Interactive Editing of Discrete Chebyshev Nets - Supplements

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1. Proofs of propositions

1.1. Proof of proposition 1

Proof

$$\begin{aligned}(L_\mathbf{e}/L)^p + (L/L_\mathbf{e})^p - 2 &= \frac{((L_\mathbf{e}/L)^p-1)^2}{(L_\mathbf{e}/L)^p} \\ &= \frac{(L_\mathbf{e}/L-1)^2((L_\mathbf{e}/L)^{p-1}+(L_\mathbf{e}/L)^{p-2}+\cdots+1)^2}{(L_\mathbf{e}/L)^p},\end{aligned} \tag{1}$$

$$\begin{aligned}(L_\mathbf{e}/L + L/L_\mathbf{e} - 2)^p &= \frac{(L_\mathbf{e}/L-1)^{2p}}{(L_\mathbf{e}/L)^p} \\ &= \frac{(L_\mathbf{e}/L-1)^2(L_\mathbf{e}/L-1)^{2p-2}}{(L_\mathbf{e}/L)^p},\end{aligned} \tag{2}$$

We consider $f_1(x) = (x^{p-1}+x^{p-2}+\cdots+x+1)^2$, $f_2(x) = (x-1)^{2p-2}$. When $x \geq 1$, $f_1(x) > (x^{p-1})^2 > (x-1)^{2p-2}$. When $0 < x < 1$, $f_1(x) > 1 > (1-x)^{2p-2}$. Hence we get $f_1(x) > f_2(x)$ when $x > 0$. Then $(L_\mathbf{e}/L)^p + (L/L_\mathbf{e})^p - 2 \geq (L_\mathbf{e}/L + L/L_\mathbf{e} - 2)^p$. And the equality holds when $L_\mathbf{e}/L = 1$. □

1.2. Proof of proposition 2

Proof First, we have

$$\begin{aligned}\nabla^2 \hat{f} = &\frac{\partial \bar{\mathbf{g}}}{\partial \mathbf{x}}^T \nabla^2 h_{\text{inc}} \frac{\partial \bar{\mathbf{g}}}{\partial \mathbf{x}} + \sum_i \frac{\partial h_{\text{inc}}}{\partial u_i} \nabla^2 g_i^+ \\ &+ \frac{\partial \underline{\mathbf{g}}}{\partial \mathbf{x}}^T \nabla^2 h_{\text{dec}} \frac{\partial \underline{\mathbf{g}}}{\partial \mathbf{x}} + \sum_i \frac{\partial h_{\text{dec}}}{\partial v_i} \nabla^2 g_i^- \geq 0,\end{aligned} \tag{3}$$

where $u_i = \bar{g}_i(\mathbf{x})$ and $v_i = \underline{g}_i(\mathbf{x})$, $\forall i$. Hence $\hat{f}$ is convex.
Next, since $g_i(\mathbf{x}_0) = \bar{g}_i(\mathbf{x}_0;\mathbf{x}_0) = \underline{g}_i(\mathbf{x}_0;\mathbf{x}_0)$, $\nabla g_i(\mathbf{x}_0) = \nabla \bar{g}_i(\mathbf{x}_0;\mathbf{x}_0) = \nabla \underline{g}_i(\mathbf{x}_0;\mathbf{x}_0)$, we get

$$\begin{aligned}f(\mathbf{x}_0) &= \hat{f}(\mathbf{x}_0;\mathbf{x}_0), \\ \nabla f(\mathbf{x}_0) &= \nabla \hat{f}(\mathbf{x}_0;\mathbf{x}_0).\end{aligned}$$

Finally, since $\underline{g}_i(\mathbf{x};\mathbf{x}_0) \leq g_i(\mathbf{x}) \leq \bar{g}_i(\mathbf{x};\mathbf{x}_0)$, we have

$$\hat{f}(\mathbf{x};\mathbf{x}_0) \geq h_{\text{inc}}(\mathbf{g}(\mathbf{x})) + h_{\text{dec}}(\mathbf{g}(\mathbf{x})) = f(\mathbf{x}).$$

