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# Supplementary Material: An Interactive Approach for Identifying Structure Definitions

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**Abstract**

*In this supplementary material, we present an application of the proposed approach to finding a structure definition for center lines of tropical cyclones. The definition builds on the vortex core regions. Here, we analyze two methods for core line detection, i.e., extremum and centroid methods. We were able to find an appropriate computational procedure as well as reasonable parameter ranges that lead to reduced variability of the derived center lines.*

## S1. Example applications

### S1.1. Core line of a Tropical Cyclone (TC)

Structure definitions often build on each other. The second example demonstrates this situation. A TC core line is a structure that describes position and course of a TC. It is located in or close to the center of the TC vortex and represents another mental concept that is not explicitly given by data fields. Representation of the TC domain in cylindrical coordinates that are centered at the vortex core line is very useful for studying TC structure and dynamics (see [PMOK12, DPK*21]). Quantitative results, however, highly depend on a TC core line.

#### S1.1.1. Core line computation

Here, we extract TC core lines based on the TC vortex core regions that were discussed in the main part of the paper. In addition to the indicator quantities, the mathematical definition of the center should be considered. We focus on two methods for core line computation. Both methods compute one point per horizontal slice.

**Extremum method:** This method identifies the center of a vortex as the extremum of an indicator quantity [SWH05]. Usually, the maximum is used; in some cases, e.g., for pressure fields, the minimum value may be used.

**Centroid method:** This method depends on a search radius $R$ [NMT14]. The location of the minimum pressure on a constant height surface was used as first guess, and the centroid was calculated within a circle of radius $R$ around the first guess:

$$\bar{x}(R) = \frac{\sum\limits_{r<R} x_i \omega_i}{\sum\limits_{r<R} \omega_i} \quad \text{and} \quad \bar{y}(R) = \frac{\sum\limits_{r<R} y_i \omega_i}{\sum\limits_{r<R} \omega_i}, \tag{1}$$

where $x_i$, $y_i$ are grid points within a circle of radius $R$ and $\omega_i$ is the value of indicator quantity at this point. In order to determine the center based on the quantity that should be minimized, inverted values could be used, e.g., pressure deficit $p' = \bar{p} - p$. Potential

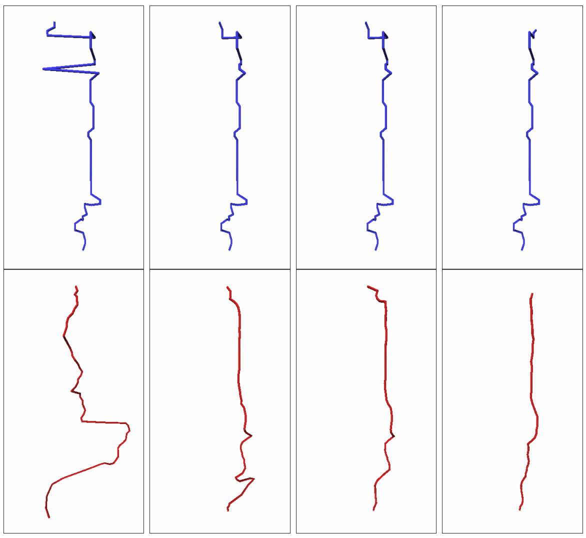

**Figure S1:** *Lines extracted by maximum (blue lines, top row) and centroid method (red lines, bottom row) using - from left to right - $\Omega > 0.5$, $\Omega > 0.65$, $\Omega > 0.7$ and $\Omega > 0.8$.*

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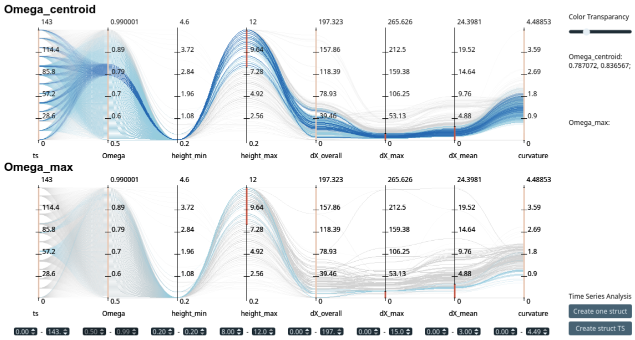


