stochastic analysis on manifolds. The preprocessing on shapes is introduced in Section **??**, to make the

shape manifold finite dimensional. In Section 3, we introduce the stochastic curve development for a human activity process as a mapping from a manifold to a flat space. In Sections 4 and 5, we construct a connection on the shape manifold, and derive the corresponding curve development result for a given human activity.

2. Background

To study human activity as a random process on a shape manifold, we use the shape manifold in [KSWM04]. We numerically compute a connection on shape manifold to map a random process from a manifold to a flat space as in [Hsu01]. In the end, our statistical modeling in the flat space helps achieve activity classification. To proceed, we first provide in Section a brief review of the required background in differential geometry to allow us to define a shape manifold as a working space. We also describe the required tools of parallel transportation and curve development on a shape manifold. For clarity and easy cross-referencing, we summarize the notation adopted throughout the paper in Table 2.

2.1. Shape Manifold

According to [KSWM04], a planar shape is a simple and closed curve in $\mathbb{R}^2$

$$\alpha(s) : I \to \mathbb{R}^2, \quad (1)$$

where an arc-length parameterization is adopted. A shape is represented by a *direction index function* $\theta(t)$. With such a parameterization, $\theta(s)$ may be associated to the shape by

$$\frac{\partial \alpha}{\partial s} = e^{j\theta(s)}. \quad (2)$$

The ambient space of the manifold of θ is an affine space based on $\mathbb{L}^2$. Thus we have,

$$\theta \in A(\mathbb{L}^2). \quad (3)$$

The restriction of a shape to be a closed and simple curve, and invariant over rigid Euclidean transformations. The shape manifold M is defined by a level function ϕ as ,

$$\phi(\theta) = (\int_0^{2\pi} \theta ds, \int_0^{2\pi} \cos(\theta) ds, \int_0^{2\pi} \sin(\theta) ds) \quad (4)$$

$$M = \phi^{-1}(\pi, 0, 0) \quad (5)$$

One of the most important properties of M is that the tangent space TM is well defined. Such a property not only simplifies the analysis, but also makes possible the incremental computation,

$$T_\theta M = \{f \in \mathbb{L}^2 | f \perp span\{1, \cos(\theta), \sin(\theta)\}\} \quad (6)$$

[†] Note that errors in segmentation clearly imply errors down the processing stream, and this investigation is a research topic in and of itself, and is left for future work.

Table 1: *Notation Table*

M	a manifold
H	connection as horizontal field in principle fiber bundle
f	a mapping between manifolds
f_*	a push forward between the corresponding tangent spaces
γ	a curve on manifold
γ_*	the tangent along the curve γ
$\tilde{\gamma}$	a horizontal lift of curve γ
β	a curve development of γ in Euclidean space
V	a vector field defiend on a manifold
$\tilde{V}$	a horizontal lift of vector field V
(P,G,M)	a principal fiber bundle P based on the manifold M with the group actor G
$\mathbb{F}(M)$	linear frame bundle
X	a manifold valued random process
W	a stochastic development of X, a Euclidean valued random process
U	a horizontal lift of X, a Fiber Bundle valued random process

2.2. Connection on Manifold

To study a curve on manifold, we should first overcome the complexity due to the curvature of the manifold. For example, a geodesic in a flat space may simply be described by a straight line. On a shape manifold, however, the "straightness" of a curve is not as straight forward as that in the Euclidean space. In differential geometry, one defines "straightness" by the covariant derivative along a curve. A curve is a geodesic if the covariant derivative of a curve's tangent along the curve is zero everywhere. More generally, the concept of a connection defines the relation between a tangent space along a curve on a manifold. Such a relation may be used to determine the parallelism of two tangent vectors in two different tangent spaces. A connection may be defined in ways to be written in different forms, one of which we exploit to develop our stochastic modeling of a shape sequence on a manifold. Specifically, we proceed to describe the parallelism between tangent spaces by specify parallel of basis pair at different tangent space. Such a form of connection is much simpler to calculate and takes place in a so-called principal fiber bundle as a horizontal vector field.