Therefore, $\hat{f}$ is a convex majorizer of f at $\mathbf{x}_0$. □

1.3. Proof of proposition 3

Proof Since

$$\begin{aligned}\tilde{f}(\mathbf{x};\mathbf{x}_0) &= (h_{\text{inc}} + h_{\text{dec}})([\mathbf{g}](\mathbf{x};\mathbf{x}_0);\mathbf{u}_0) \\ &= h_{\text{inc}}([\mathbf{g}](\mathbf{x};\mathbf{x}_0)) + h_{\text{dec}}([\mathbf{g}](\mathbf{x};\mathbf{x}_0)).\end{aligned}$$

and

$$\underline{g}_i(\mathbf{x};\mathbf{x}_0) \leq [g_i](\mathbf{x};\mathbf{x}_0) \leq \bar{g}_i(\mathbf{x};\mathbf{x}_0),$$

$$h_{\text{inc}}([\mathbf{g}](\mathbf{x};\mathbf{x}_0) \leq h_{\text{inc}}(\bar{\mathbf{g}}(\mathbf{x};\mathbf{x}_0)),\ h_{\text{dec}}([\mathbf{g}](\mathbf{x};\mathbf{x}_0) \leq h_{\text{dec}}(\underline{\mathbf{g}}(\mathbf{x};\mathbf{x}_0)).$$

Then $\tilde{f}(\mathbf{x};\mathbf{x}_0) \leq \hat{f}(\mathbf{x};\mathbf{x}_0)$.
Since $\nabla g_i(\mathbf{x}_0) = \nabla \bar{g}_i(\mathbf{x}_0;\mathbf{x}_0) = \nabla \underline{g}_i(\mathbf{x}_0;\mathbf{x}_0)$,

$$\frac{\partial h}{\partial u_i} \leq \frac{\partial h_{\text{inc}}}{\partial u_i},\ \frac{\partial h_{\text{dec}}}{\partial u_i} \leq \frac{\partial h}{\partial u_i},$$

we have $\nabla^2 \tilde{f}(\mathbf{x}_0;\mathbf{x}_0) \leq \nabla^2 \hat{f}(\mathbf{x}_0;\mathbf{x}_0)$. □

2. More comparisons

2.1. More comparisons with [SPSH*17]

We provide more comparisons with [SPSH*17] in Table 1. We only show the numbers of iterations until $dL_{\max} < 10^{-6}$ because the time cost of each iteration of [SPSH*17] is similar to that of ours, and we only want to show the differences between the solvers. Then we compare two methods more precisely by considering the adaptive p strategy. We choose the small $p = 2$ and the large $p = 50$. The first one is using [SPSH*17]'s solver for $p = 2$ and $p = 50$, which is called adaptive [SPSH*17]. The second one is using our solver for $p = 2$ and $p = 50$. From this table we find that our solver is more efficient than [SPSH*17]'s. Besides, the result meshes of [SPSH*17]'s solver are noisy as shown in Figure 8. We also provide three resulting meshes and graphs (Figure 1, Figure 2, Figure 3).

2.2. More parameters testing

We test three parameters (i.e., ε_{dL1}, ε_{dL2}, and ε_4) in our workflow with more options. The results are in Table 2. For most parameters, the time costs are similar.

2.3. More comparisons

We provide more comparisons with [SA07], [BDS*12], D_{p4}, D_{p8} and [SPSH*17] on 15 models. The results are in Table 3. We show $dL_{\max}$ and corresponding time cost in this table. The results are similar to what we show in article.

Model		#iter	
Name	Vertices	Adaptive [SPSH*17]	Ours
Figure 19 Armadillo	7192	24	18
Figure 11 Bunny	9062	24	19
Figure 7 Bird	1307	15	10
Figure 14 Chair	13456	45	42
Figure 5 Dear	1222	25	16
Figure 12 Elephant	6055	37	21
Figure 10 Human	17699	36	32
Figure 9 Kitten	10108	41	36
Figure 17 Santa	1488	20	16
Figure 17 Teddy	6777	23	16

Table 1: *More comparisons with [SPSH*17]. We show 10 models which have been shown in article, and consider the adaptive p strategy. The deformations are the same as above. We choose the termination threshold $\varepsilon_3 = 10^{-6}$, and show the corresponding numbers of iterations in the table.*