**Figure S2:** *Comparing two TC core line extraction methods using parallel coordinates plot.*

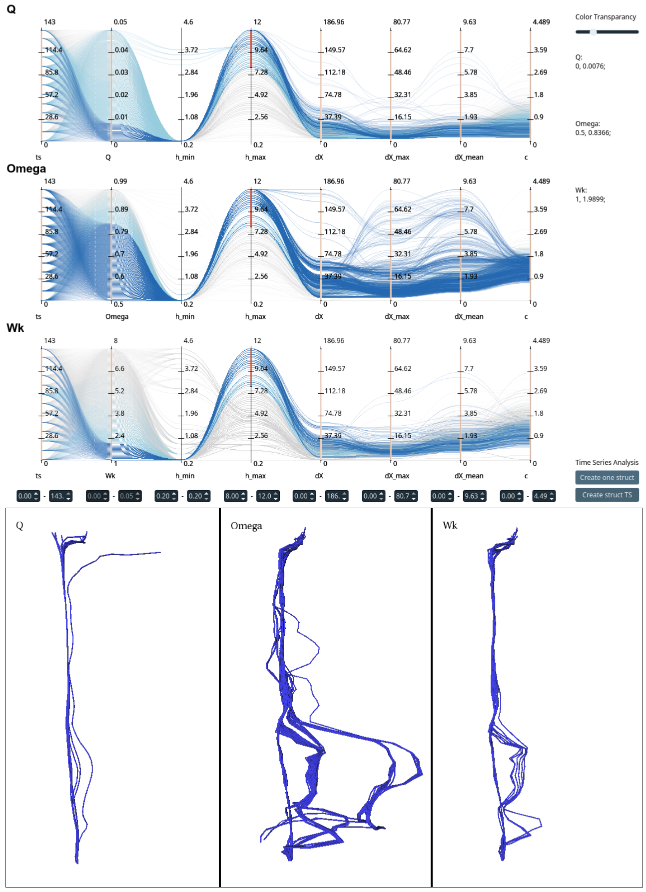


**Figure S3:** *Parallel coordinates window and core lines identified by $Q$, $\Omega$ and $W_k$ using only height restrictions.*

vorticity (PV) was proposed [NMT14] as indicator $\omega$ considering only points within the circle with positive PV. We combine this with the extracted structures, i.e., instead of determining the first guess center and using various radii $R$, we apply Eq. 1 on the TC vortex core regions extracted using indicator quantities $Q$, $\Omega$, and $W_k$.

### S1.1.2. Geometrical attributes for a line

With the same motivation as in Sect. 4.1.2 in the main text, we determined the following attributes for line-like features to help identify structures matching our mental image of a core line:

| | |
|---|---|
| $h_{min}$ | minimum height |
| $h_{max}$ | maximum height |
| $dX$ | overall horizontal displacement |
| $\overline{dX}$ | mean of horizontal displacement |
| $\max(dX)$ | maximum horizontal displacement |
| $c$ | line curvature [Leg99] |

*Minimum* and *maximum height* are the smallest and largest $z$-coordinates over all line points. These attributes correspond to $h_{min}$ and $h_{max}$ defined for the TC vortex core region. Further properties, such as *horizontal displacement* and *line curvature*, describe the smoothness of a line. *Horizontal displacement* is computed as Euclidean distance between two 2D points $X_{z_i}$ and $X_{z_j}$, i.e.,

$$dX_{ij} = d(X_{z_i}, X_{z_j}),$$

where $z_i$ and $z_j$ are the corresponding horizontal levels. *Overall horizontal displacement* is measured between the start and end points of the line. *Mean* and *maximum horizontal displacement* are the average and maximum values of $dX$, respectively, calculated for every two consecutive levels. Finally, *line curvature* is defined as

$$c(A,B,C) = 4\frac{\text{Area of the triangle}(A,B,C)}{d(A,B)d(B,C)d(C,A)} = 2\frac{sin(\angle ABC)}{d(C,A)},$$

where $A$, $B$ and $C$ are points that represent two consecutive line sections and $d(\cdot,\cdot)$ is the Euclidean distance in $\mathbb{R}^3$ [Leg99].