Definition 1 (Principal Fiber Bundle) A principal fiber bundle is a set (P,G,M), where P,M are C^∞ manifolds, and G is a Lie group such that
(1) G acts freely on the right of P, $P \times G \to P$. For $g \in G$, we shall also write R_g for the map $g : P \to P$
(2) M is the quotient space of P by an equivalence relation under G (any shape subjected to a $g \in G$ is equivalent to itself), and the projection $\pi : P \to M$ is C^∞, so for $m \in M$, G is simply transitive on $\pi^{-1}(m)$
(3) P is locally trivial

One example of a principal bundle is the linear frame bundle $\mathbb{F}(M)$, which will be extensively used in this paper. For any point u in $\mathbb{F}(M)$ can be written as

$$u = (x,b) \qquad (7)$$

where $x \in M$ and $b = e_1, e_2, ..., e_n$ is a basis of the associated tangent space T_xM. The group G acting on a fibre is $GL(n))$

Definition 2 (Connection) A connection on the principal bundle (P,G,M) is a d-dimensional distribution H on P, where $d = dim(M)$, such that
(1) $H \in C^\infty$
(2) for every $p \in P$, $H_p + V_p = P_p$
(3) for every $p \in P$, $g \in G$, $(R_g)_* H_p = H_{pg}$.
where V_p is the vertical subspace, $Y \in P_p$ is vertical if and only if $\pi_* Y = 0$

With the definition of a connection on a manifold in hand, we can achieve a horizontal lift from a manifold to a linear frame bundle. More specifically, in this operation we find a "horizontal" curve in a linear frame bundle corresponding to a curve on a manifold. As a result, we discover the appropriate distribution[‡] of bases along the curve accomplishing the parallel transport of a shape to its next evolution in the sequence. With the parallel bases along a curve, the tangent of a curve can be represented by its corresponding coefficients. By preserving the coefficients, we can move all the tangents to a single tangent plane. Such an operation is called a parallel transport along a curve. If we reintegrate all the parallelly transported tangents in that single tangent plane, we have a curve in the flat space. This is referred to as *a curve development* in the tangent plane.

[‡] In geometry on manifolds, a distribution is a collection of subspaces of tangent spaces.

In a manifold equipped with a connection, we can construct under certain conditions a 1-1 mapping from a curve on the manifold to a curve in the principal fiber bundle. We subsequently facilitate the curve development onto $\mathbb{R}^d$. In Figure 1, we illustrate the definition of H in one particular principal fiber bundle, the frame bundle $\mathbb{F}(M)$. H_p could be understood as a subspace of $T_p\mathbb{F}(M)$ which is smoothly defined for all $p \in \mathbb{F}(M)$.
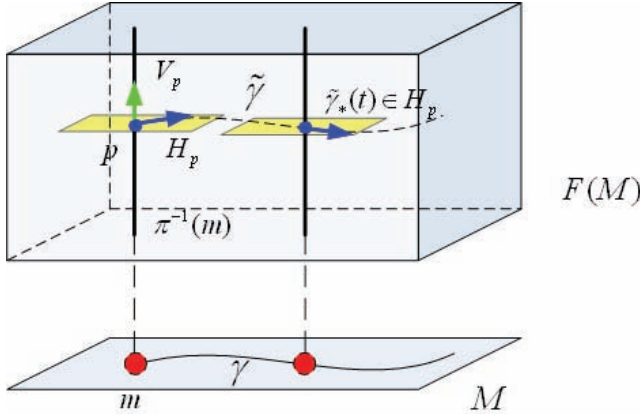
Figure 1: *A connection H as a distribution in $F(M)$, where the yellow plane represents the H_p as the subspace of $T_pF(M)$. $\tilde{\gamma}$ is the horizontal lift of curve γ on manifold M. $\tilde{\gamma}_*$ is the tangent along $\tilde{\gamma}$, such that $\forall t, \tilde{\gamma}_* \in H_{\gamma(\tilde{t})}$*