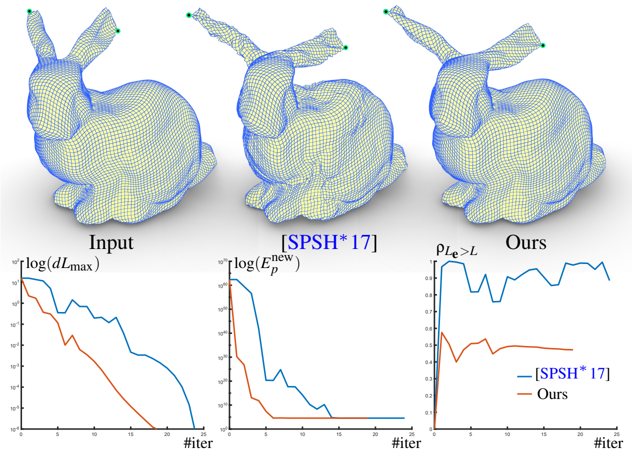

Figure 1: *Comparison with [SPSH*17] on a Bunny model.*

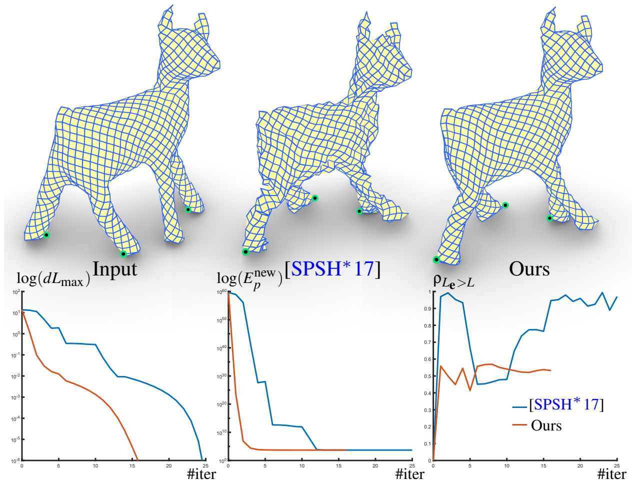

Figure 2: *Comparison with [SPSH*17] on a Dear model.*

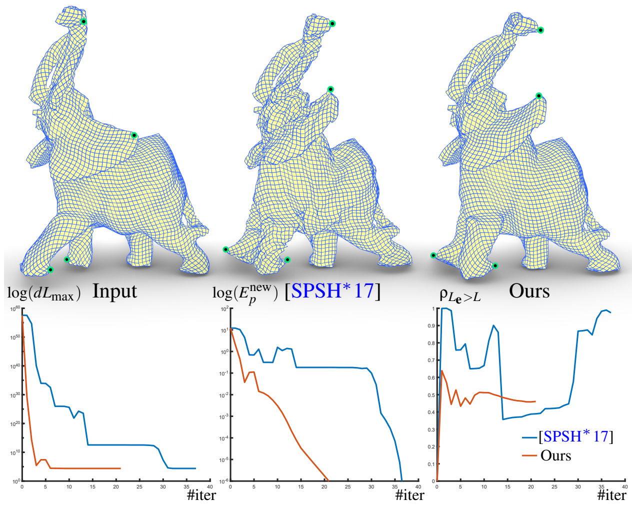

Figure 3: *Comparison with [SPSH*17] on an Elephant model.*

ε_{dL1}		ε_{dL2}		ε_4	
Value	Time(s)	Value	Time(s)	Value	Time(s)
0.05	0.303	5×10^{-4}	0.205	1×10^{-6}	0.199
0.04	0.331	4×10^{-4}	0.203	9×10^{-7}	0.203
0.03	0.304	3×10^{-4}	0.198	8×10^{-7}	0.193
0.02	0.198	2×10^{-4}	0.198	7×10^{-7}	0.194
0.01	0.197	1×10^{-4}	0.198	6×10^{-7}	0.190
0.009	0.196	9×10^{-5}	0.197	5×10^{-7}	0.190
0.008	0.197	8×10^{-5}	0.197	4×10^{-7}	0.193
0.007	0.195	7×10^{-5}	0.194	3×10^{-7}	0.190
0.006	0.197	6×10^{-5}	0.195	2×10^{-7}	0.193
0.005	0.196	5×10^{-5}	0.194	1×10^{-7}	0.203

Table 2: *More comparisons using different parameters for Figure 17 Teddy. We run our workflow until $dL_{\max} < 10^{-6}$ and show the time cost.*

References

[BDS*12] BOUAZIZ S., DEUSS M., SCHWARTZBURG Y., WEISE T., PAULY M.: Shape-up: Shaping discrete geometry with projections. *Comput. Graph. Forum 31*, 5 (2012), 1657–1667. 2, 3

[SA07] SORKINE O., ALEXA M.: As-rigid-as-possible surface modeling. In *Symposium on Geometry processing* (2007), vol. 4, pp. 109–116. 2, 3