### S1.1.3. Analysis and results

First, we compare the two line extraction methods, that is, the extremum with the centroid method. Fig. S1 shows some examples of the extracted structures using indicator $\Omega$. The lines extracted using the extremum method show less variation; however, most of them are of zigzag form. At the same time, the centroid method provides smoother lines the shape of which can drastically change along with the parameters, as can be seen in Fig. S1, bottom row, left two images. The visualization of this example suggests that the structures extracted with the centroid method are better than those extracted with the extremum method. Incorporating parameter space exploration with structure visualization can help objectify this observations. Interestingly, no common interval can be found for the extremum method for the same parameters that derive reasonable TC centers as with using the centroid method, see Fig. S1. Lines extracted for this parameter setting can be found in Fig. S5. To depict multiple lines at once, we used the easiest but at the same time most informative technique where we display the lines together without further adjustments. The main issue with this visualization occurs if lines perfectly overlap. Therefore, this approach leads to an indistinct representation of structures found by the extremum method, while it worked well for the lines defined by the centroid method.

Next, we focus on the centroid method and utilize the PC tool to investigate the parameter space for three selected indicator quantities. First, we applied the height restrictions $h_{min} = 0.2\,\text{km}$ and

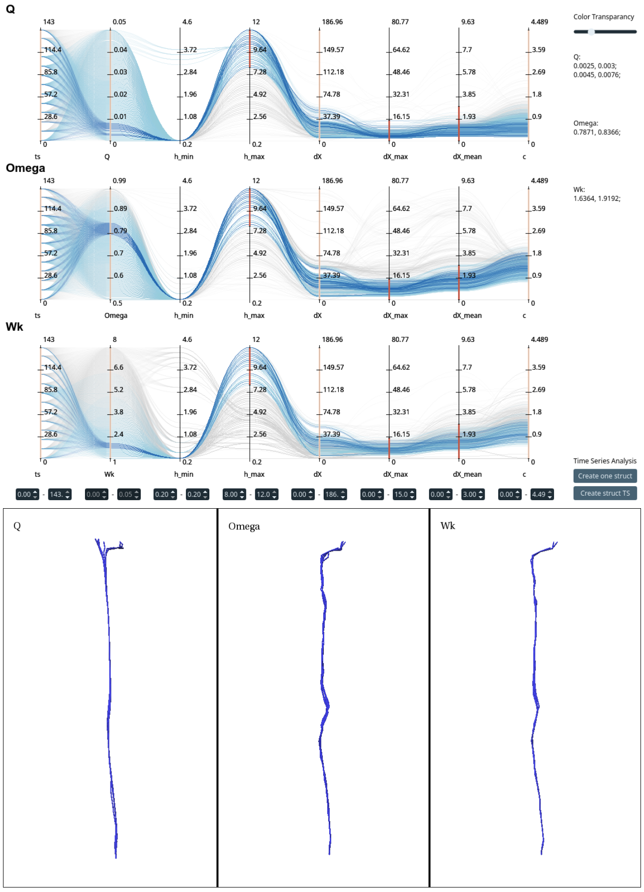


**Figure S4:** *Parallel coordinates window and core lines extracted by selecting proper parameter values for three indicator quantities $Q$, $\Omega$ and $W_k$.*