Definition 3 (Horizontal Lift) Let γ be a piecewise C^∞ curve in M, $\gamma : [0,1] \rightarrow M$. Let $p \in \pi^{-1}(\gamma(0))$. Then there exists a unique lift $\tilde{\gamma}$ of γ such that $\tilde{\gamma}_* \in H$ and $\tilde{\gamma}(0) = p$

Definition 4 (Curve Development) Let γ be a piecewise C^∞ curve in M starting at m, $b = (m, e_1, e_2, ..., e_N)$ a basis at m. Let $\tilde{\gamma}$ be the horizontal lift of γ in $\mathbb{F}(M)$. Then by writing,

$$\beta = \int_0^t \tilde{\gamma}^{-1} \gamma_* ds \qquad (8)$$

where

$$(\tilde{\gamma}^{-1} \gamma_*)_i(s) = < e_i(s), \gamma_*(s) > \qquad (9)$$

we define a curve in $\mathbb{R}^N$ which is called a development of X into $\mathbb{R}^N$ with respect to b. And $b \circ \beta$ is a curve in T_mM called the development of X in T_mM. $b \circ Y$ is independent of of the choice of basis b at m. A conceptual distribution is demonstrated in Figure (2)

Definition 5 (Parallel Transport) Let γ be a piecewise C^∞ curve in M equipped with a connection H. Let $\tilde{\gamma}$ be the horizontal lift of γ. Let V be the vector field defined along the curve γ. Thus the parallel transport of $V(\gamma(t))$ from $\gamma(t)$ to $\gamma(t+h)$ can be defined as the mapping between tangent planes $\tau : T_{t1}M \rightarrow T_{t2}M$,

$$\tau_h(V(\gamma(t))) = \tilde{\gamma}(t+h) \circ \tilde{\gamma}(t)^{-1} \circ V(\gamma(t)) \qquad (10)$$
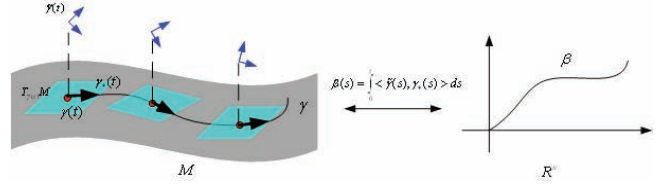
Figure 2: *The curve development of γ is $\int_0^t < \tilde{\gamma}(s)^{-1}, \gamma_*(s) > ds$, where $\tilde{\gamma}(s)^{-1}, \tilde{\gamma}(s)$ are the dual basis and basis of $T_{\gamma_s}M$. γ_* is the tangent along $\gamma \in M$*

A more expanded and detailed discussion of connections may be found in [BC94] [KN96].

2.3. Stochastic Horizontal Lift and Development

The horizontal lift and curve development mentioned in the last section may also be extended to a stochastic setting. According to [Hsu01], a stochastic horizontal lift and its development may be defined as follows,

Definition 6 (Stochastic Horizontal Lift and Development) (1) An $\mathbb{F}(M)$-valued semimartingale U is said to be horizontal if there exists an $\mathbb{R}^d$-valued semi-functional W such that,

$$dU_t = \sum_{i=1}^d H_i(U_t) \circ dW_t^i \qquad (11)$$

where H_i is the fundamental horizontal vector field that span H.
(2) Let W be an R^d-value semimartingale and U_0 an $\mathbb{F}(M)$-valued, F_0-measurement random variable. The solution U of SDE (11) is called the stochastic development of W in $\mathbb{F}(M)$. Its projection $X = \pi U$ is called a stochastic development of W in M.
(3) Let X be an M-valued semimartingale. An $\mathbb{F}(M)$-value horizontal semimartingale U such that its projection $\pi U = X$ is called a stochastic horizontal lift of X.