[SPSH*17] SHTENGEL A., PORANNE R., SORKINE-HORNUNG O., KOVALSKY S. Z., LIPMAN Y.: Geometric optimization via composite majorization. *ACM Transactions on Graphics (TOG) 36*, 4 (2017), 1–11. 1, 2, 3

Model		[SA07]		[BDS* 12]		D_{p4}		D_{p8}		[SPSH* 17]		Ours	
Name	Vertices	dL_{max}	Time(s)	dL_{max}	Time(s)	dL_{max}	Time(s)	dL_{max}	Time(s)	dL_{max}	Time(s)	dL_{max}	Time(s)
Figure 8 Rabbit	4011	0.074	1.049	9.41×10^{-5}	0.277	8.48×10^{-4}	0.817	0.034	0.831	6.80×10^{-8}	0.227	8.39×10^{-7}	0.103
Figure 19 Shirt	8592	5.01×10^{-3}	2.221	2.17×10^{-5}	0.524	0.020	1.568	0.277	1.563	4.76×10^{-7}	0.566	9.91×10^{-7}	0.437
Figure 10 Human	17699	0.147	4.783	4.38×10^{-4}	1.198	9.70×10^{-3}	4.650	0.055	4.672	9.21×10^{-8}	1.501	9.82×10^{-7}	0.903
Figure 19 Horse	11153	0.339	3.057	4.34×10^{-3}	0.671	0.066	2.696	0.212	2.692	5.26×10^{-8}	1.193	9.52×10^{-7}	1.012
Figure 14 Chair	13456	0.270	3.528	1.04×10^{-3}	0.838	0.016	2.990	0.049	2.960	8.96×10^{-10}	1.058	9.79×10^{-7}	0.847
Figure 11 Bunny	9062	0.063	2.448	3.98×10^{-4}	0.565	6.92×10^{-7}	1.375	1.03×10^{-5}	2.288	6.78×10^{-9}	0.335	7.40×10^{-7}	0.323
Figure 19 Armadillo	7192	0.022	1.904	3.49×10^{-4}	0.455	9.97×10^{-7}	0.679	3.17×10^{-4}	1.430	6.89×10^{-9}	0.340	8.84×10^{-7}	0.208
Figure 5 Dear	1222	0.043	0.413	3.74×10^{-5}	0.167	8.56×10^{-7}	0.051	1.61×10^{-4}	0.205	1.27×10^{-8}	0.043	6.84×10^{-7}	0.031
Figure 2 Dragon	3462	0.132	0.948	1.29×10^{-3}	0.236	5.76×10^{-7}	0.402	3.14×10^{-4}	0.603	4.00×10^{-8}	0.206	8.69×10^{-7}	0.155
Figure 16 Sparse panda	972	0.096	0.323	1.05×10^{-3}	0.140	2.52×10^{-3}	0.186	0.015	0.186	1.63×10^{-9}	0.034	5.47×10^{-7}	0.037
Figure 16 Middle panda	3895	0.156	1.052	5.96×10^{-4}	0.267	7.16×10^{-4}	0.825	0.022	0.828	9.92×10^{-8}	0.118	9.61×10^{-7}	0.113
Figure 16 Dense panda	15477	0.268	4.742	8.27×10^{-4}	0.984	2.17×10^{-3}	3.846	0.014	3.849	5.00×10^{-7}	1.082	9.07×10^{-7}	0.718
Figure 9 Kitten	10108	0.303	2.677	1.64×10^{-3}	0.651	8.84×10^{-7}	2.197	1.54×10^{-3}	2.617	1.64×10^{-7}	0.882	9.19×10^{-7}	0.712
Figure 19 Botijo	9224	0.186	2.641	7.66×10^{-4}	0.596	8.89×10^{-7}	1.561	4.36×10^{-4}	2.163	5.17×10^{-7}	0.514	6.41×10^{-7}	0.338
Figure 7 Bird	1307	0.011	0.400	8.25×10^{-6}	0.180	8.32×10^{-7}	0.065	2.10×10^{-4}	0.222	5.17×10^{-8}	0.039	5.58×10^{-7}	0.026

Table 3: *More comparisons with [SA07], [BDS* 12], D_{p4}, D_{p8} and [SPSH* 17] on 15 models we have used in this article. These methods stop either when $dL_{max} < 10^{-6}$ or when they converge.*