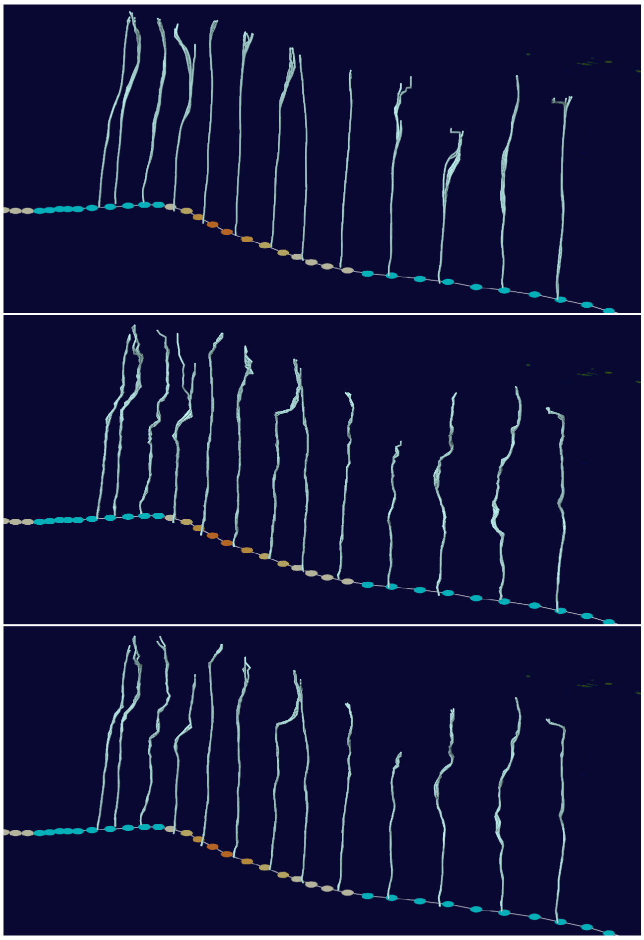

**Figure S5:** *Core lines detected using the centroid method for $Q$ (top), $\Omega$ (middle), and $W_k$ (bottom), showing 9, 11, 5 lines, respectively. Selected time frame: 02-07 September.*

$h_{max} > 8\,\text{km}$ to ensure that derived lines have a desired height. Extracted structures still have a high variation (especially the ones found with $\Omega$), see Fig. S3. Taking horizontal displacement parameters into consideration (see Fig. S4), i.e., $\overline{dX} < 3\,\text{km}$ and $\max dX < 15\,\text{km}$, leads to intervals listed in the following table.

| indicator | core region | center line |
|---|---|---|
| $Q$ | [0.002,0.0076] | [0.0025,0.03] ∪ [0.0045,0.0076] |
| $\Omega$ | [0.7574,0.8316] | [0.7871,0.8366] |
| $W_k$ | [1.4949,1.9899] | [1.6364,1.9192] |

Identified intervals for TC vortex core region and TC center line overlap but do not perfectly match. Overall the intervals derived for center line definition are more strongly restricted.

Finally, extracted lines for the selected time frame are shown in Fig. S5. For each time step, 9, 11, and 5 lines represent the selected intervals corresponding to $Q$, $\Omega$, and $W_k$, respectively.

As a result we find that possible structure definitions of center lines are given per horizontal slice by $x_\omega = (\bar{x}_\omega, \bar{y}_\omega)$ with

$$\bar{x}_\omega = \frac{\sum_{i \in C_\omega} x_i \omega_i}{\sum_{i \in C_\omega} \omega_i} \quad \text{and} \quad \bar{y}_\omega = \frac{\sum_{i \in C_\omega} y_i \omega_i}{\sum_{i \in C_\omega} \omega_i},$$

where $i$ are the indices of grid points $(x_i, y_i)$ within one of the volumes

$$C_Q = \{x \in \mathbb{R}^3 \,|\, Q(x) > \tau_Q; \quad \tau_Q \in [0.0025, 0.03] \cup [0.0045, 0.0076]\},$$
$$C_\Omega = \{x \in \mathbb{R}^3 \,|\, \Omega(x) > \tau_\Omega; \quad \tau_\Omega \in [0.7871, 0.8366]\},$$
$$C_{W_k} = \{x \in \mathbb{R}^3 \,|\, W_k(x) > \tau_{W_k}; \tau_{W_k} \in [1.6364, 1.9192]\}.$$

The core lines extracted on the basis of $Q$ are generally somewhat smoother than those resulting from $\Omega$ and $W_k$.

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