With the horizontal lift, a random process X_t on a manifold may be lifted to a random process U_t in an associated frame bundle. By the development, the random process in the frame bundle may then be mapped to a Euclidean process W. Under fixed initial conditions, such a two step operation will establish a 1-1 correspondence of a random process on a manifold to one in a flat space. More importantly, the well defined correspondence provides an alternative way to study a random process on a manifold as a more convenient analysis in the Euclidean space. A more detailed discussion of this development will be given in Section 3

3. Dynamic of Human Activity on Shape Manifold

In this paper, a human activity as captured in a video clip is represented by a shape/silhouette time sequence. Using the shape representation in Section 2, a shape sequence is hence

viewed as a path on a shape manifold. For a given activity, for example running, there are many different representative paths on the shape manifold. To construct a model which allows such a variability, we view these paths as realizations of an underlying random process on the manifold. The curvature of a manifold, however, generally renders the classical stochastic analysis in a flat space unapplicable.

Hsu in [Hsu01] proposes *an efficient analysis framework* to construct an invertible mapping from a manifold-valued random process to a Euclidean-valued random process. The essence of the mapping is to compute a Euclidean process that can drive a stochastic differential equation (SDE) to generate a manifold-valued semi-martingale.[§] In a Euclidean space the semi-martingales are much simpler to analyze. In contrast to the orthogonal projection method onto a tangent space around a mean, Hsu's theory provides a *one to one correspondence* between a process on a manifold and one on a Euclidean space. This improced accuracy of representation is primarily due to the so-called *"rolling without sliding" property* of a parallel transport.

As in [Hsu01], our mapping of a random process to a flat space proceeds composite in two steps: firstly, by a horizontal lift, i.e. a manifold-value process X_t may be mapped to a frame bundle valued random process U_t. Then by development, the frame bundle valued random process is mapped to a Euclidean valued random process. Any random process on the shape manifold, may then be written as a solution to some $SDE(V, X_0, Z)$. Generally we have

$$X_t = X_0 + \int_0^t \sum_i V_i(X_s) \circ dZ_s^i, \qquad (12)$$

where, X_0 is the initial condition, V_i is a smooth vector field defined on M and Z_s^i is a Euclidean valued random process driving Equation (12). The stochastic integration here is Stratonovich integration. More intuitively Equation 12 can be understood as $dX_t = \sum_i V_i(X_s) \circ dZ_s^i$. Thus the dynamic described by equation 12 is characterized by both vector field V and the driving process Z_t. However, the form of Euclidean process Z is variant to different selection of V. In contrast to our goal to construct a $1 - 1$ mapping from manifold process to Euclidean process, there is no one one correspondence such as $X \leftrightarrow Z$ without giving knowledge about V. Previous work in [Hsu01] solve this problem by setting V equal to the horizontal lift of X_t. Given the uniqueness of horizontal lift, the resulting driving process Z_t will have the

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[§] A martingale is a stochastic process X_t whose conditional expectation at some time t given the process up to $s \leq t$ is its expectation at s or $E(X_t|X_s), \forall s < t,$, is equal to $E(X_s)$. A semi-martingale is a decomposable random process into a local martingale together with a finite variation differentiable process starting at zero (e.g. a Brownian motion).

one one correspondence to X_t. Let vector field U_t be the horizontal lift of X_t in $F(M)$, Equation 12 may be rewritten as

$$X_t = X_0 + \int_0^t \sum_i U_s^i \circ dW_s^i. \qquad (13)$$

According to the definition of frame bundle and stochastic horizontal lift in section 2 we know

$$U_t = \{e_1, e_2, \cdots, e_i, \cdots, e_n\} \qquad (14)$$

where e_i is the basis of $T_{X_t}M$. In Equation 13 the differential dX_t is represented in a selected basis U_t with corresponding driving process W_t. For an orthogonal basis one can write ,

$$dW_t = U_t^{-1} \circ dX_t \qquad (15)$$

$\forall i = 1, 2, 3, \cdots$ Equation 15 can be represented in vector as

$$dW_t^i = <e_i, dX_t> \qquad (16)$$

Such rewrite of Equation 12 provides a representation of the random process X_t on a manifold in terms of the Euclidean random process W_t, which is the stochastic anti-development of X_t according to the definition in section 2.3. Given a connection H, U_t is uniquely determined for X_t up to a group action along the fiber $\pi^{-1}(X_t)$. So the mapping from X_t to W_t is a *1-1 mapping* if the initial condition $U_{t=0}$ is given.

In the above discussion, we provide a $1 - 1$ mapping from $X_t \in M$ to $W_t \in R^{dim(M)}$. The critical point for implementing such a mapping is the specific form of the connection H which we discuss in Section 4.

4. Flat Connection on a Shape Manifold

The construction of H is critical to the implementation of a horizontal lift and a curve development. In theory there may exist many different H for a given manifold. Once the concrete form of H is determined, the geometry of a manifold is specified accordingly. Among different kind of connection, we adopt the flat connection for the efficiency of calculate. The implementation of the flat connection H proceeds by constructing a smooth section $\sigma : M \to \mathbb{F}(M)$ in the linear frame bundle $\mathbb{F}(M)$. Let $\pi' : M \times G \to G$ be the projection from the linear frame bundle to the general linear group. Let $u = (m \in M, g \in GL(N)) \in F(M)$. We define π' as the following,

$$\pi'(u) = g\sigma(m)^{-1} \qquad (17)$$

We know given any two point u_1, u_2 along the fiber there exist a unique $g \in GL(n)$ such that

$$u_1 = R_g \circ u_2 \qquad (18)$$

So $\pi'(u)$ hence implies that the matrix transformation that maps the basis $\sigma(m)$ to g.

Given π', the flat horizontal vector field can be defined as the kernel of the pushforward of π' :

$$H_p = Ker(\pi'_*) \tag{19}$$

where $\pi'_* : TF(M) \to TG$ is the push forward of π'. To interpret equation 19, we zoom into a single point $u \in F(M)$. Let $u = (m_0, g_0)$. Let curve $\tilde{\gamma}(t) \in (m, \pi'(m_0)\sigma(m)) \subset F(M)$. Thus we have

$$\pi'(\tilde{\gamma}(t)) = \pi'(m_0) \tag{20}$$

Consequently,

$$\pi'_*(\frac{\partial \tilde{\gamma}(t)}{t}) = 0 \tag{21}$$

Thus $H_p(u)$ is actually the tangent on the subspace $(m, \sigma(m)\pi'(m_0)) \subset F(M)$.

In the implementation of the smooth section σ, we need to assign each point $\theta \in M$ a basis $\{e_k\}_{k=1,2,3...}$ for the tangent space $T_\theta M$. From Section 2, we know that the tangent space of M can be written as

$$T_\theta \hat{M} = \{v \in S | v \perp span\{1, \cos(\theta), \sin(\theta)\}\}. \tag{22}$$

The essential form of the basis in TM is a set of partial differential operators.[¶] Firstly the basis of TM is represented by the basis of $\{B_i\}$ in L^2, which is orthogonal to subspace $span\{1, \cos(\theta), \sin(\theta)\}$. Since $\hat{M}$ is the Fourier approximation of M, $\forall i, \{B_i\}$ is projected onto the Fourier approximation space S as BS_i. Then the basis of $T\hat{M}$ can be extracted as a linearly independent subset of bases functions BS_i. The details of this procedure implementation are as as follows,

One can show that the following set of functions is a linearly independent set,

$$\{1, cos\theta, sin\theta, ..., cosi\theta, sini\theta, ...\}, \tag{23}$$

Let B_i be the result of a Gram Schmidt orthogonalization of the above basis in ambient space as shown in figure 3

$$\{v_1, v_2, v_3, B_{i=1,2,3,...}\} = \\ ON\{1, cos\theta, sin\theta, ..., cosi\theta, sini\theta, ...\}$$

where $\{v_1, v_2, v_3\}$ are the first three basis vectors from the Gram Schmidt procedure which satisfy $span\{v_1, v_2, v_3\} = span\{1, \cos(\theta), \sin(\theta)\}$. These basis vectors are excluded because they are orthogonal to the tangent space of M. Then B_i is the ambient representation of the basis of $T_\theta M$.

As shown in figure 3, the orthogonal projection from L^2 onto S can be written as a Fourier approximation of B_i and

denoted by BS_i. Letting ϕ_j denote the Fourier basis functions, it follows that

$$BS_i = \sum_{j=1}^{N} \hat{B}_{i,j} \phi_j, \tag{24}$$

where

$$\hat{B}_{i,j} = <B_i, \phi_j> \tag{25}$$

for $0 < i < \infty$, $0 < j < N$. $\hat{B}$ is the submatrix composed of the first N columns of the full rank infinite matrix $< B_i, \phi_j >_{0<i,j<\infty}$. Thus $rank(\hat{B}) = N$, then there exists a $i(k)$ such that $0 < k < N$, and $\hat{B}_{i(k),j}$ are linearly independent. Then we can get a basis for $T_m \hat{M}$ by a Gram Schmidt procedure applied to $BS_{i(k)}$.

$$e_k = ON\{BS_{i(1)}, BS_{i(2)}, BS_{i(3)}, \cdots\}. \tag{26}$$
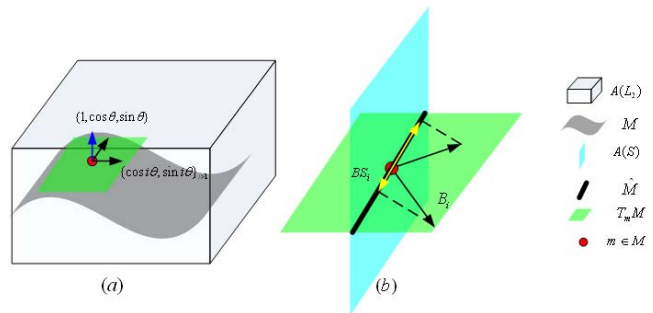
Figure 3: *implementation of σ : (a) the basis in the ambient space, which yields B_i as the result of Gram Schmidt procedure (b) the fourier approximation of B_i by BS_i.*

5. Stochastic Development of Human Activity

In the shape manifold M equipped with the flat connection H defined in Equation (19), the horizontal lift U_t of X_t with initial condition U_0 is computed as follows

$$U_t = R_{g(\sigma(X_0), U_0)} \circ \sigma(X_t) \tag{27}$$

In the ambient space $A(S)$, $\forall t, U_t, \sigma(X_t)$ can be represented by $N \times N$ invertible matrix. For example, $U_t = [e_1, e_2, \cdots, e_N]$, where $e_i \in R^N$ span the tangent space $T_m M$. In such setting U_t can be calculated as matrix multiplication in ambient space.
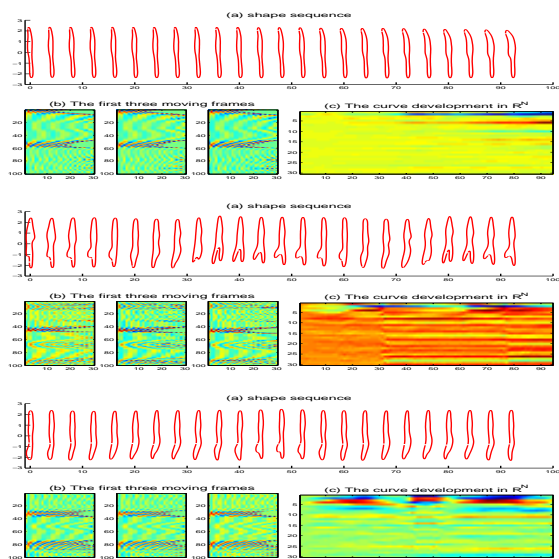
$$(U_t)_{ij} = \sum_k g(\sigma(X_0), U_0)_{ik} \hat{\sigma}(X_t)_{kj} \tag{28}$$

Consequently the development W_t of X_t can be written as,

$$W_t = \int_0^t (U_t)^{-1} dX_t \tag{29}$$

In Figure (4), a few numerical results are demonstrated for W_t for three activities: walking, running and bending.

As discussed in Section 3, under a specific connection

[¶] In the implementation computation, the real vectors in ambient space are used to represent the basis in TM.

Figure 4: *curve development of $X_t \in M$ in $\mathbb{R}^{N=30}$: (a) the original shape sequence represented by angle functions X_t (b) the horizontal lift U_1, U_2, U_3. (c) the development $(U_t)^{-1} dX_t$*

H, given the distribution of $U_0 \in \mathbb{F}(M)$, U is unique for a given X_t. Thus the driving process W_t fully characterize $SDE(X,U,W)$. Therefore human activity generated from $SDE(X,U,W)$ is only characterized by the driving process W_t which may be viewed as representative of the dynamics of the shape sequence of the random process on the manifold M.

6. Stochastic/Statistical Analysis in a Euclidean Space

As discussed in previous section 3, the random process X_t on the shape manifold is now mapped to a Euclidean random process W_t. The recognition problem of human activity is thus reduced to comparing different processes W_t in the flat space.

Given Euclidean process W_t obtained from the observed X_t, we analyze it as one with first order stationary increments. A self-covariance matrix K of the first increment dW_t of W_t is estimated from observations. We subsequently proceed to discuss activity classification by introducing a metric for K. Computing the increments of W_t to achieve stationarity also carries a potential of increasing the noise, particularly when the process is non-homogeneous. We leave the theoretical arguments on how to select the proper differentiation order for the future work and live with the first order increments in this paper.

In the following experiments, we run our algorithm on data bases: the activity classification database in [BGSB05]

The activity data in [BGSB05] includes 10 different activities. Each one has 9 video sequence for 9 different actors. The data base provide the background substraction results of video. We perform level set segmentation to extract the contour of shape. [CSYK09].

We first assume that the incremental process dW_t is stationary and ergodic. Then by the distribution of dW we can characterize different activities. We next proceed to estimate the covariance matrix $K(dW^i, dW^j)$ as the feature of choice for the underlying distribution.

$$K(dW^i, dW^j) = E((dW^i - E(dW^i))(dW^j - E(dW^j))) \tag{30}$$

where dW^i is the ith element of the vector dW. The distance between two different covariance matrices is defined by the Frobenius norm as,

$$D(K1,K2) = \|K1 - K2\|_F. \tag{31}$$

The results of $D(K1,K2)$ for the data base in [BGSB05] and CMU-Mobo [RS01] are shown in Figure 5. Using the distance matrix D, we may carry the recognition/classification task by using, for example, the leave-one-out algorithm. The nearest neighborhood algorithm is used for classification. If we let N_B be the total number of realizations of B. and $N(B,A)$ the number of realizations of B classified as A activity, we have

$$P(A|B) = \frac{N(B,A)}{N(B)} \tag{32}$$

Table 6 shows the recognition rate of both data bases.
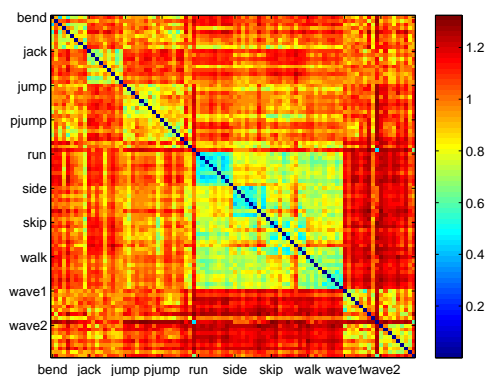
Figure 5: *Distance matrix of data base in [BGSB05];(a2)distance matrix of CMU Mobo data base;*

To perform a consistent comparison of results published in [BGSB05], we need to change our experiment to the same setting. In [BGSB05] the number of activity observations is

Table 2: *table of recognition rate. the number in () is the result in [BGSB05]*

$P(act1|act2)$	bend	jack	jump	pjump	run	side	skip	walk	wave1	wave2
bend	1	0	0	0	0	0	0	0	0	0
jack	0	1 (0.98)	0 (0.02)	0	0	0	0	0	0	0
jump	0	0(0.02)	1 (0.971)	0	0	0	0	0	0	0
pjump	0.0556	0	0	0.944(1)	0	0	0	0	0	0
run	0	0	0	0(0.108)	0.944(0.892)	0.0556	0	0	0	0
side	0	0	0	0	0	1	0	0	0	0
skip	0	0	0	0	0	0	1	0	0	0
walk	0	0	0	0(0.09)	0	0(0.09)	0	1(0.948)	0(0.35)	0
wave1	0	0	0	0(0.09)	0	0	0	0(0.019)	1(0.972)	0
wave2	0	0	0	0	0	0	0	0	0(0.09)	1(0.991)

increased by segmenting any video sequence for a given activity into many overlapped chunks. The segments are assumed independent and the classification is carried out. The performance of our proposed method is summarized in the following tables.

7. Conclusion

In this paper, we provide a systematic framework for the stochastic modeling of human activity on shape manifold. In theory, such framework is one one mapping from random process on manifold to the random process in Euclidean space. The implementation developed by adapting the general horizontal lift and curve development with flat connection to the particular shape manifold. The advantage of such mapping is that the problem of modeling on the original shape manifold is now regressed to the traditional modeling in Euclidean environment. In the resulted flat space, the representative random process of activity is modeled as stationary incremental process. The experiment on two different data bases well demonstrate the performance of the proposed modelings of activities.

References

[AC99] AGGARWAL J., CAI Q.: Human motion analysis: A reivew. *Computer Vision and Image Understanding 73* (1999), 428–440. 1

[BC94] BISHOP R. L., CRITTENDEN R. J.: *Analysis of planar shapes using geodesic paths on shape spaces*. Academic Press, 1994. 1, 4

[BGSB05] BLANK M., GORELICK L., SHECHTMAN E., BASRI M. I. R.: Action as space-time shapes. *IEEE ICCV* (2005). 1, 7, 8

[CSYK09] CHEN P., STEEN R., YEZZI A., KRIM H.: Joint brain parametric t1-map segmentation and rf inhomogeneity calibration. *International Journal of Biomedical Imaging* (2009). 7

[EL04] ELGAMMAL A. M., LEE C.-S.: Inferring 3d body pose from silhouettes using activity manifold learning. *Proceedings of the Conference on Computer Vision and Pattern Recognition* (2004), 681–688. 1

[Hsu01] HSU E. P.: *stochastic analysis on manifold,Graduate studies in mathematics*. American Mathematical Society, 2001. 2, 4, 5

[Ken84] KENDALL D.: Shape manifolds: Procrustean metrics and complex projective spaces. *Bull. London Math. Soc. 16* (1984), 81–121. 1

[KN96] KOBAYASHI S., NOMIZU K.: *Foundations of differential geometry*. John Wiley& Sons, 1996. 4

[KSWM04] KLASSEN E., SRIVASTAVA A., W MIO S. J.: Analysis of planar shapes using geodesic paths on shape spaces. *IEEE Trans. Pattern Analysis and Machine Intelligence 26* (2004), 372–383. 1, 2

[PRSV08] P T., R C., SUBRAHMANIAN V U. O.: Machine recognition of human activities: A survey. *IEEE Transactions on Circuits and Systems for Video Technology 18* (2008), 1473–1488. 1

[RS01] R.GROSS, SHI J.: The cmu motion of body (mobo) database. *Tech. Report CMU-RI-TR-01-18, Robotics Institute, Carnegie Mellon University* (2001). 1, 7

[SL02] SMALL C., LE H.: The statistical analysis of dynamic curves and sections. *Pattern Recognition 35* (2002), 1587–1609. 1

[VCC04] VEERARAGHAVAN A., CHOWDHURY A. R., CHELLAPPA R.: Role of shape and kinematics in human movement analysis. *IEEE Computer Society Conference on Computer Vision and Pattern Recognition* (2004). 1

[VRCC05] VEERARAGHAVAN A., ROY-CHOWDHURY A. K., CHELLAPPA R.: matching shape sequences in video with applications in human movement analysis. *IEEE trans. Pattern analysis and machine intelligence 27* (2005). 1